Course of lectures «Contemporary Physics: Part1»

Lecture №9

Fluid Mechanics.

Pressure. Variation of Pressure with Depth. Pressure Measurements. Buoyant Forces and Archimedes's Principle. Fluid Dynamics. Bernoulli's Equation.

Fluid Mechanics

A **fluid** is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by the walls of a container. Both liquids and gases are fluids.

In our treatment of the mechanics of fluids, we do not need to learn any new physical principles to explain such effects as the buoyant force acting on a submerged object and the dynamic lift acting on an airplane wing. First, we consider the mechanics of a fluid at rest—that is, *fluid statics*. We then treat the mechanics of fluids in motion— that is, *fluid dynamics*. We can describe a fluid in motion by using a model that is based upon certain simplifying assumptions.

Pressure

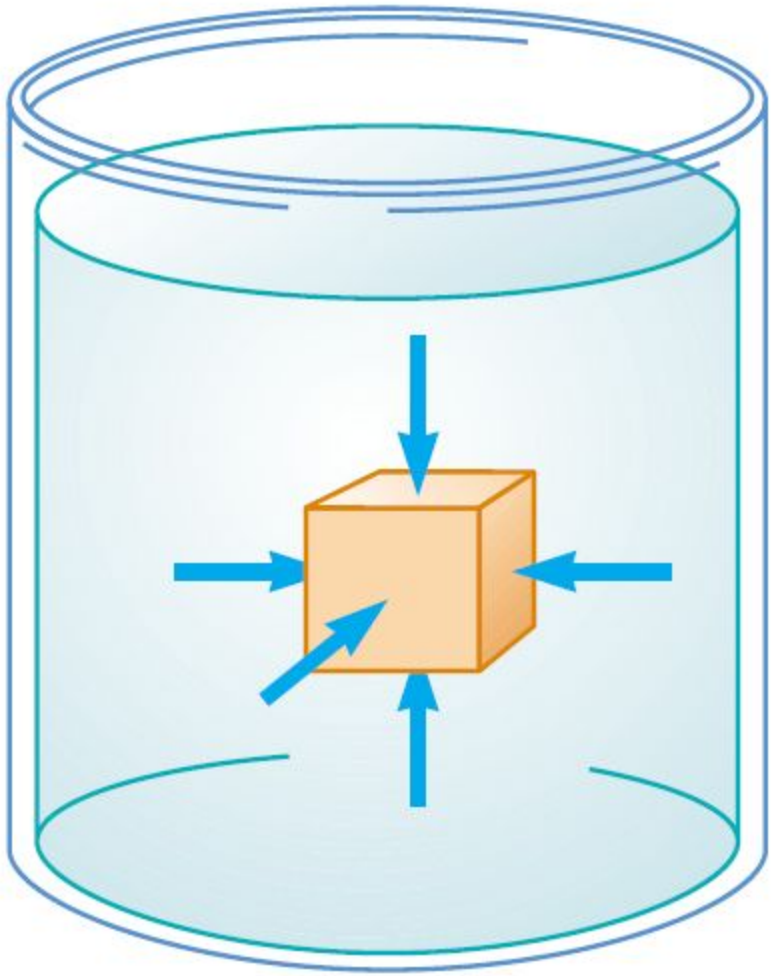


Figure 7.1 At any point on the surface of a submerged object, the force exerted by the fluid is perpendicular to the surface of the object. The force exerted by the fluid on the walls of the container is perpendicular to the walls at all points.

Pressure

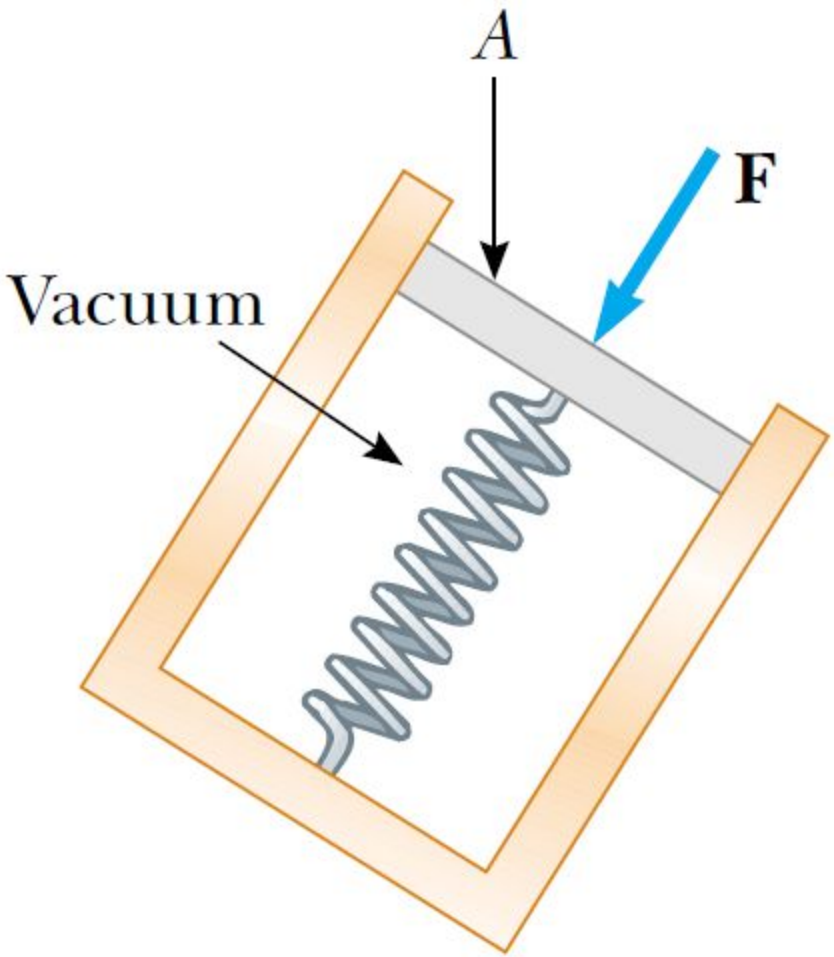


Figure 7.2 A simple device for measuring the pressure exerted by a fluid.

If F is the magnitude of the force exerted on the piston and A is the surface area of the piston, then the **pressure** P of the fluid at the level to which the device has been submerged is defined as the ratio F/A:

$$P \equiv \frac{F}{A} \qquad (7.1)$$

Note that pressure is a scalar quantity because it is proportional to the magnitude of the force on the piston.

If the pressure varies over an area, we can evaluate the infinitesimal force dF on an infinitesimal surface element of area dA as

$$dF = P\,dA \qquad \textbf{(7.2)}$$

where P is the pressure at the location of the area dA. The pressure exerted by a fluid varies with depth. Therefore, to calculate the total force exerted on a flat vertical wall of a container, we must integrate Equation 7.2 over the surface area of the wall.

Because pressure is force per unit area, it has units of newtons per square meter (N/m^2) in the SI system. Another name for the SI unit of pressure is **pascal** (Pa):

$$1\ \text{Pa} \equiv 1\ \text{N/m}^2 \qquad \textbf{(7.3)}$$

Snowshoes keep you from sinking into soft snow because they spread the downward force you exert on the snow over a large area, reducing the pressure on the snow surface.

