Course of lectures «Contemporary Physics: Part2»

Lecture №8

Introduction to Quantum Physics. Blackbody Radiation and Planck's Hypothesis. The Photoelectric Effect. The Compton Effect. The Nature of Electromagnetic Waves. The Wave Properties of Particles. A New Model: The Quantum Particle. The Double-Slit Experiment Revisited. The Uncertainty Principle.

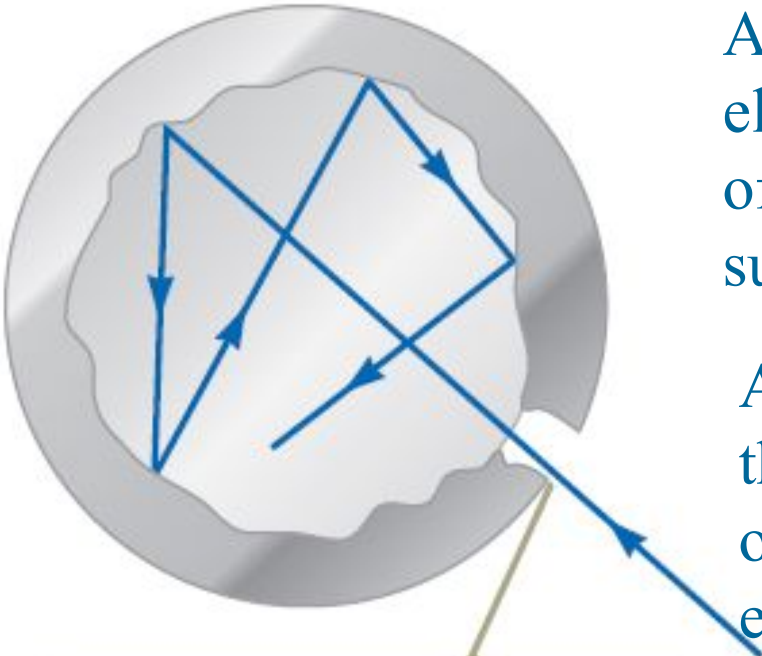
Einstein's special theory of relativity when dealing with particle speeds comparable to the speed of light. As the 20th century progressed, many experimental and theoretical problems were resolved by the special theory of relativity. For many other problems, however, neither relativity nor classical physics could provide a theoretical answer. Attempts to apply the laws of classical physics to explain the behavior of matter on the atomic scale were consistently unsuccessful. For example, the emission of discrete wavelengths of light from atoms in a hightemperature gas could not be explained within the framework of classical physics.

As physicists sought new ways to solve these puzzles, another revolution took place in physics between 1900 and 1930. A new theory called *quantum mechanics* was highly successful in explaining the behavior of particles of microscopic size. Like the special theory of relativity, the quantum theory requires a modification of our ideas concerning the physical world.

The first explanation of a phenomenon using quantum theory was introduced by Max Planck. Many subsequent mathematical developments and interpretations were made by a number of distinguished physicists, including Einstein, Bohr, de Broglie, Schrцdinger, and Heisenberg. Despite the great success of the quantum theory, Einstein frequently played the role of its critic, especially with regard to the manner in which the theory was interpreted.

Because an extensive study of quantum theory is beyond the scope of this book, this chapter is simply an introduction to its underlying principles.

