

**Quick Quiz 4.1** Suppose you are standing directly behind someone who steps back and accidentally stomps on your foot with the heel of one shoe. Would you be better off if that person were (a) a large professional basketball player wearing sneakers (b) a petite woman wearing spike-heeled shoes?

**Quick Quiz 4.2** A grandfather clock depends on the period of a pendulum to keep correct time. Suppose a grandfather clock is calibrated correctly and then a mischievous child slides the bob of the pendulum downward on the oscillating rod. Does the grandfather clock run (a) slow (b) fast (c) correctly?

The background features several large, overlapping, colorful swirls in shades of purple, green, and blue. Scattered throughout are numerous small, yellow, triangular shapes that resemble sunbeams or confetti.

# **Course of lectures «Contemporary Physics: Part1»**

*Lecture №8*

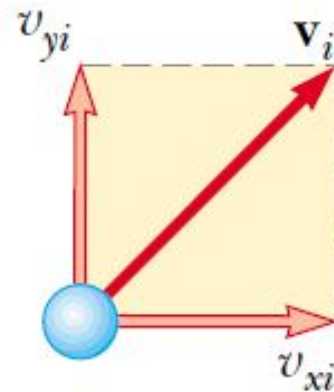
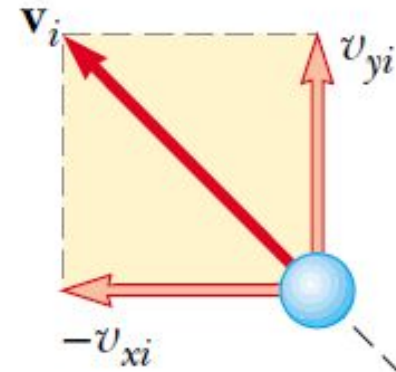
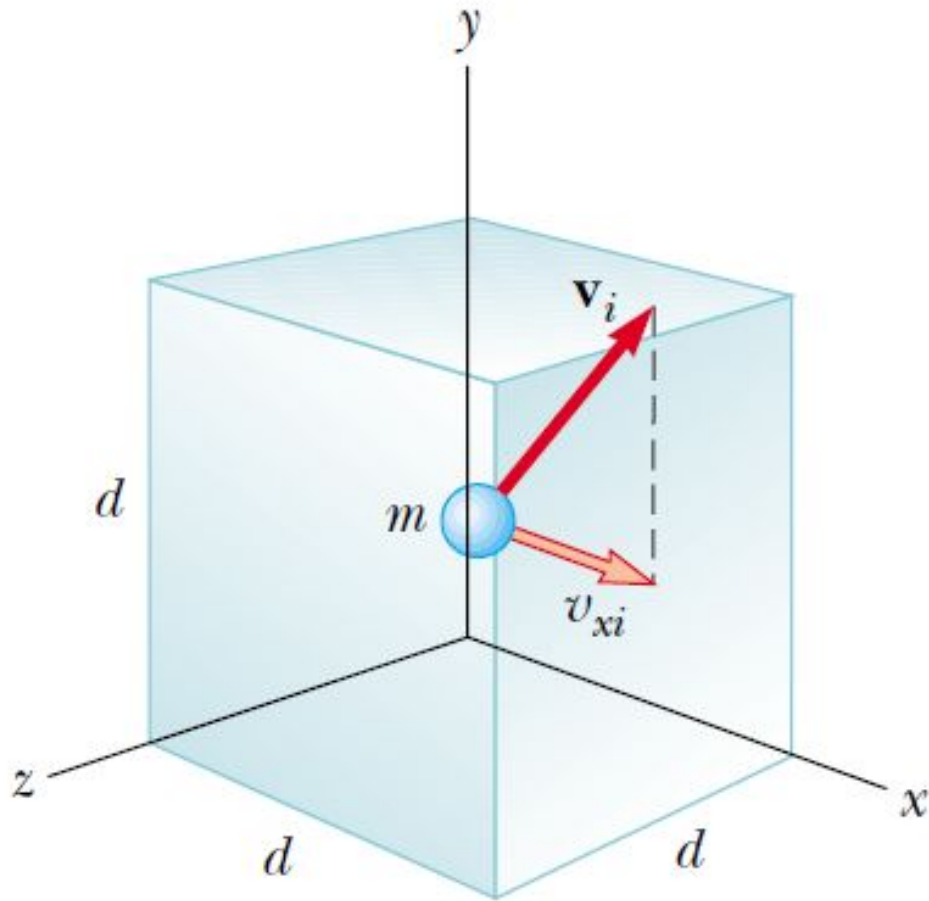
**The Kinetic Theory of Gases.  
Heat Engines, Entropy, and the Second  
Law of Thermodynamics.**

# Molecular Model of an Ideal Gas

The macroscopic description of model of ideal gas make the following assumptions:

1. **The number of molecules in the gas is large, and the average separation between them is large compared with their dimensions.**
2. **The molecules obey Newton's laws of motion, but as a whole they move randomly.**
3. **The molecules interact only by short-range forces during elastic collisions.**
4. **The molecules make elastic collisions with the walls.**
5. **The gas under consideration is a pure substance; that is, all molecules are identical.**

# Molecular Model of an Ideal Gas



# Molecular Model of an Ideal Gas

$$\Delta p_{xi} = -mv_{xi} - (mv_{xi}) = -2mv_{xi}$$

$$\bar{F}_{i, \text{ on molecule}} \Delta t_{\text{collision}} = \Delta p_{xi} = -2mv_{xi}$$

$$\Delta t = \frac{2d}{v_{xi}} \quad \bar{F}_i \Delta t = -2mv_{xi}$$

$$\bar{F}_i = \frac{-2mv_{xi}}{\Delta t} = \frac{-2mv_{xi}^2}{2d} = \frac{-mv_{xi}^2}{d}$$

$$\bar{F}_{i, \text{ on wall}} = -\bar{F}_i = -\left(\frac{-mv_{xi}^2}{d}\right) = \frac{mv_{xi}^2}{d}$$

# Molecular Model of an Ideal Gas

$$\bar{F} = \sum_{i=1}^N \frac{m v_{xi}^2}{d} = \frac{m}{d} \sum_{i=1}^N v_{xi}^2$$

$$F = \frac{m}{d} \sum_{i=1}^N v_{xi}^2 \quad \overline{v_x^2} = \frac{\sum_{i=1}^N v_{xi}^2}{N}$$

$$F = \frac{m}{d} N \overline{v_x^2} \quad (10.1)$$

# Molecular Model of an Ideal Gas

$$v_i^2 = v_{xi}^2 + v_{yi}^2 + v_{zi}^2$$

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$$

$$\overline{v^2} = 3\overline{v_x^2}$$

$$F = \frac{N}{3} \left( \frac{\overline{mv^2}}{d} \right)$$

# Molecular Model of an Ideal Gas

$$P = \frac{F}{A} = \frac{F}{d^2} = \frac{1}{3} \left( \frac{N}{d^3} \overline{mv^2} \right) = \frac{1}{3} \left( \frac{N}{V} \right) \overline{mv^2}$$

$$P = \frac{2}{3} \left( \frac{N}{V} \right) \left( \frac{1}{2} \overline{mv^2} \right) \quad (10.2)$$

The pressure of a gas is proportional to the number of molecules per unit volume and to the average translational kinetic energy of the molecules,  $\frac{1}{2} \overline{mv^2}$ .



# Molecular Interpretation of Temperature

$$PV = \frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right)$$

$$PV = Nk_B T$$

$$T = \frac{2}{3k_B} \left(\frac{1}{2}m\overline{v^2}\right) \quad (10.3)$$

This result tells us that temperature is a direct measure of average molecular kinetic energy.