Variation of Pressure with Depth

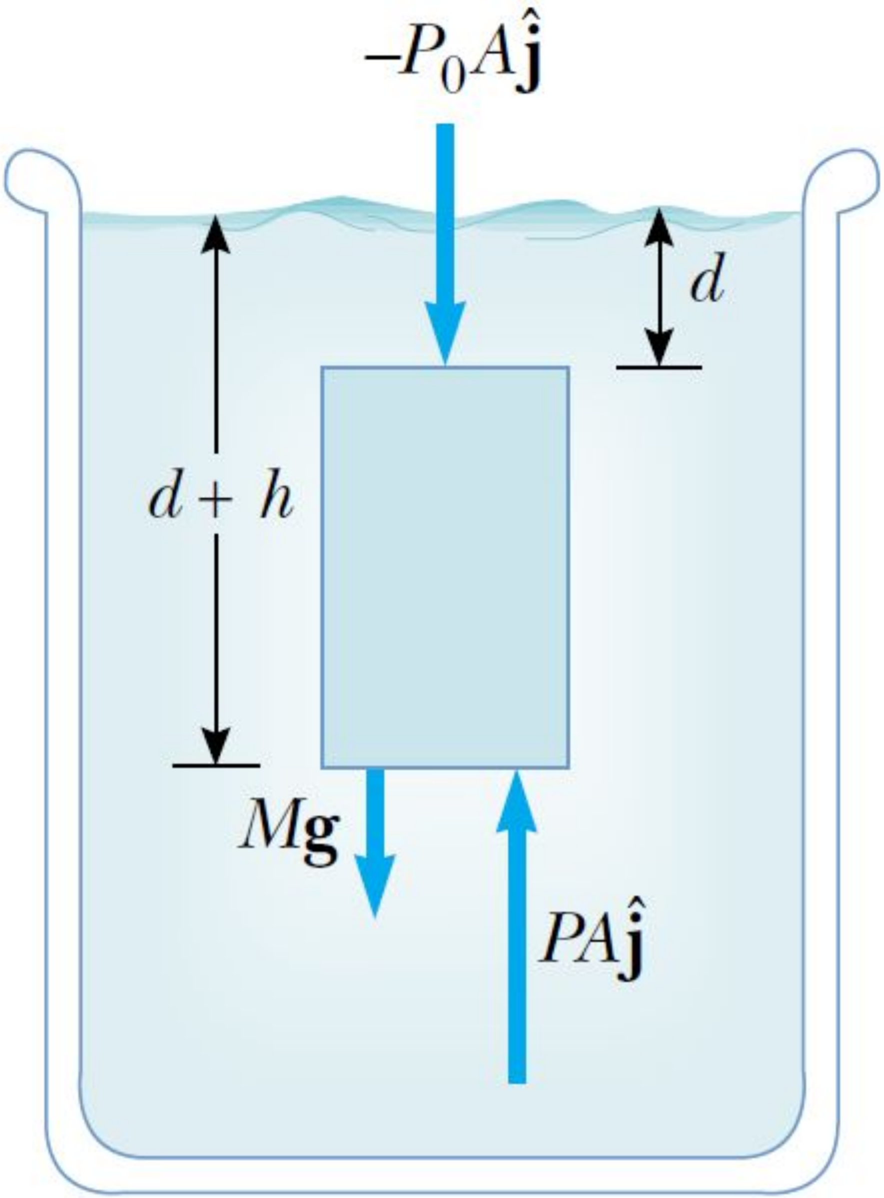
Table 7.1

$$\rho \equiv \frac{m}{V}$$

Densities of Some Common Substances at Standard Temperature (0°C) and Pressure (Atmospheric)

Substance	$\rho\ (kg/m^3)$	Substance	$\rho\ (kg/m^3)$
Air	1.29	Ice	0.917×10^3
Aluminum	2.70×10^3	Iron	7.86×10^3
Benzene	0.879×10^3	Lead	11.3×10^3
Copper	8.92×10^3	Mercury	13.6×10^3
Ethyl alcohol	0.806×10^3	Oak	0.710×10^3
Fresh water	1.00×10^3	Oxygen gas	1.43
Glycerin	1.26×10^3	Pine	0.373×10^3
Gold	19.3×10^3	Platinum	21.4×10^3
Helium gas	1.79×10^{-1}	Seawater	1.03×10^3
Hydrogen gas	8.99×10^{-2}	Silver	10.5×10^3

Variation of Pressure with Depth

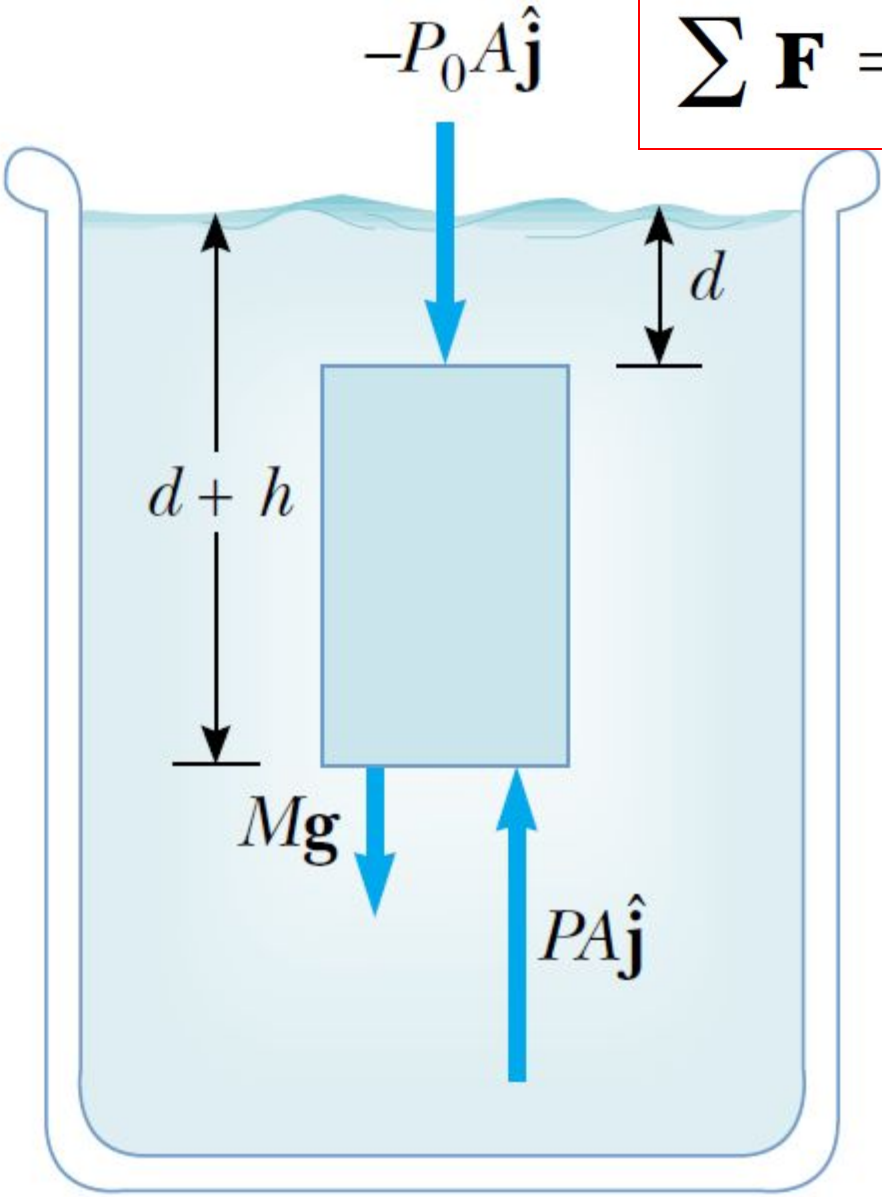


$$M = \rho V = \rho A h$$

$$Mg = \rho A h g$$

Figure 7.3 A parcel of fluid (darker region) in a larger volume of fluid is singled out. The net force exerted on the parcel of fluid must be zero because it is in equilibrium.

Variation of Pressure with Depth



$$\sum \mathbf{F} = PA\hat{\mathbf{j}} - P_0A\hat{\mathbf{j}} - M\mathbf{g}\hat{\mathbf{j}} = 0$$