Blackbody Radiation and Planck's Hypothesis

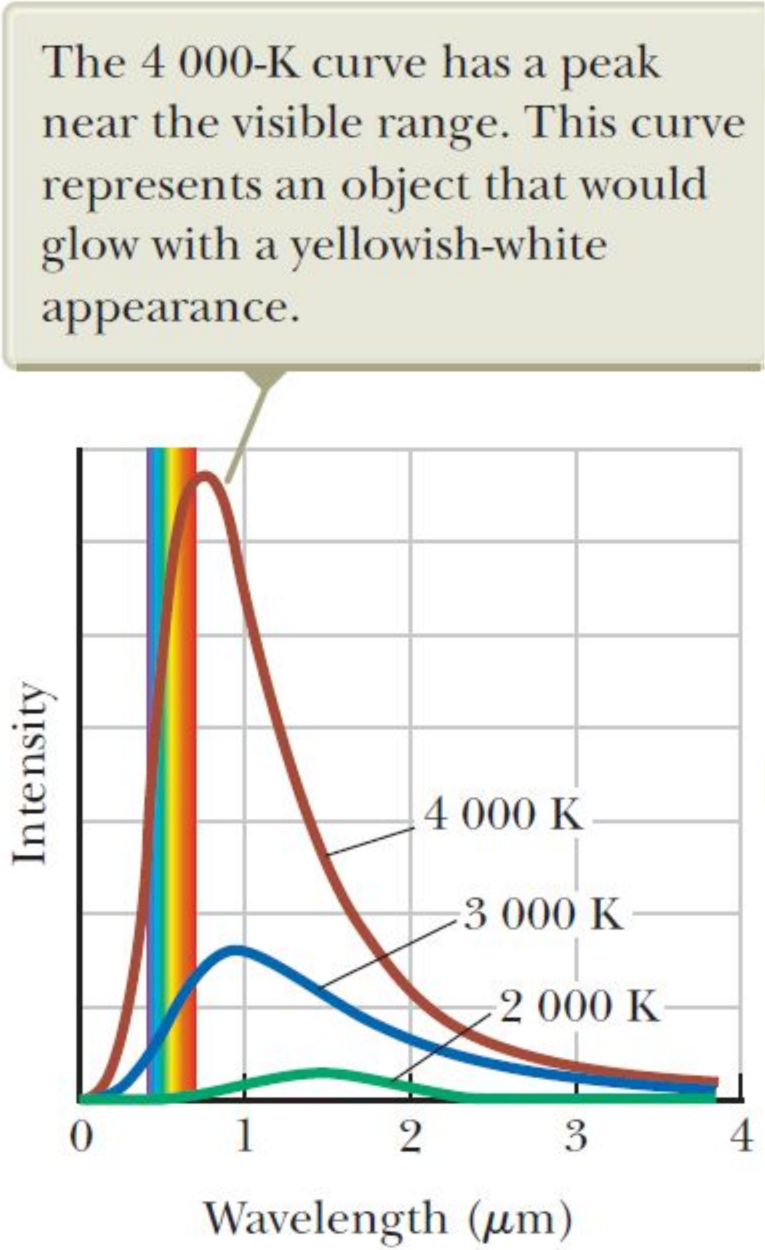


The opening to a cavity inside a hollow object is a good approximation of a black body: the hole acts as a perfect absorber.

An object at any temperature emits electromagnetic waves in the form of **thermal radiation** from its surface.

A **black body** is an ideal system that absorbs all radiation incident on it. The electromagnetic radiation emitted by the black body is called **blackbody radiation.**

Blackbody Radiation and Planck's Hypothesis



The following two consistent experimental findings were seen as especially significant how the intensity of blackbody radiation varies with temperature and wavelength:

1. The total power of the emitted radiation increases with temperature. The **Stefan's law:**

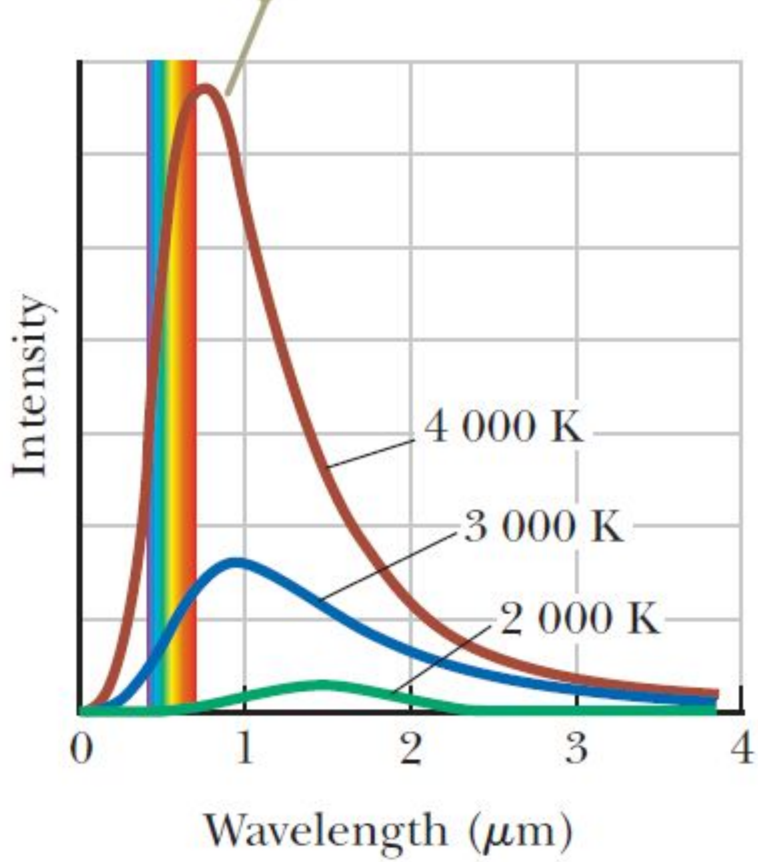
$$\boxed{P = \sigma A e T^4}$$

$$\sigma = 5.670 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$$

where P is the power in watts radiated at all wavelengths from the surface of an object, A is the surface area of the object in square meters, e is the emissivity of the surface, and T is the surface temperature in kelvins. For a black body, the emissivity is e=1 exactly.