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_B T \quad (10.4)$$

$$\frac{1}{2}m\overline{v_x^2} = \frac{1}{2}k_B T \quad (10.5)$$

# Molecular Interpretation of Temperature

$$\frac{1}{2}m\overline{v_y^2} = \frac{1}{2}k_B T$$

$$\frac{1}{2}m\overline{v_z^2} = \frac{1}{2}k_B T$$

## Theorem of equipartition of energy

each degree of freedom contributes  $\frac{1}{2}k_B T$  to the energy of a system, where possible degrees of freedom in addition to those associated with translation arise from rotation and vibration of molecules.

$$K_{\text{tot trans}} = N\left(\frac{1}{2}m\overline{v^2}\right) = \frac{3}{2}Nk_B T = \frac{3}{2}nRT \quad (10.6)$$

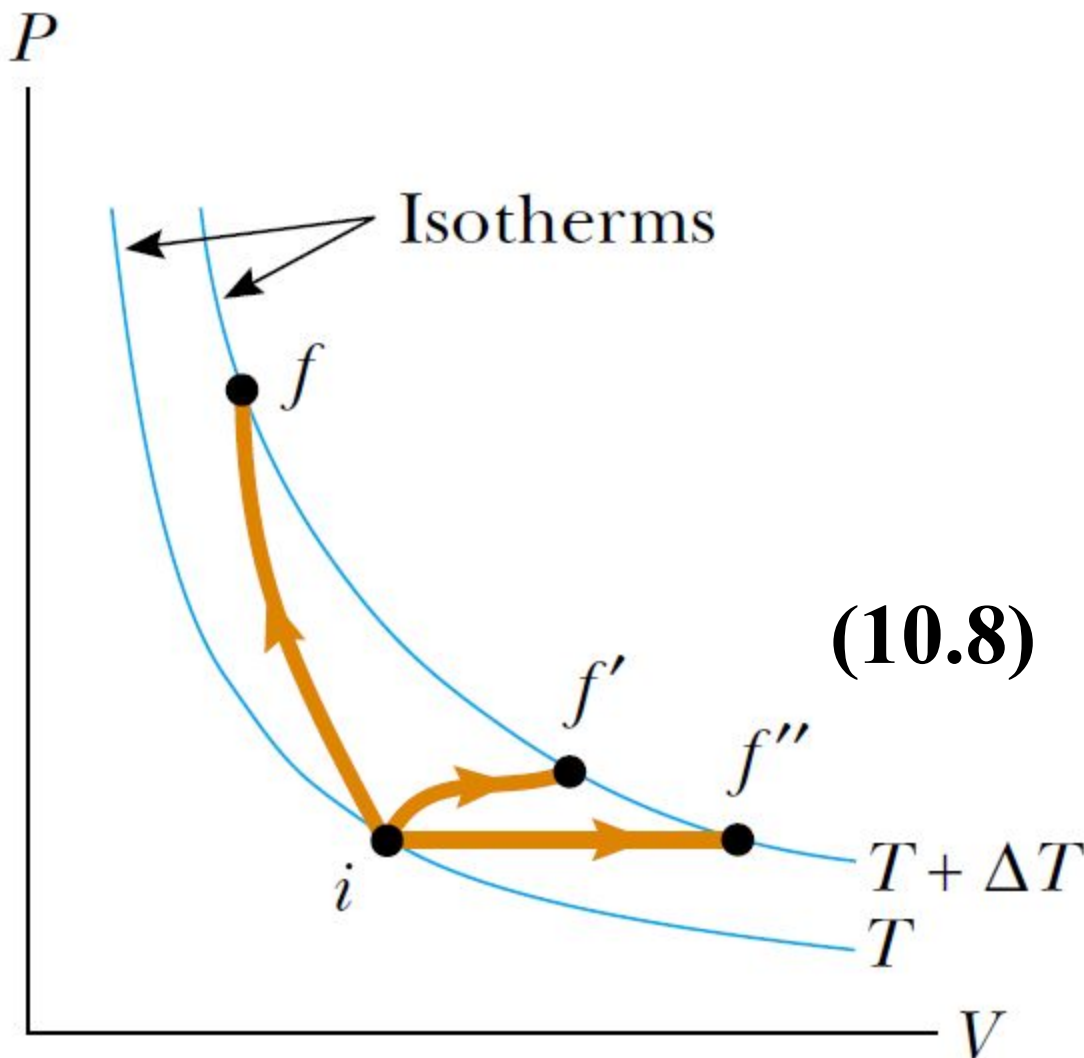
$$v_{\text{rms}} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3RT}{M}} \quad (10.7)$$

# Molecular Interpretation of Temperature

**Table 10.1**

Some rms Speeds		
Gas	Molar mass (g/mol)	$v_{\text{rms}}$ at 20°C (m/s)
H <sub>2</sub>	2.02	1 902
He	4.00	1 352
H <sub>2</sub> O	18.0	637
Ne	20.2	602
N <sub>2</sub> or CO	28.0	511
NO	30.0	494
O <sub>2</sub>	32.0	478
CO <sub>2</sub>	44.0	408
SO <sub>2</sub>	64.1	338

# Molar Specific Heat of an Ideal Gas



$$\Delta T = T_f - T_i$$

$$Q = \Delta E_{\text{int}} - W,$$

molar specific heats:

(10.8)

$$Q = nC_V \Delta T$$

$$Q = nC_P \Delta T$$

(10.9)

$$E_{\text{int}} = K_{\text{tot trans}} = \frac{3}{2} N k_B T = \frac{3}{2} n R T \quad (10.10)$$

# Molar Specific Heat of an Ideal Gas

If energy is transferred by heat to a system at *constant volume*, then no work is done on the system. From the first law of thermodynamics, we see that

$$Q = \Delta E_{\text{int}} \quad (10.11)$$

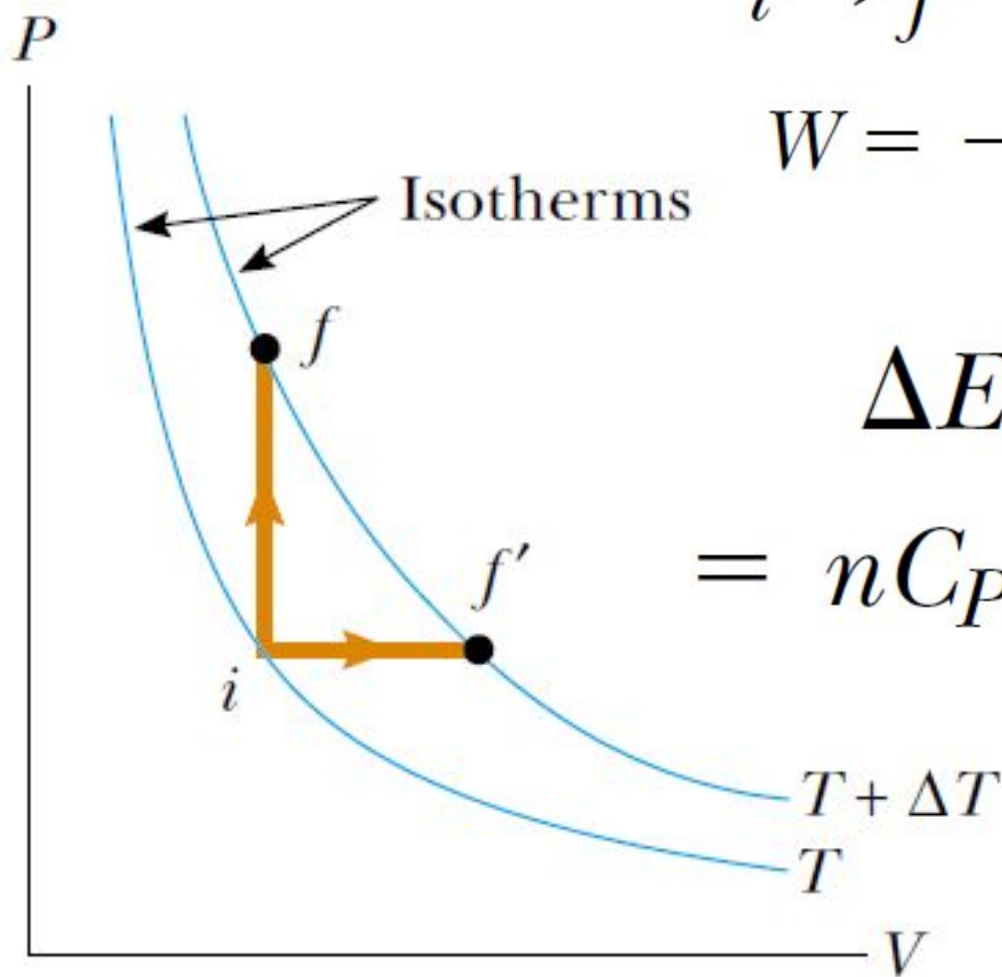
$$\Delta E_{\text{int}} = nC_V \Delta T \quad (10.12)$$