$$PA - P_0A - \rho Ahg = 0$$

$$PA - P_0A = \rho Ahg$$

$$P = P_0 + \rho gh \qquad (7.4)$$

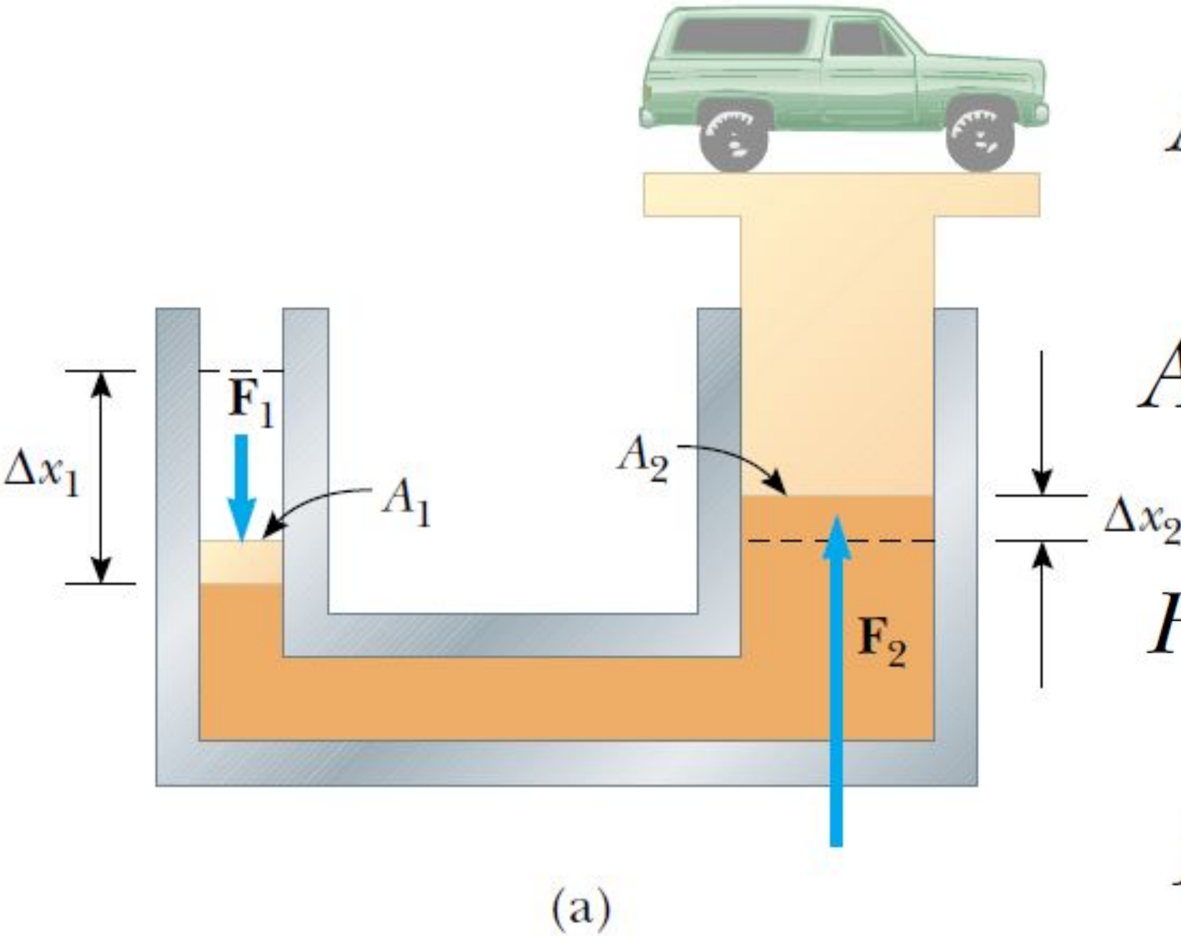
That is, **the pressure *P* at a depth *h* below a point in the liquid at which the pressure is P_0 is greater by an amount ρgh.**

If the liquid is open to the atmosphere and P_0 is the pressure at the surface of the liquid, then P_0 is atmospheric pressure. In our calculations and working of end-of-chapter problems, we usually take atmospheric pressure to be

$$P_0 = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

In view of the fact that the pressure in a fluid depends on depth and on the value of P_0, any increase in pressure at the surface must be transmitted to every other point in the fluid. This concept was first recognized by the French scientist Blaise Pascal (1623–1662) and is called **Pascal's law: a change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container**.

$$P = F_1/A_1 = F_2/A_2$$



$$A_1 \Delta x_1 = A_2 \Delta x_2$$

$$A_2/A_1 = \Delta x_1/\Delta x_2$$

$$F_2/F_1 = \Delta x_1/\Delta x_2$$

$$F_1 \Delta x_1 = F_2 \Delta x_2$$

Figure 7.4 (a) Diagram of a hydraulic press. Because the increase in pressure is the same on the two sides, a small force F_1 at the left produces a much greater force F_2 at the right.

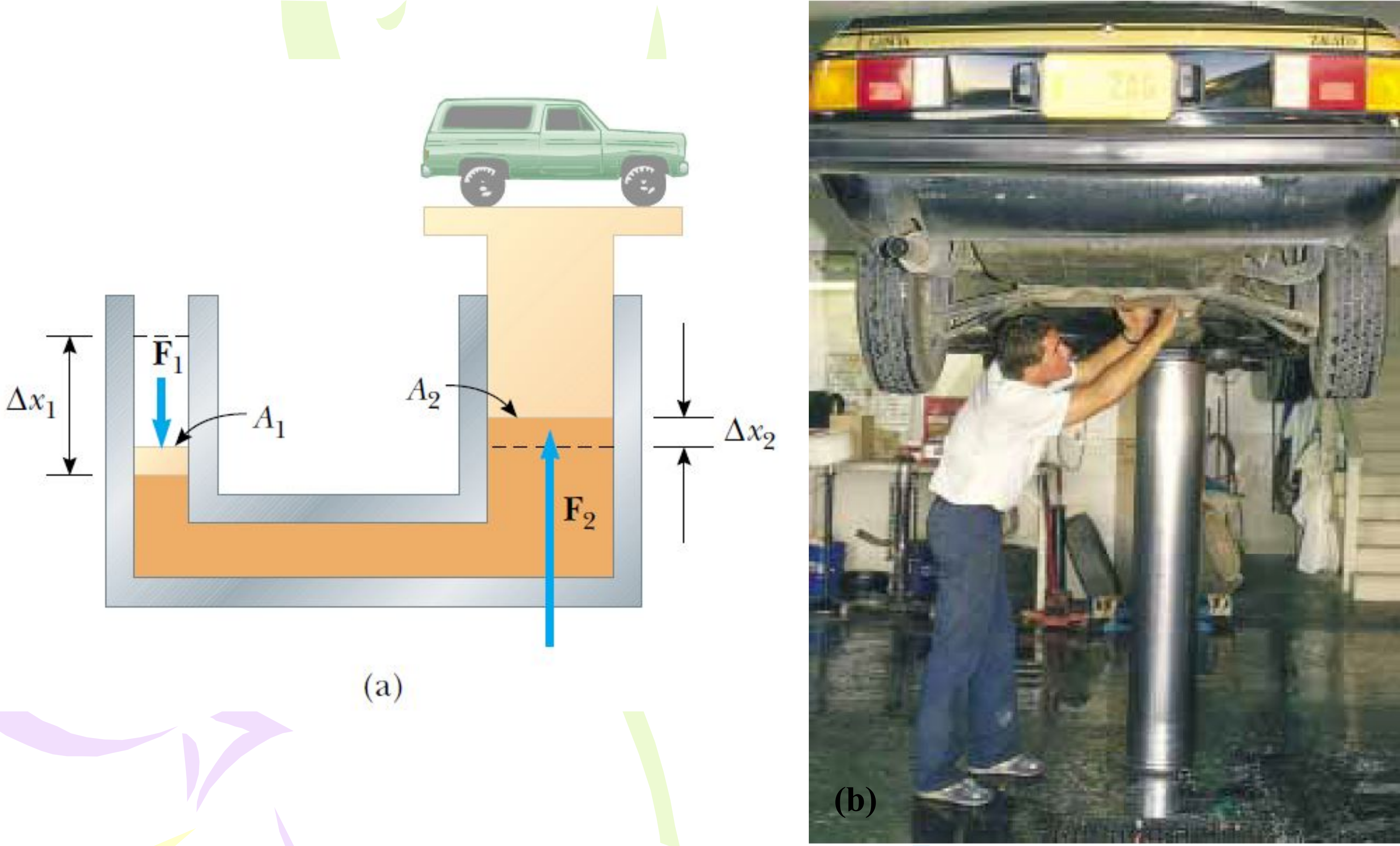
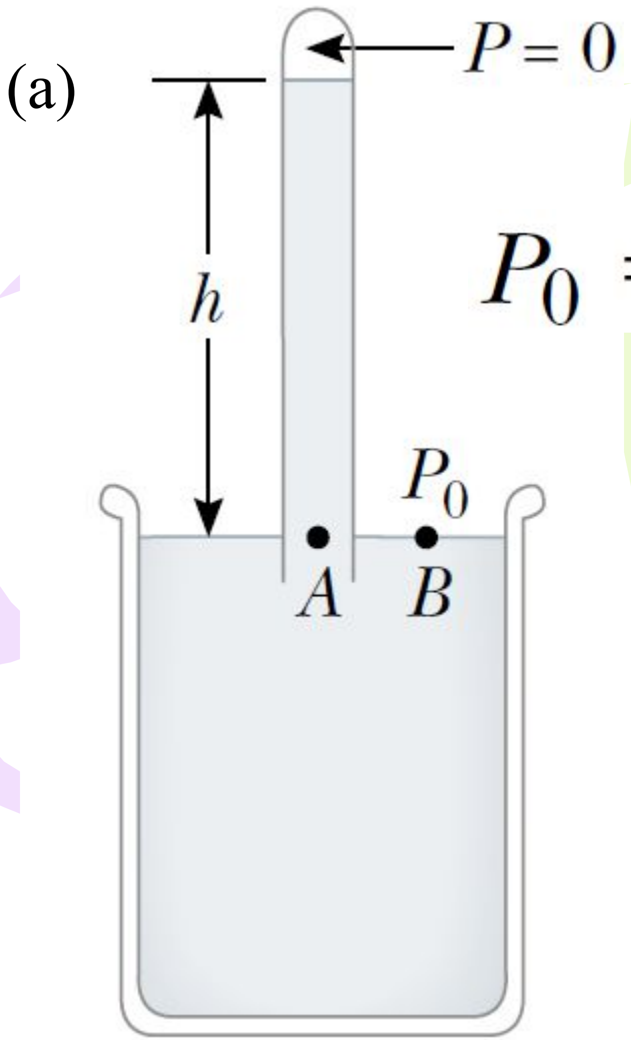


Figure 7.4 (a) Diagram of a hydraulic press. Because the increase in pressure is the same on the two sides, a small force F_1 at the left produces a much greater force F_2 at the right. (b) A vehicle undergoing repair is supported by a hydraulic lift in a garage.

Pressure Measurements



$$P_0 = \rho_{Hg} g h$$

$$P_0 = 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

$$P_0 = \rho_{Hg} g h \longrightarrow$$

$$h = \frac{P_0}{\rho_{Hg} g} =$$

$$= 0.760 \text{ m}$$

Figure 7.5 (a) a mercury barometer.

Pressure Measurements

(b)

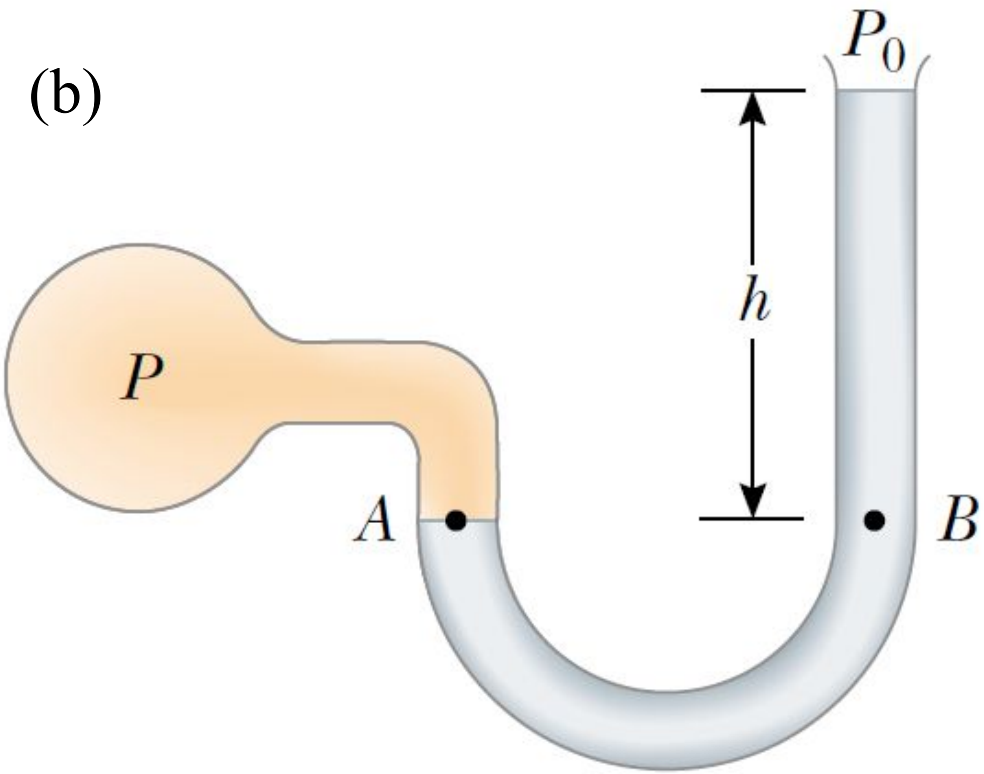


Figure 7.5 (b) an open-tube manometer.

$$P = P_0 + \rho g h$$

The difference in pressure $P - P_0$ is equal to $\rho g h$. The pressure P is called the **absolute pressure**, while the difference $P - P_0$ is called the **gauge pressure**. For example, the pressure you measure in your bicycle tire is gauge pressure.

Buoyant Forces and Archimedes's Principle

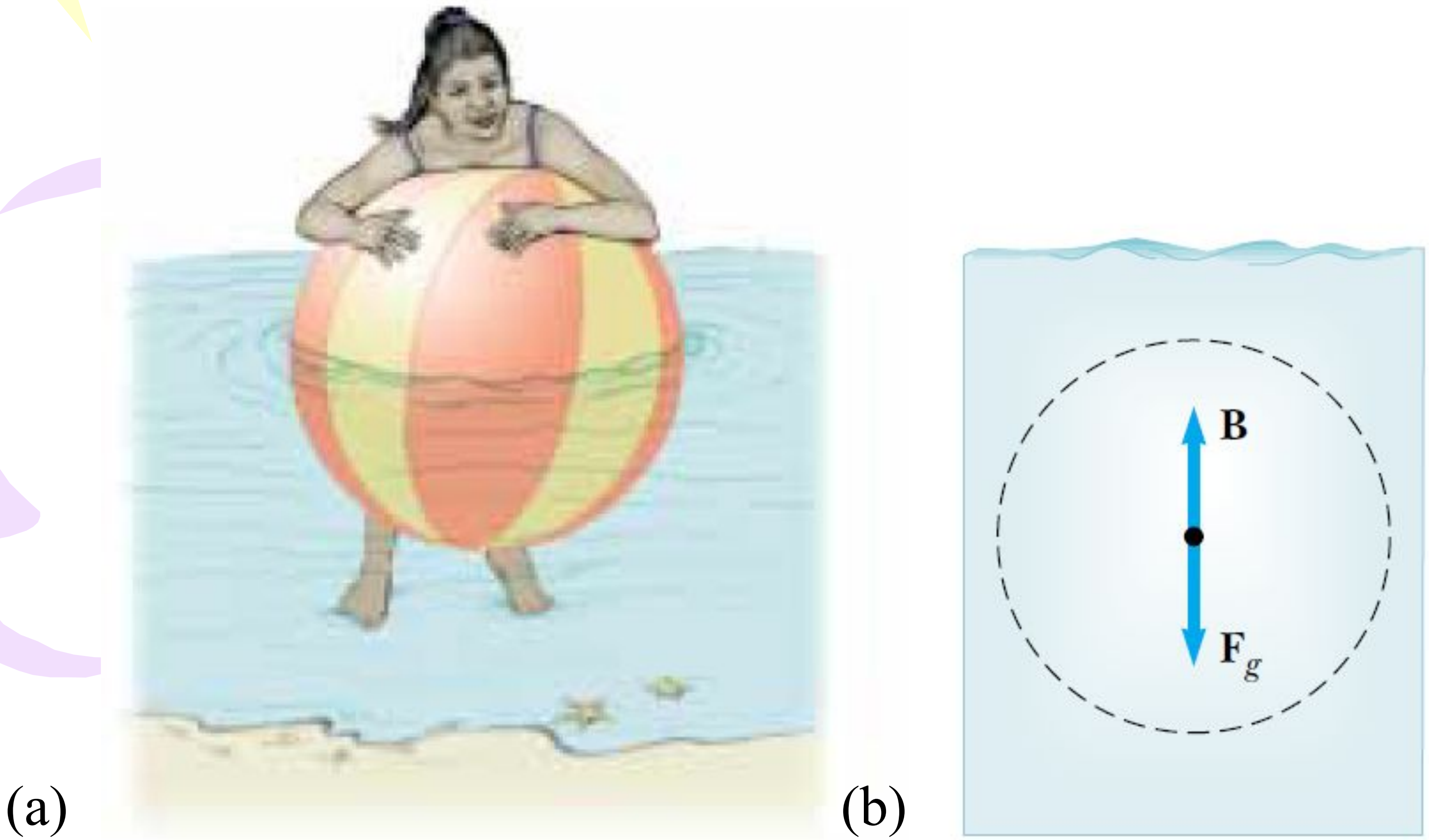


Figure 7.6 (a) A swimmer attempts to push a beach ball underwater. (b) The forces on a beach ball–sized parcel of water. The buoyant force **B** on a beach ball that replaces this parcel is exactly the same as the buoyant force on the parcel.

Buoyant Forces and Archimedes's Principle

The upward force exerted by a fluid on any immersed object is called a **buoyant force**.

The magnitude of the buoyant force always equals the weight of the fluid displaced by the object. This statement is known as **Archimedes's principle**.