$$E_{\text{int}} = nC_V T$$

$$(10.13) \quad C_V = \frac{1}{n} \frac{dE_{\text{int}}}{dT}$$

$$C_V = \frac{3}{2}R \quad (10.14)$$

# Molar Specific Heat of an Ideal Gas



$$i \rightarrow f' \quad Q = nC_p \Delta T.$$

$$W = -P \Delta V$$

$$\begin{aligned} \Delta E_{\text{int}} &= Q + W = \\ &= nC_p \Delta T + (-P \Delta V) \end{aligned}$$

**(10.15)**

# Molar Specific Heat of an Ideal Gas

$$nC_V \Delta T = nC_P \Delta T - nR \Delta T$$

$$C_P - C_V = R \quad (10.16)$$

$$\gamma = \frac{C_P}{C_V} = \frac{5R/2}{3R/2} = \frac{5}{3} = 1.67 \quad (10.17)$$

# Adiabatic Processes for an Ideal Gas

An **adiabatic process** is one in which no energy is transferred by heat between a system and its surroundings.

$$PV^\gamma = \text{constant} \quad (10.18)$$

**Home work:**

**Proof That  $PV^\gamma = \text{Constant}$  for an Adiabatic Process**



# The Boltzmann Distribution Law

$$v_{\text{rms}} = \sqrt{v^2} = \sqrt{\frac{3k_{\text{B}}T}{m}} = \sqrt{\frac{3RT}{M}} \quad (10.7)$$

The number density  $n_V(E)$

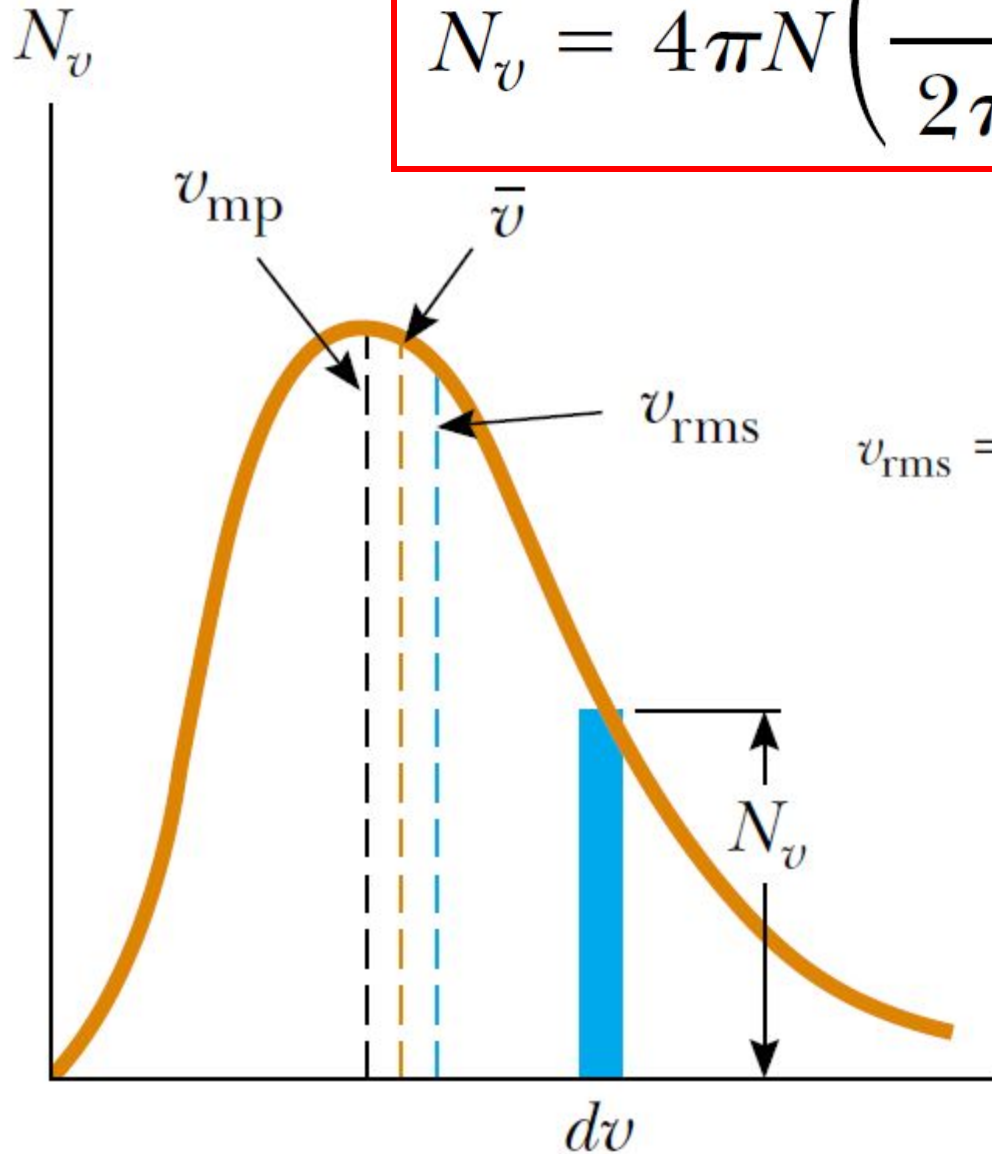
$$n_V(E) dE \quad E \text{ and } E + dE.$$

$$n_V(E) = n_0 e^{-E/k_{\text{B}}T} \quad (10.19)$$

The **Boltzmann distribution law**, is important in describing the statistical mechanics of a large number of molecules. It states that the **probability of finding the molecules in a particular energy state varies exponentially as the negative of the energy divided by  $k_{\text{B}}T$ .**