Buoyant Forces and Archimedes's Principle

$$B = (P_b - P_t)A =$$

$$= (\rho_{fluid} gh)A = \rho_{fluid} gV$$

(7.5)

$$B = Mg$$

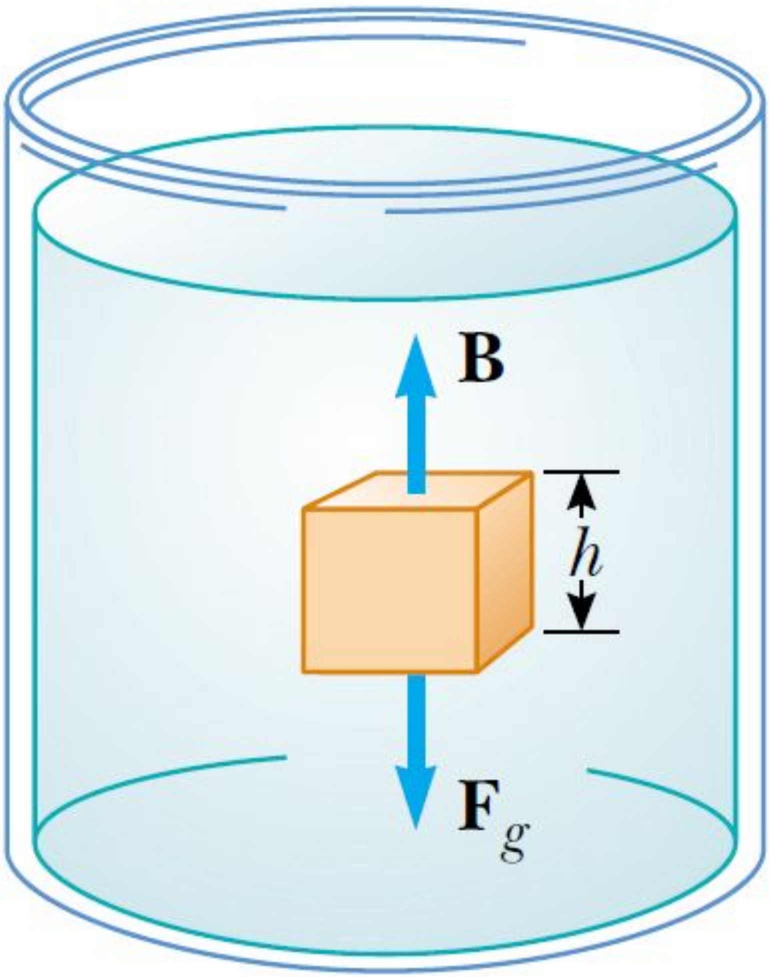


Figure 7.7 The external forces acting on the cube of liquid are the gravitational force $\mathbf{F}_g$ and the buoyant force **B**. Under equilibrium conditions, $B = F_g$.

Case 1: Totally Submerged Object

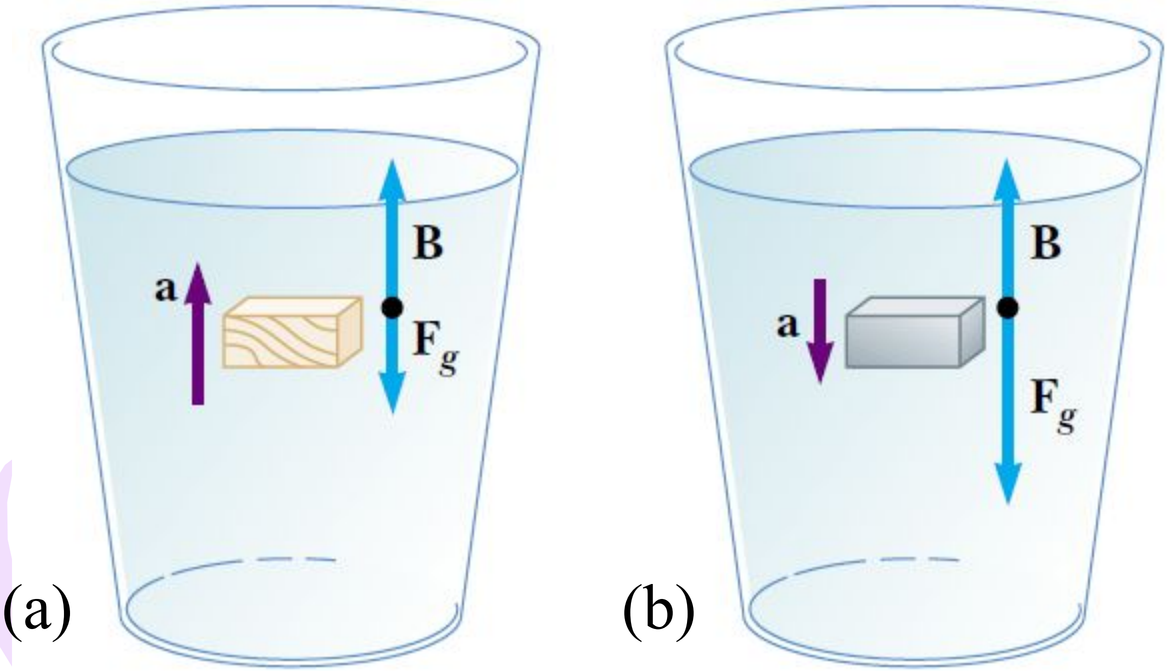


Figure 7.8 The external forces acting on the cube of liquid are the gravitational force $\mathbf{F}_g$ and the buoyant force **B**. Under equilibrium conditions, $B = F_g$.

$$B = \rho_{\text{fluid}}\, gV = \rho_{\text{fluid}}\, gV_{\text{obj}},$$

$$F_g = Mg = \rho_{\text{obj}}\, gV_{\text{obj}},$$

$$B - F_g = (\rho_{\text{fluid}} - \rho_{\text{obj}})\, gV_{\text{obj}}$$

Case 1: Totally Submerged Object

Thus, **the direction of motion of an object submerged in a fluid is determined only by the densities of the object and the fluid.**

Case 2: Floating Object

Figure 7.9 An object floating on the surface of a fluid experiences two forces, the gravitational force $\mathbf{F}_g$ and the buoyant force **B**. Because the object floats in equilibrium, $B = F_g$.

$$V_{\text{obj}}, \qquad \rho_{\text{obj}} < \rho_{\text{fluid}},$$

$$B = \rho_{\text{fluid}} g V_{\text{fluid}}.$$

Case 2: Floating Object

$$F_g = Mg = \rho_{\text{obj}}\, g V_{\text{obj}}, \quad F_g = B,$$

$$\rho_{\text{fluid}}\, g V_{\text{fluid}} = \rho_{\text{obj}}\, g V_{\text{obj}},$$

$$\frac{V_{\text{fluid}}}{V_{\text{obj}}} = \frac{\rho_{\text{obj}}}{\rho_{\text{fluid}}} \qquad \textbf{(7.6)}$$

This equation tells us that **the fraction of the volume of a floating object that is below the fluid surface is equal to the ratio of the density of the object to that of the fluid.**

Fluid Dynamics

When fluid is in motion, its flow can be characterized as being one of two main types. The flow is said to be **steady**, or **laminar**, if each particle of the fluid follows a smooth path, such that the paths of different particles never cross each other. In steady flow, the velocity of fluid particles passing any point remains constant in time.

Figure 7.10 Laminar flow around an automobile in a test wind tunnel.

Fluid Dynamics

Above a certain critical speed, fluid flow becomes **turbulent**; turbulent flow is irregular flow characterized by small whirlpool-like regions, as shown in Figure 7.11.

Figure 7.11 Hot gases from a cigarette made visible by smoke particles. The smoke first moves in laminar flow at the bottom and then in turbulent flow above.

Fluid Dynamics

The term **viscosity** is commonly used in the description of fluid flow to characterize the degree of internal friction in the fluid. This internal friction, or *viscous force,* is associated with the resistance that two adjacent layers of fluid have to moving relative to each other. Viscosity causes part of the kinetic energy of a fluid to be converted to internal energy. This mechanism is similar to the one by which an object sliding on a rough horizontal surface loses kinetic energy.

Fluid Dynamics

Because the motion of real fluids is very complex and not fully understood, we make some simplifying assumptions in our approach. In our model of **ideal fluid flow**, we make the following four assumptions:

1. **The fluid is nonviscous**. In a nonviscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.
2. **The flow is steady**. In steady (laminar) flow, the velocity of the fluid at each point remains constant.
3. **The fluid is incompressible**. The density of an incompressible fluid is constant.
4. **The flow is irrotational**. In irrotational flow, the fluid has no angular momentum about any point. If a small paddle wheel placed anywhere in the fluid does not rotate about the wheel's center of mass, then the flow is irrotational.

Fluid Dynamics

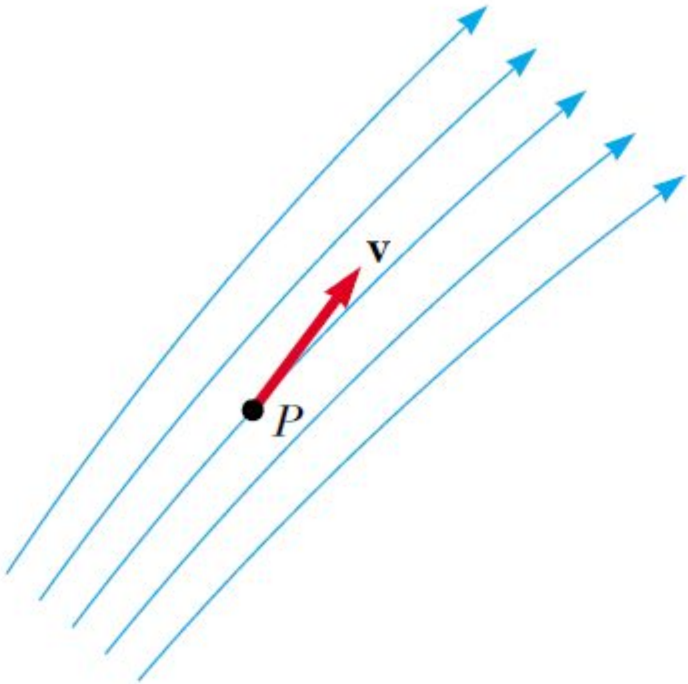
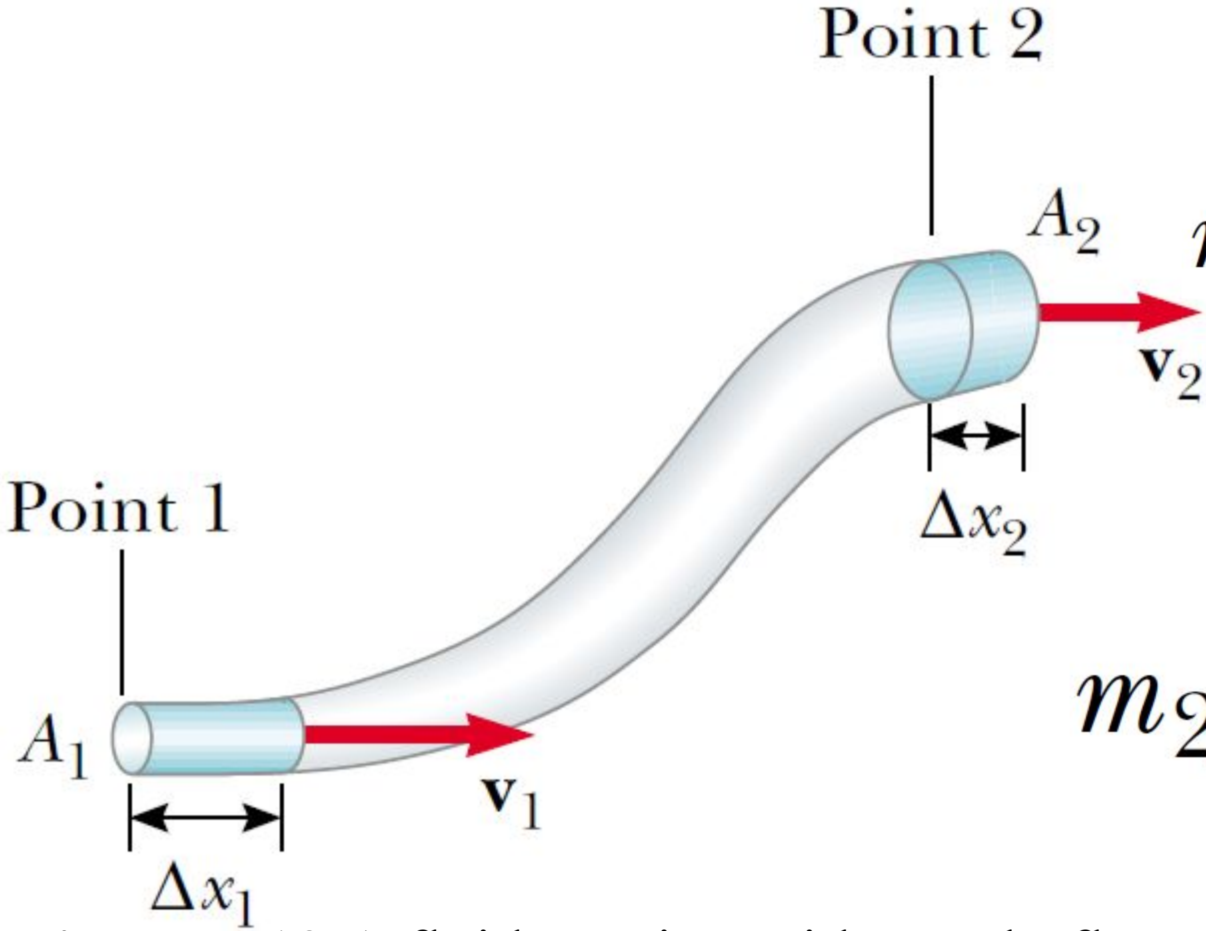


Figure 7.12 A particle in laminar flow follows a streamline, and at each point along its path the particle's velocity is tangent to the streamline.

The path taken by a fluid particle under steady flow is called a **streamline**. The velocity of the particle is always tangent to the streamline, as shown in Figure 7.12. A set of streamlines like the ones shown in Figure 7.12 form a *tube of flow*. Note that fluid particles cannot flow into or out of the sides of this tube; if they could, then the streamlines would cross each other.

Fluid Dynamics



$$\Delta x_1 = v_1 \Delta t$$

$$m_1 = \rho A_1 \Delta x_1 =$$

$$= \rho A_1 v_1 \Delta t$$

$$m_2 = \rho A_2 v_2 \Delta t$$

Figure 7.13 A fluid moving with steady flow through a pipe of varying cross-sectional area. The volume of fluid flowing through area A_1 in a time interval Δt must equal the volume flowing through are A_2 in the same time interval. Therefore, $A_1 v_1 = A_2 v_2$.