# Distribution of Molecular Speeds

$$N_v = 4\pi N \left( \frac{m}{2\pi k_B T} \right)^{3/2} v^2 e^{-mv^2/2k_B T}$$



**(10.20)**

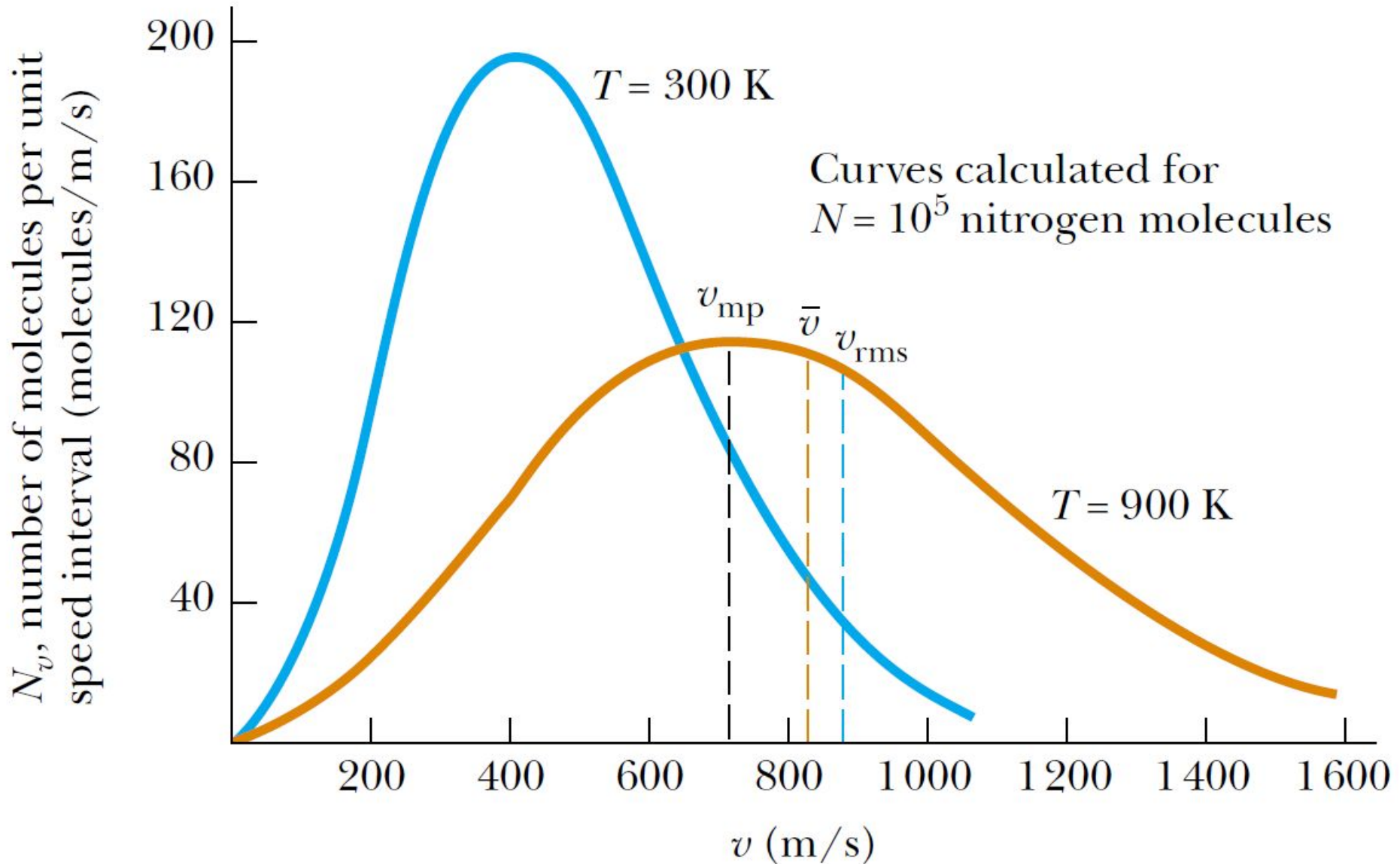
$$v_{\text{rms}} = \sqrt{v^2} = \sqrt{\frac{3k_B T}{m}} = 1.73 \sqrt{\frac{k_B T}{m}}$$

$$\bar{v} = \sqrt{\frac{8k_B T}{\pi m}} = 1.60 \sqrt{\frac{k_B T}{m}}$$

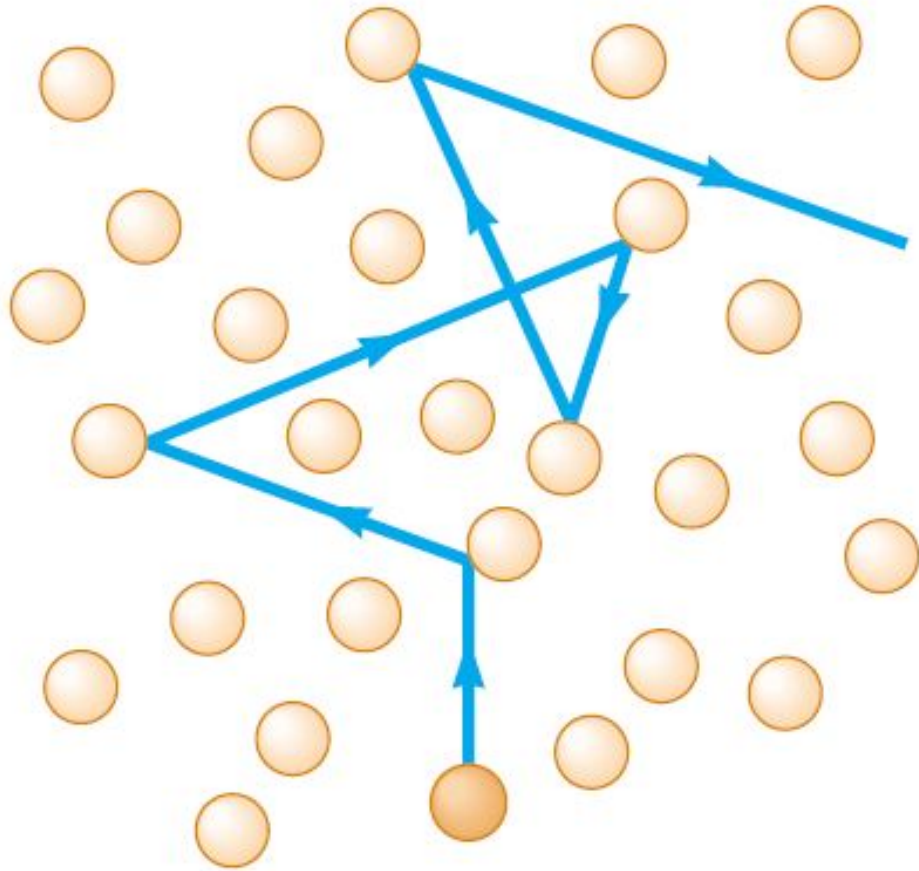
$$v_{\text{mp}} = \sqrt{\frac{2k_B T}{m}} = 1.41 \sqrt{\frac{k_B T}{m}}$$