Fluid Dynamics

$$m_1 = m_2, \quad \rho A_1 v_1 = \rho A_2 v_2;$$

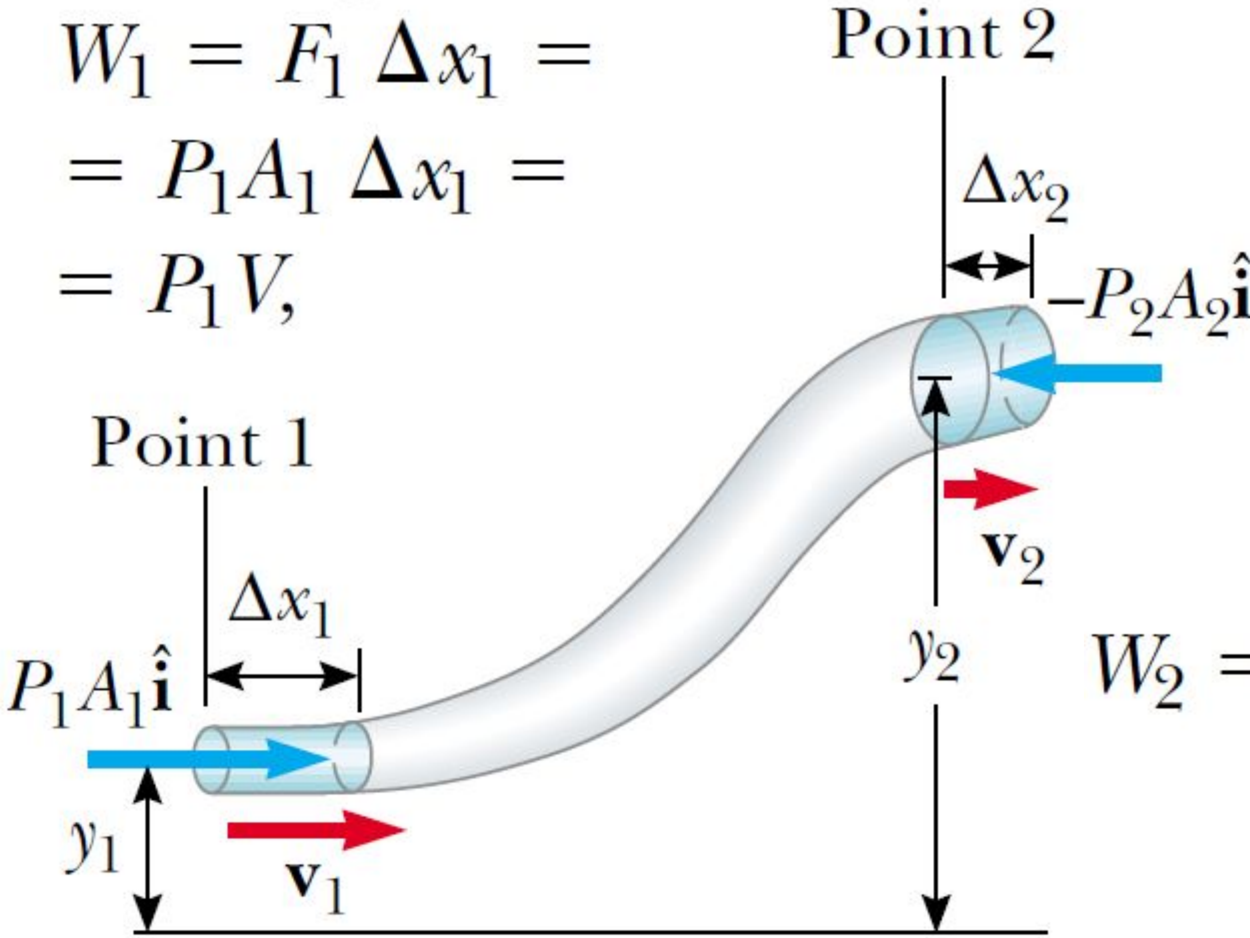
This expression is called the equation of continuity for fluids. It states that

$$A_1 v_1 = A_2 v_2 = \text{constant} \tag{7.7}$$

the product of the area and the fluid speed at all points along a pipe is constant for an incompressible fluid.

Bernoulli's Equation

$$W_1 = F_1 \Delta x_1 = \\ = P_1 A_1 \Delta x_1 = \\ = P_1 V,$$



$$W_2 = -P_2 A_2 \Delta x_2 = \\ = -P_2 V.$$

Figure 7.14 A fluid in laminar flow through a constricted pipe. The volume of the shaded portion on the left is equal to the volume of the shaded portion on the right.

Bernoulli's Equation

$$W = (P_1 - P_2)V$$

$$\Delta K = \frac{1}{2}m{v_2}^2 - \frac{1}{2}m{v_1}^2$$

$$\Delta U = mgy_2 - mgy_1$$

$$W = \Delta K + \Delta U.$$

$$(P_1 - P_2)V = \frac{1}{2}mv_2^2 - \frac{1}{2}m{v_1}^2 + mgy_2 - mgy_1$$

Bernoulli's Equation

$$\rho = m/V,$$

$$P_1 - P_2 = \tfrac{1}{2}\rho v_2{}^2 - \tfrac{1}{2}\rho v_1{}^2 + \rho g y_2 - \rho g y_1$$

$$P_1 + \tfrac{1}{2}\rho v_1{}^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2{}^2 + \rho g y_2 \qquad \textbf{(7.8)}$$

This is Bernoulli's equation as applied to an ideal fluid. It is often expressed as

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant} \qquad \textbf{(7.9)}$$

Bernoulli's Equation

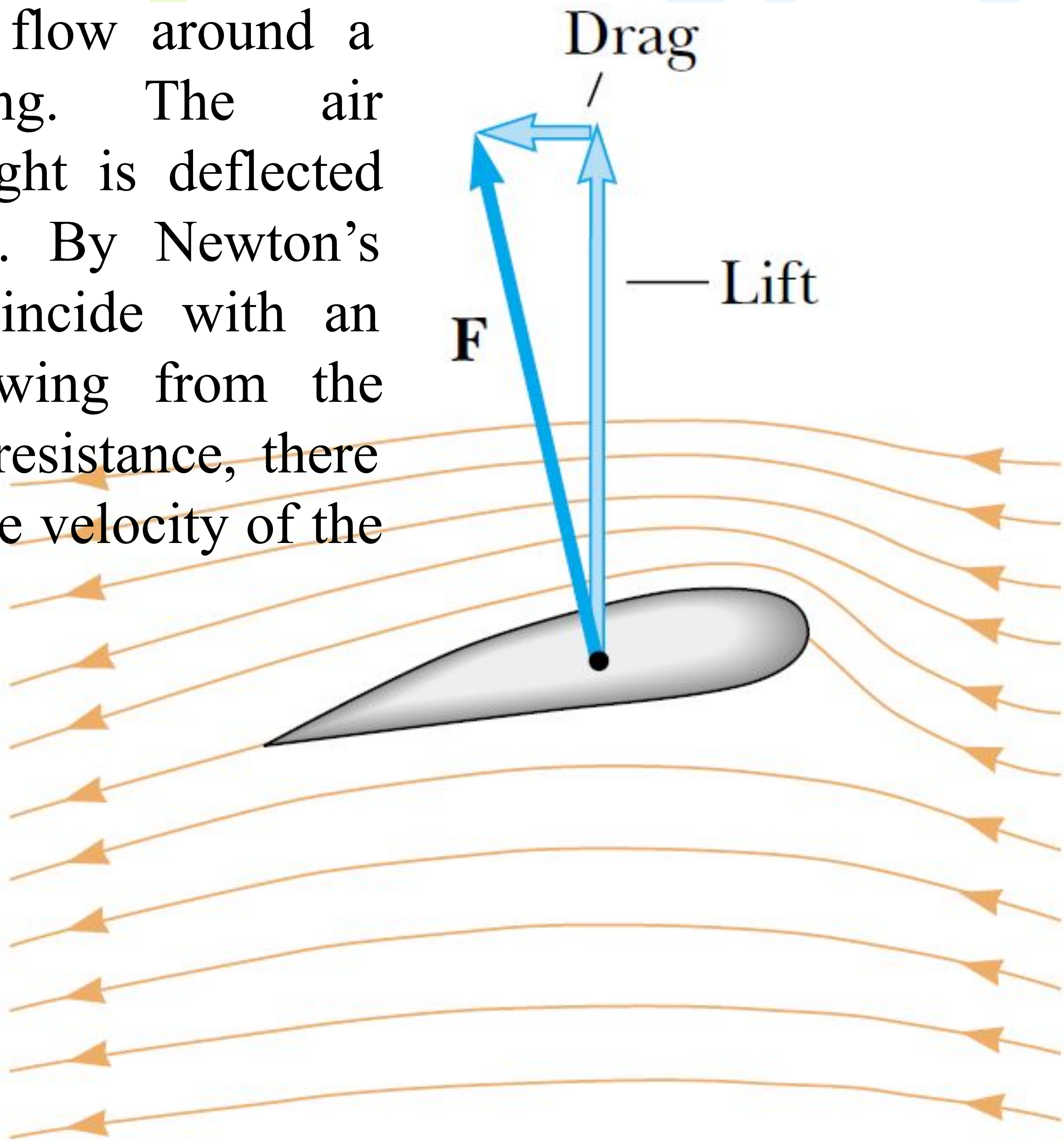
$$v_1 = v_2 = 0$$

$$P_1 - P_2 = \rho g(y_2 - y_1) = \rho g h$$

This *Bernoulli effect* explains the experience with the truck on the highway at the opening of this section. As air passes between you and the truck, it must pass through a relatively narrow channel. According to the continuity equation, the speed of the air is higher. According to the Bernoulli effect, this higher speed air exerts less pressure on your car than the slower moving air on the other side of your car. Thus, there is a net force pushing you toward the truck!

Other Applications of Fluid Dynamics

Figure 7.15 Streamline flow around a moving airplane wing. The air approaching from the right is deflected downward by the wing. By Newton's third law, this must coincide with an upward force on the wing from the air—*lift*. Because of air resistance, there is also a force opposite the velocity of the wing— *drag*.