$$v_{\text{rms}} > \bar{v} > v_{\text{mp}}$$

# Distribution of Molecular Speeds

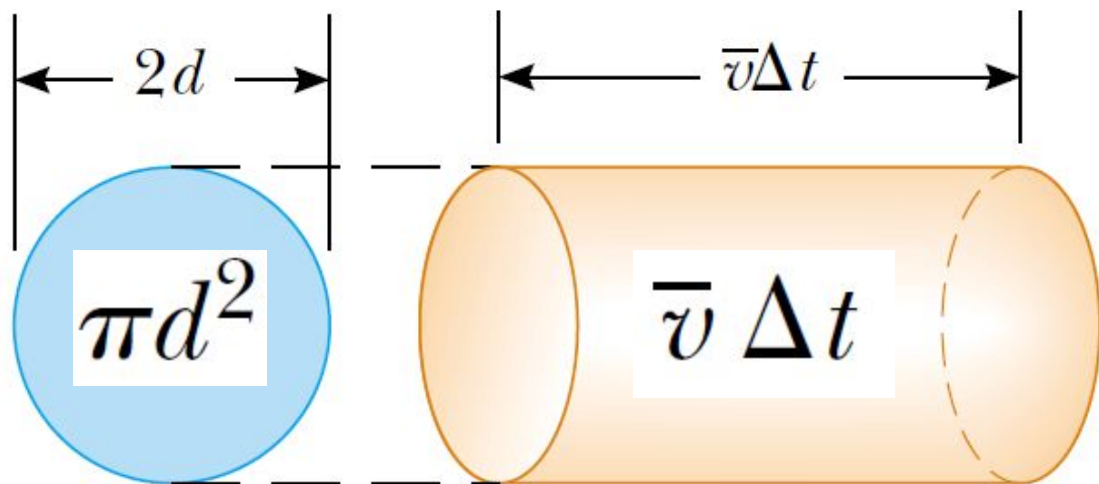
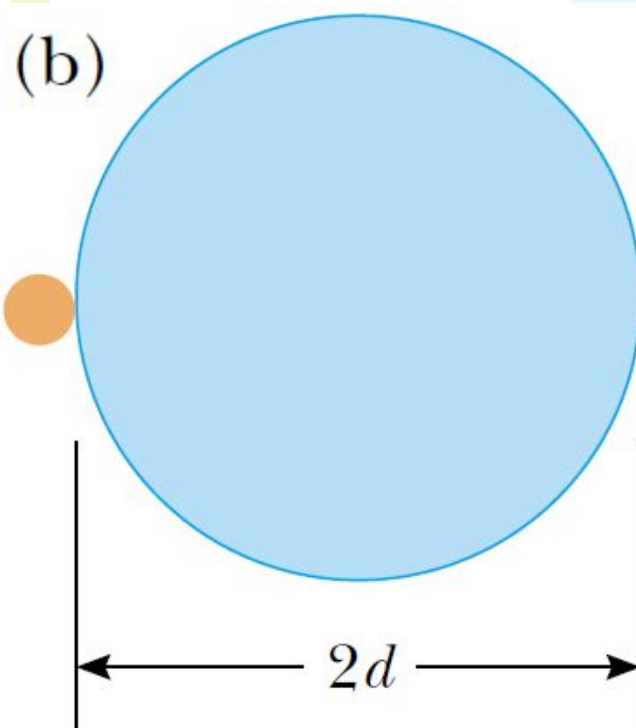
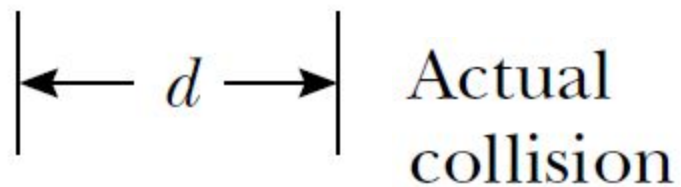
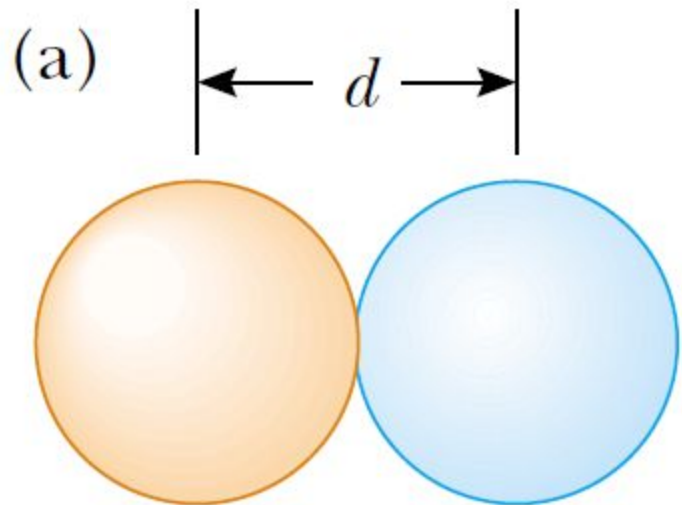


# Mean Free Path



The average distance between collisions is called the **mean free path**.

# Mean Free Path



$$\pi d^2 \bar{v} \Delta t$$

$$(\pi d^2 \bar{v} \Delta t) n_V$$

# Mean Free Path

$$\ell = \frac{\bar{v} \Delta t}{(\pi d^2 \bar{v} \Delta t) n_V} = \frac{1}{\pi d^2 n_V}$$

Collision frequency  $f$ , is

$$f = \pi d^2 \bar{v} n_V$$

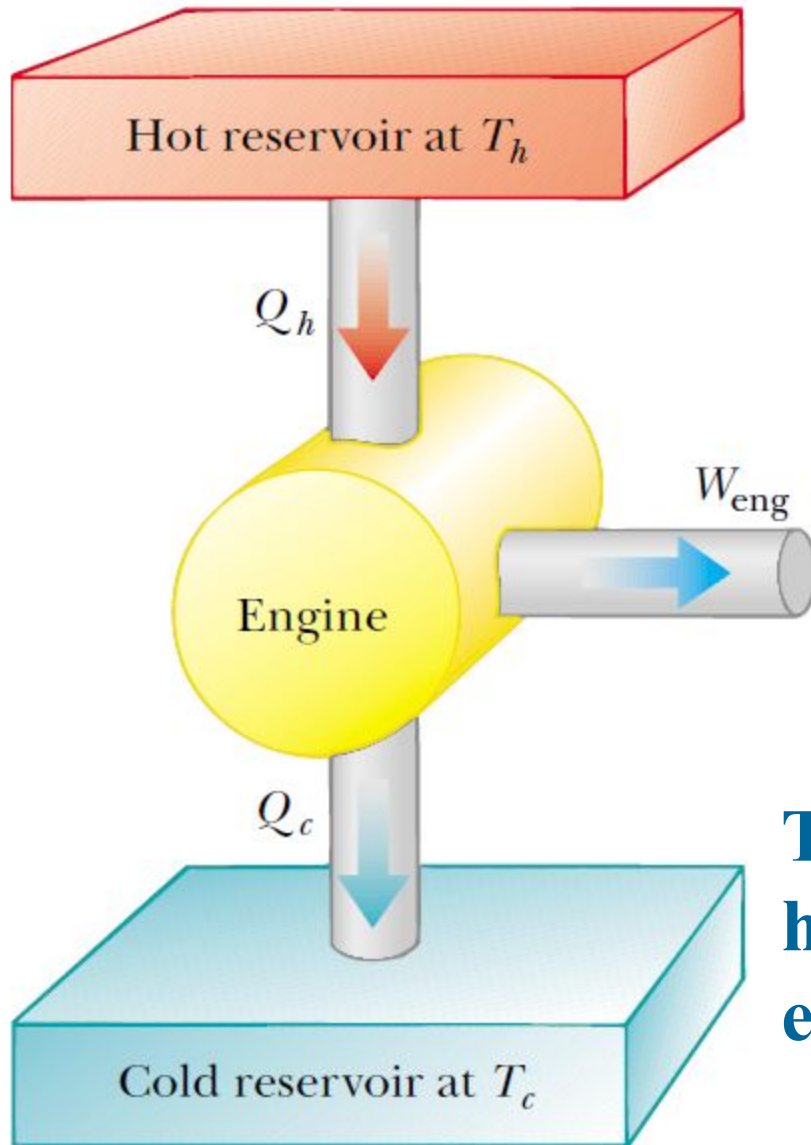
$$\ell = \frac{1}{\sqrt{2} \pi d^2 n_V}$$

**(10.21)**

$$f = \sqrt{2} \pi d^2 \bar{v} n_V = \frac{\bar{v}}{\ell}$$

**(10.22)**

# Heat Engines and the Second Law of Thermodynamics



A **heat engine** is a device that takes in energy by heat and, operating in a cyclic process, expels a fraction of that energy by means of work.

$$\Delta E_{\text{int}} = 0$$

$$\Delta E_{\text{int}} = Q + W = Q - W_{\text{eng}},$$

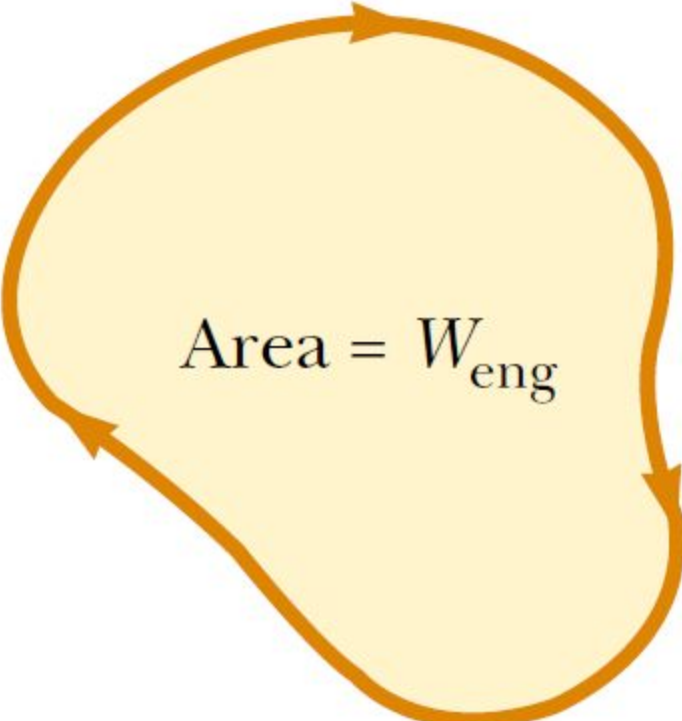
The net work  $W_{\text{eng}}$  done by a heat engine is equal to the net energy  $Q_{\text{net}}$  transferred to it.

# Heat Engines and the Second Law of Thermodynamics

$$Q_{\text{net}} = |Q_h| - |Q_c|; \text{ therefore,}$$

$$W_{\text{eng}} = |Q_h| - |Q_c| \quad (10.23)$$

The net work done in a cyclic process is the area enclosed by the curve representing the process on a  $PV$  diagram.



Area =  $W_{\text{eng}}$

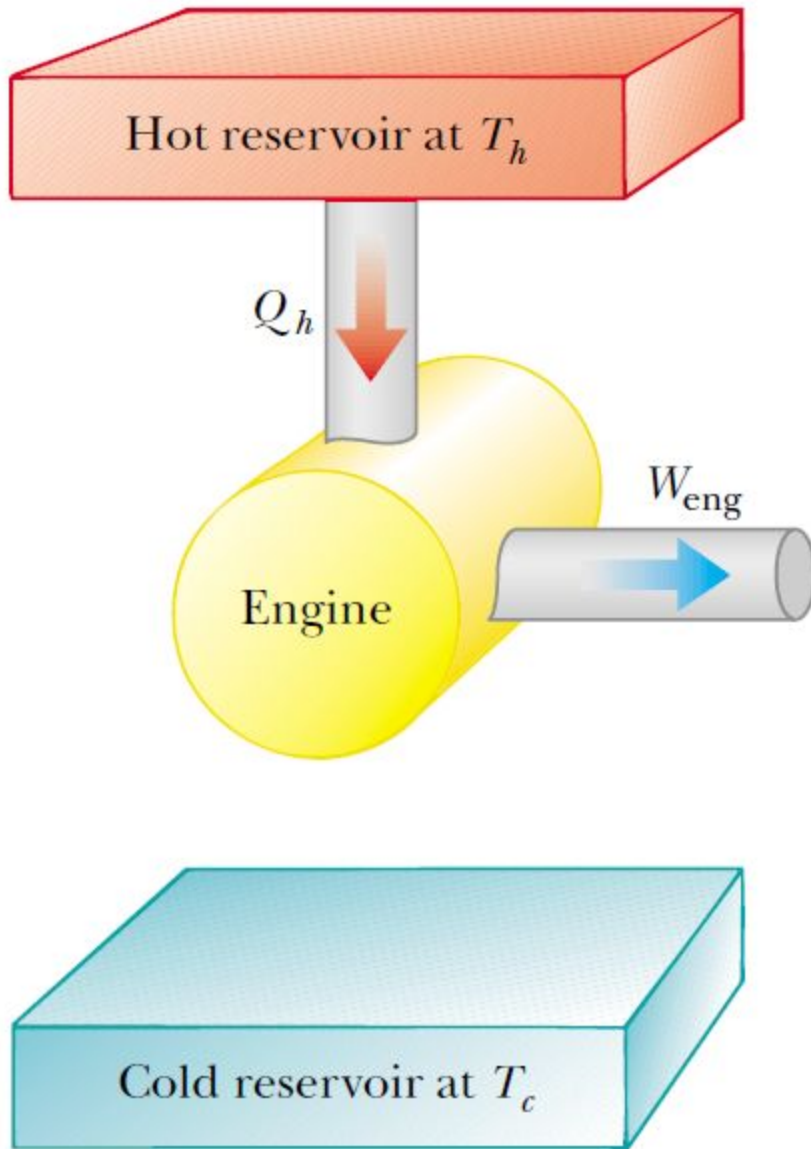
The thermal efficiency

(10.24)

$$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{|Q_h| - |Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|}$$



# Heat Engines and the Second Law of Thermodynamics

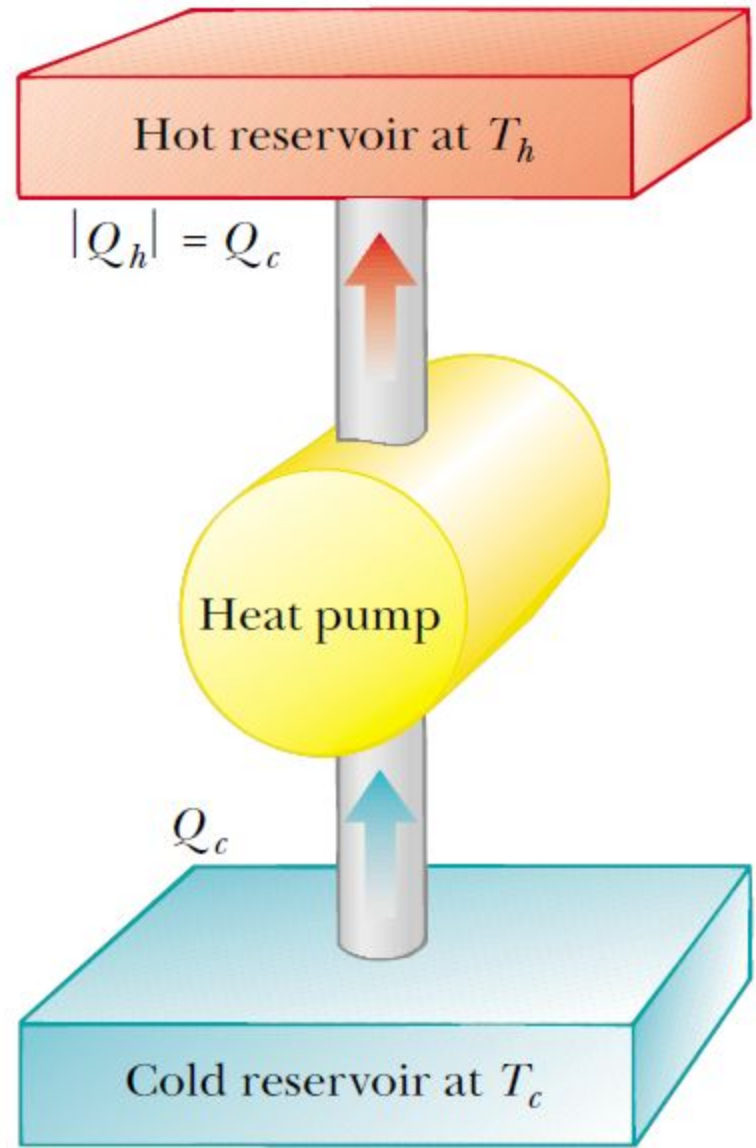
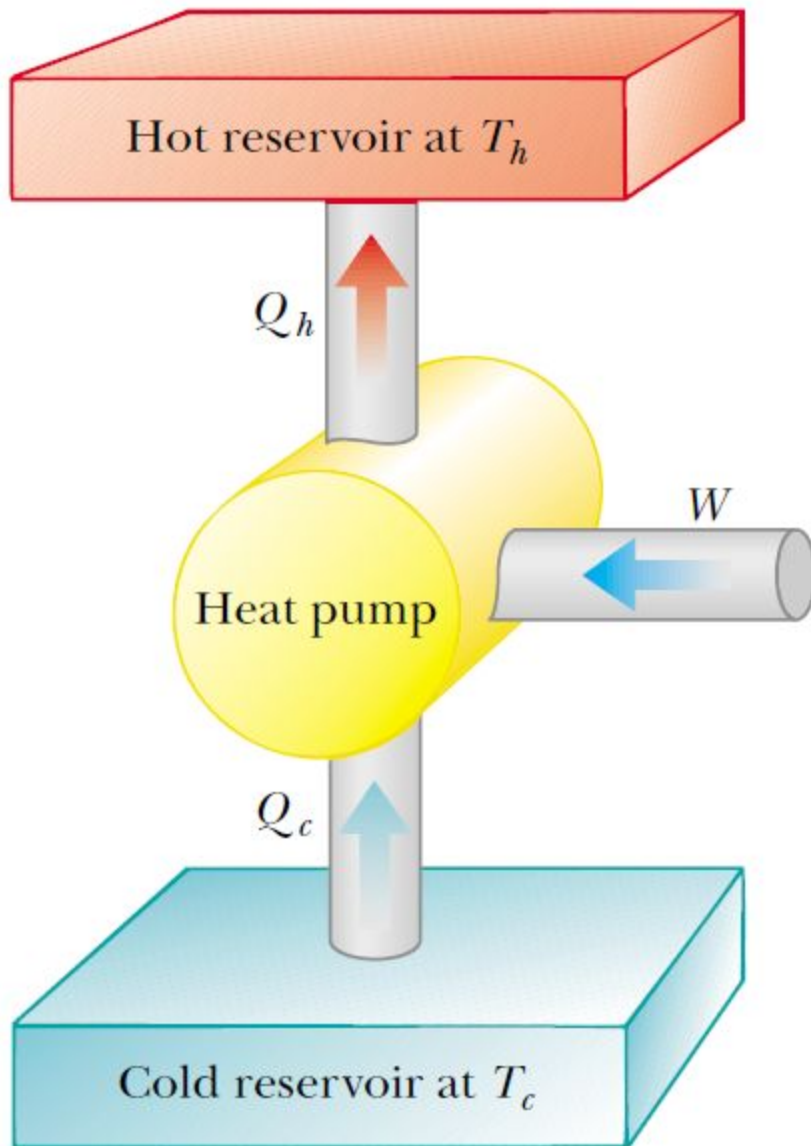