Other Applications of Fluid Dynamics

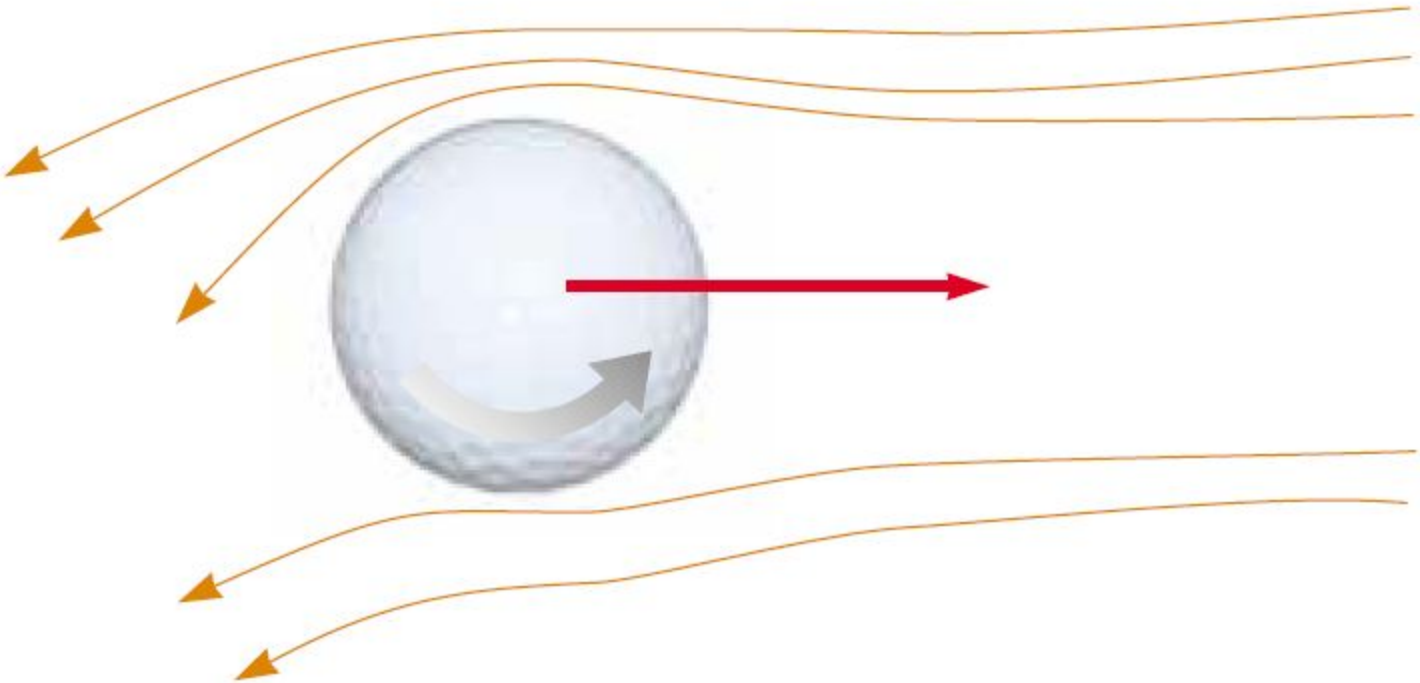


Figure 7.16 Because of the deflection of air, a spinning golf ball experiences a lifting force that allows it to travel much farther than it would if it were not spinning.

Other Applications of Fluid Dynamics

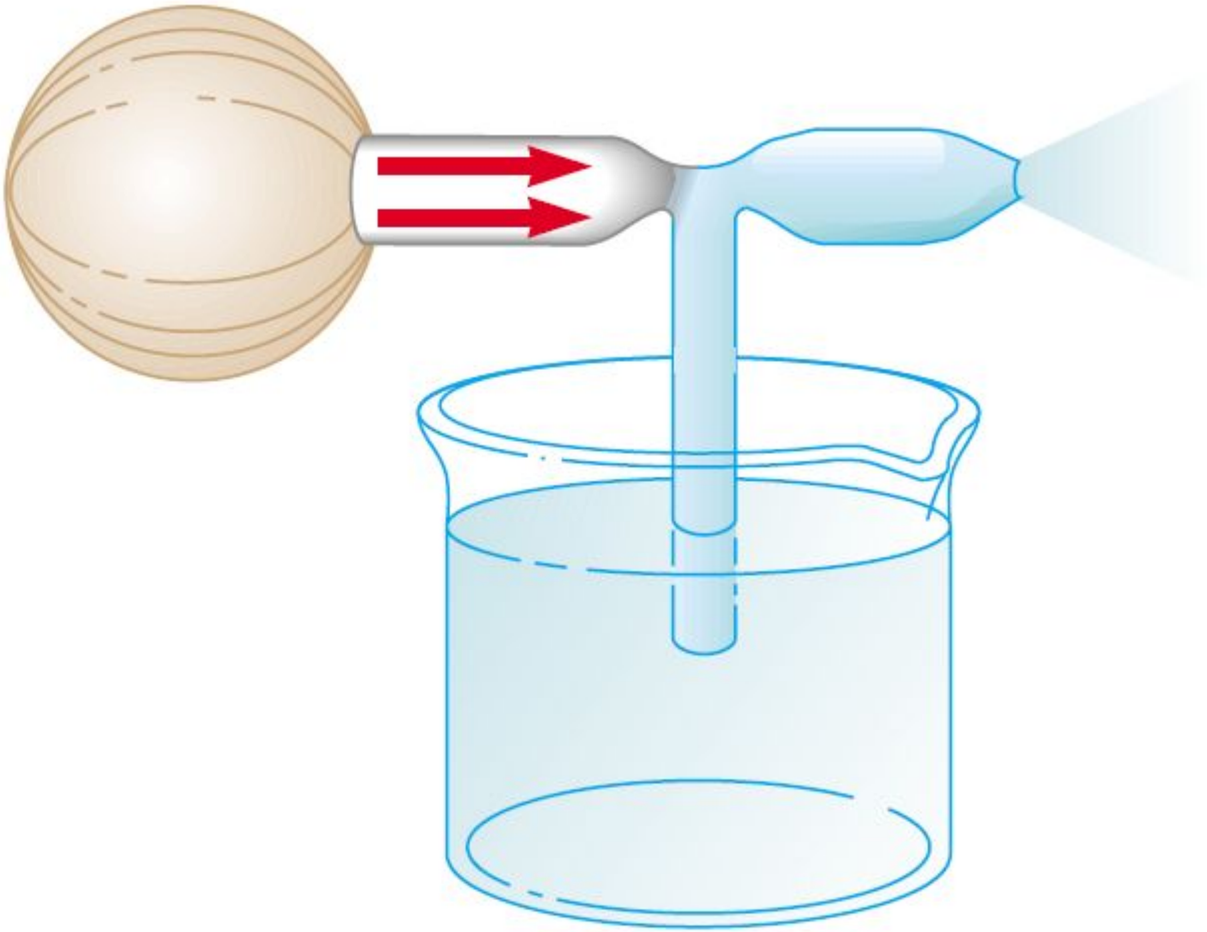


Figure 7.17 A stream of air passing over a tube dipped into a liquid causes the liquid to rise in the tube.

Quick Quiz 7.1

Suppose you are standing directly behind someone who steps back and accidentally stomps on your foot with the heel of one shoe. Would you be better off if that person were (a) a large professional basketball player wearing sneakers (b) a petite woman wearing spike-heeled shoes?

Quick Quiz 7.2

The pressure at the bottom of a filled glass of water (ρ=1 000 kg/m^3) is P. The water is poured out and the glass is filled with ethyl alcohol ρ=806 kg/m^3). The pressure at the bottom of the glass is (a) smaller than P (b) equal to P (c) larger than P (d) indeterminate.

Quick Quiz 7.3

Several common barometers are built, with a variety of fluids. For which of the following fluids will the column of fluid in the barometer be the highest? (a) mercury (b) water (c) ethyl alcohol (d) benzene

Quick Quiz 7.4

An apple is held completely submerged just below the surface of a container of water. The apple is then moved to a deeper point in the water. Compared to the force needed to hold the apple just below the surface, the force needed to hold it at a deeper point is (a) larger (b) the same (c) smaller (d) impossible to determine.

Quick Quiz 7.5

You observe two helium balloons floating next to each other at the ends of strings secured to a table. The facing surfaces of the balloons are separated by 1–2 cm. You blow through the small space between the balloons. What happens to the balloons? (a) They move toward each other. (b) They move away from each other. (c) They are unaffected.

Quick Quiz 7.6

You tape two different soda straws together end-to-end to make a longer straw with no leaks. The two straws have radii of 3 mm and 5 mm. You drink a soda through your combination straw. In which straw is the speed of the liquid the highest? (a) whichever one is nearest your mouth (b) the one of radius 3 mm (c) the one of radius 5 mm (d) Neither—the speed is the same in both straws.