The impossible engine

On the basis of the fact that efficiencies of real engines are well below 100%, the **Kelvin–Planck form of the second law of thermodynamics** states the following:

It is impossible to construct a heat engine that, operating in a cycle, produces no effect other than the input of energy by heat from a reservoir and the performance of an equal amount of work.

# Heat Pumps and Refrigerators



Impossible heat pump

# Heat Pumps and Refrigerators

The **Clausius statement** states:

It is impossible to construct a cyclical machine whose sole effect is to transfer energy continuously by heat from one object to another object at a higher temperature without the input of energy by work.

In simpler terms, **energy does not transfer spontaneously by heat from a cold object to a hot object.**

The effectiveness of a heat pump is described in terms of a number called the **coefficient of performance (COP)**.

$$\text{COP (heating mode)} \equiv \frac{\text{energy transferred at high temperature}}{\text{work done by heat pump}} = \frac{|Q_h|}{W}$$

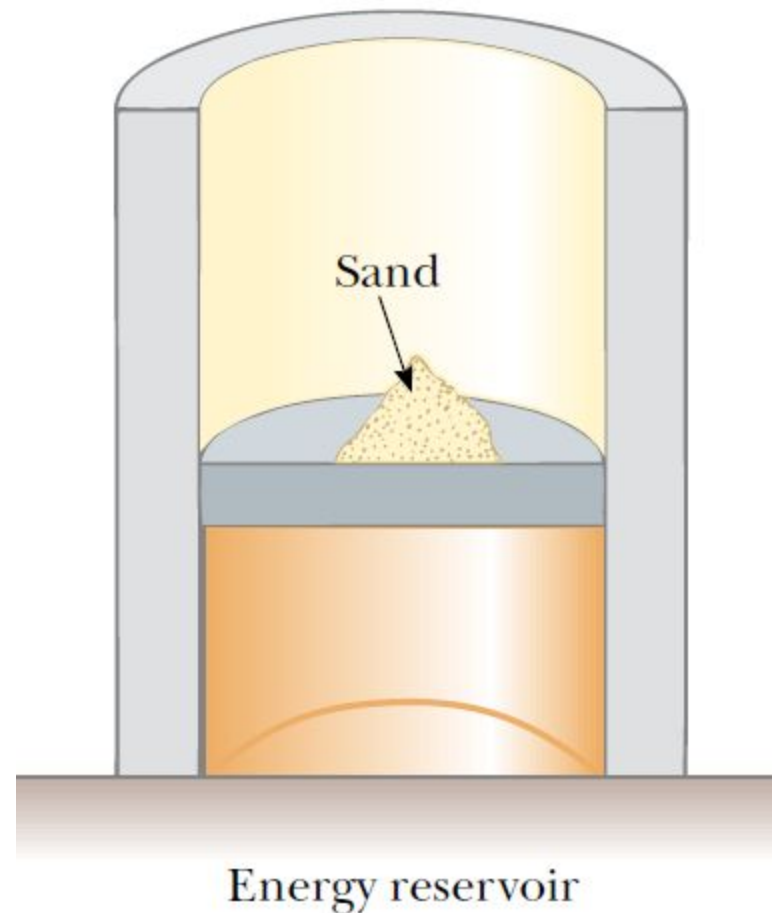
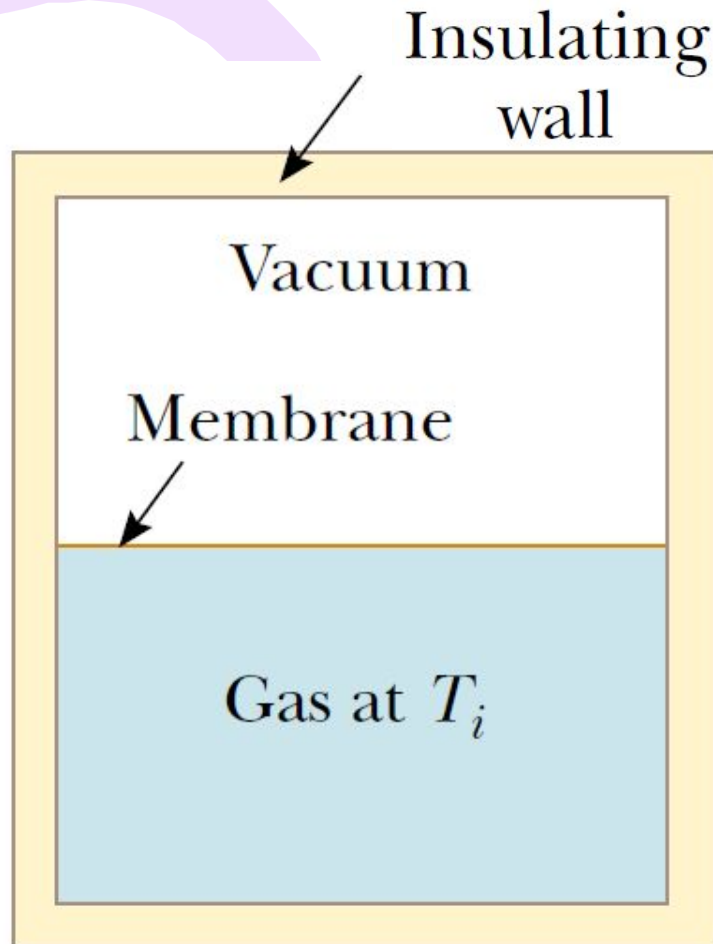
**(10.25)**

$$\text{COP (cooling mode)} = \frac{|Q_c|}{W}$$

**(10.26)**

# Reversible and Irreversible Processes

In a **reversible** process, the system undergoing the process can be returned to its initial conditions along the same path on a  $PV$  diagram, and every point along this path is an equilibrium state. A process that does not satisfy these requirements is **irreversible**.

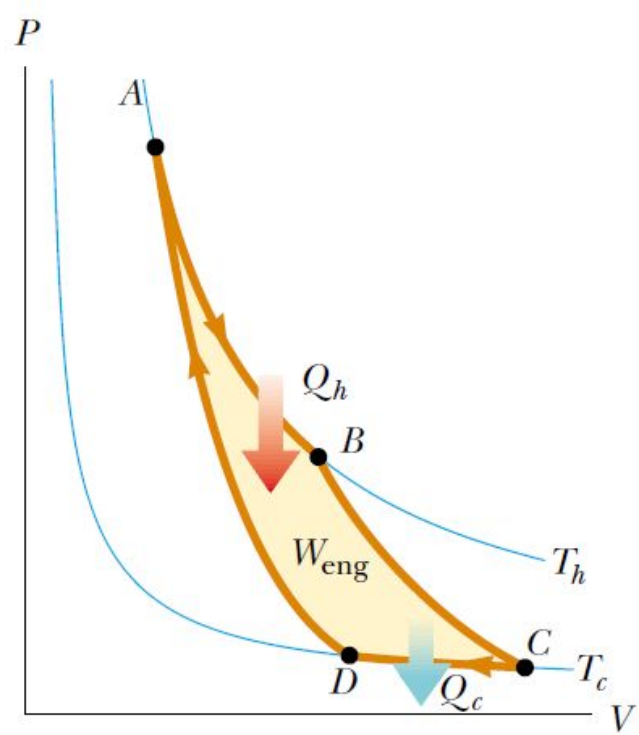
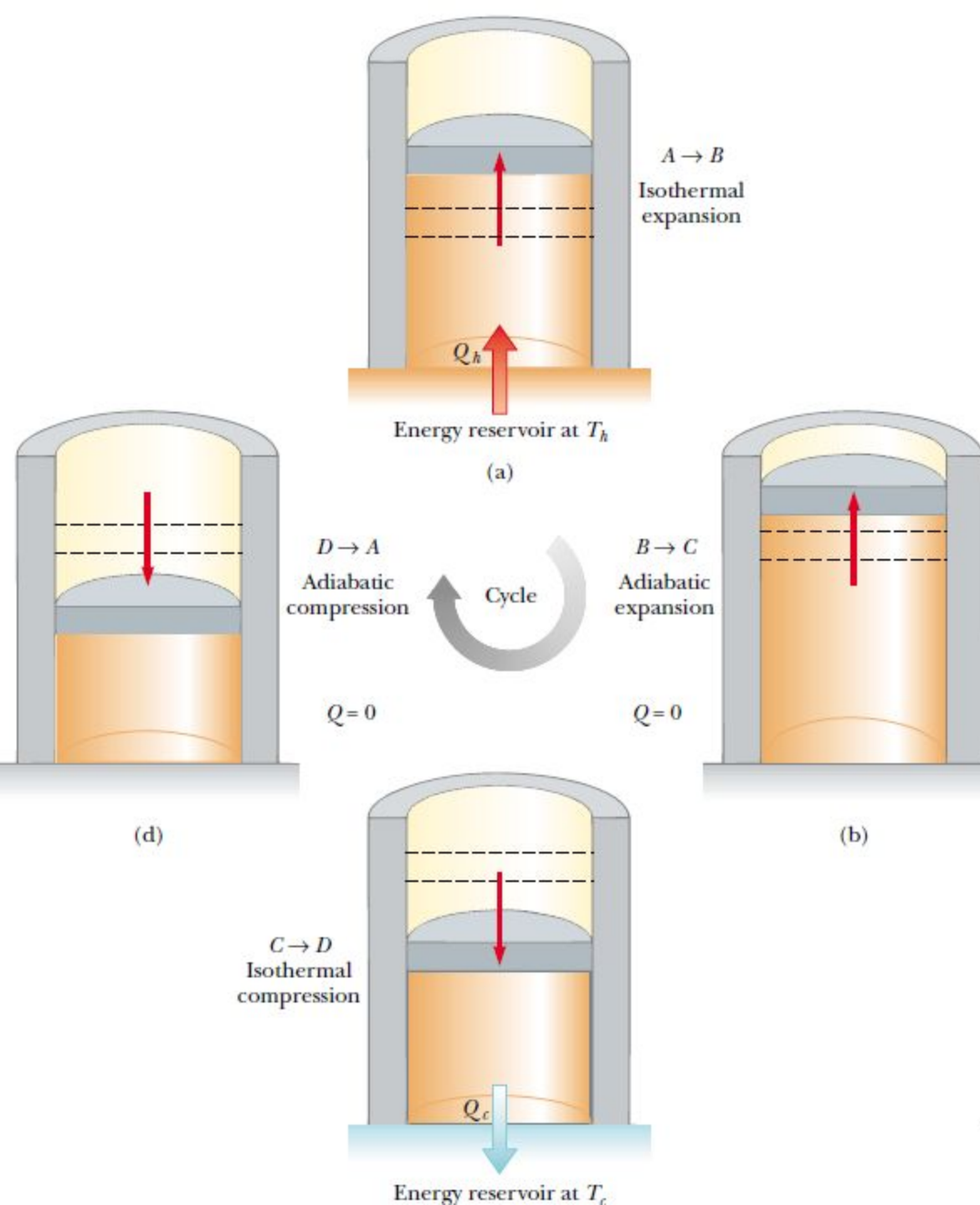


# The Carnot Engine

French engineer named Sadi Carnot showed that a heat engine operating in an ideal, reversible cycle— called a **Carnot cycle**—between two energy reservoirs is the most efficient engine possible.

**Carnot's theorem** can be stated as follows:

No real heat engine operating between two energy reservoirs can be more efficient than a Carnot engine operating between the same two reservoirs.



$$e = \frac{W_{eng}}{|Q_h|} = \frac{|Q_h| - |Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|}$$

$$\frac{|Q_c|}{|Q_h|} = \frac{T_c}{T_h}$$

# The Carnot Engine

Hence, the thermal efficiency of a Carnot engine is

$$e_C = 1 - \frac{T_c}{T_h} \quad (10.27)$$

This result indicates that **all Carnot engines operating between the same two temperatures have the same efficiency.**

# Entropy

**Isolated systems tend toward disorder and that entropy is a measure of this disorder.**

We distinguish between *microstates* and *macrostates* of a system. A **microstate** is a particular configuration of the individual constituents of the system.

**Macrostate** is a description of the conditions of the system from a macroscopic point of view and makes use of macroscopic variables such as pressure, density, and temperature for gases.

Because entropy is a measure of disorder, an alternative way of stating this is **the entropy of the Universe increases in all real processes.**



# Entropy

$$dS = \frac{dQ_r}{T} \quad (10.28)$$

The change in entropy during a process depends only on the end points and therefore is independent of the actual path followed. Consequently, the entropy change for an irreversible process can be determined by calculating the entropy change for a reversible process that connects the same initial and final states.

$$\Delta S = \int_i^f dS = \int_i^f \frac{dQ_r}{T} \quad (10.29)$$

# Entropy

Let us consider the changes in entropy that occur in a Carnot heat engine that operates between the temperatures  $T_c$  and  $T_h$ . In one cycle, the engine takes in energy  $Q_h$  from the hot reservoir and expels energy  $Q_c$  to the cold reservoir.

$$\Delta S = \frac{|Q_h|}{T_h} - \frac{|Q_c|}{T_c}$$

$$\frac{|Q_c|}{|Q_h|} = \frac{T_c}{T_h}$$

$$\Delta S = 0$$

$$\oint \frac{dQ_r}{T} = 0 \quad (10.30)$$

## Quick Quiz 5.3

If you are asked to make a very sensitive glass thermometer, which of the following working liquids would you choose? (a) mercury (b) alcohol (c) gasoline (d) glycerin

## Quick Quiz 5.4

Two spheres are made of the same metal and have the same radius, but one is hollow and the other is solid. The spheres are taken through the same temperature increase. Which sphere expands more? (a) solid sphere (b) hollow sphere (c) They expand by the same amount. (d) not enough information to say