Blackbody Radiation and Planck's Hypothesis

The 4 000-K curve has a peak near the visible range. This curve represents an object that would glow with a yellowish-white appearance.



2. The peak of the wavelength distribution shifts to shorter wavelengths as the temperature increases. This behavior is described by the following relationship, called **Wien's displacement law:**

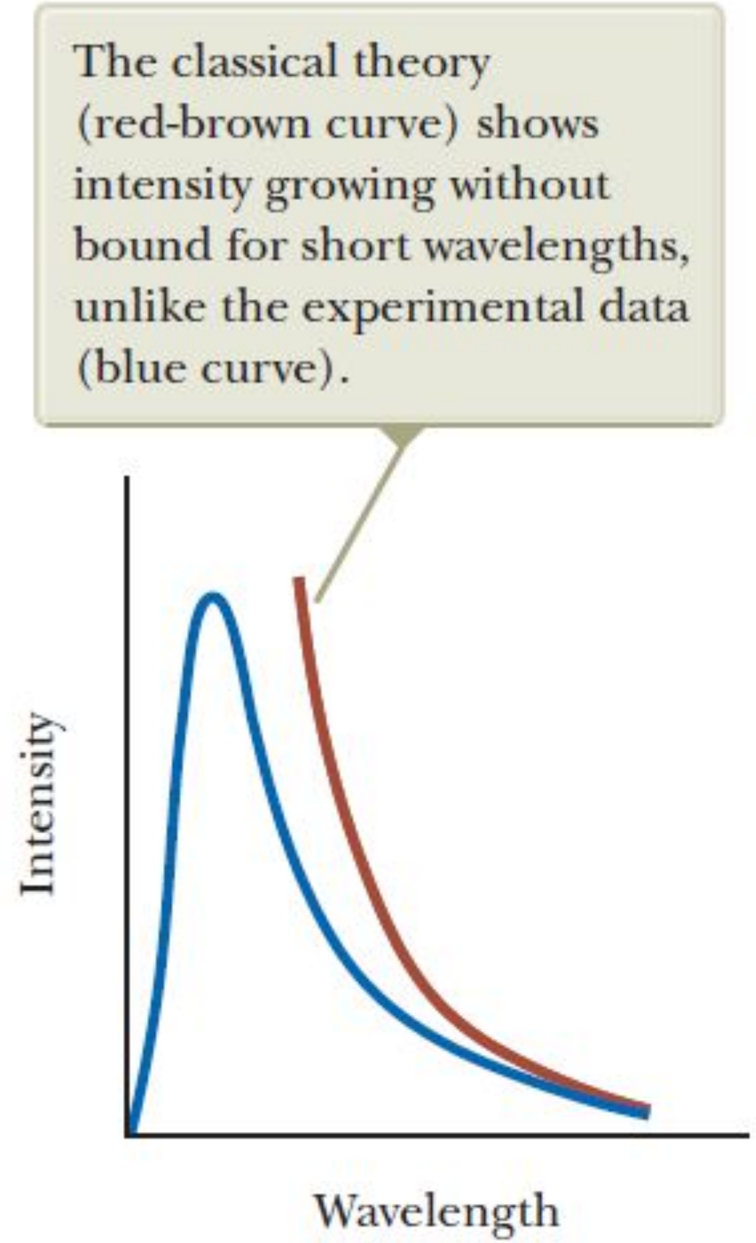
$$\boxed{\lambda_{max} T = 2.898 \times 10^{-3}\ \mathrm{m \cdot K}}$$

Where λ_{max} is the wavelength at which the curve peaks and T is the absolute temperature of the surface of the object emitting the radiation. The wavelength at the curve's peak is inversely proportional to the absolute temperature; that is, as the temperature increases, the peak is "displaced" to shorter wavelengths.

Blackbody Radiation and Planck's Hypothesis

To describe the distribution of energy from a black body, we define *I(λ,T)dl* to be the intensity, or power per unit area, emitted in the wavelength interval *dλ*. The result of a calculation based on a classical theory of blackbody radiation known as the **Rayleigh–Jeans law is**

$$\boxed{I(\lambda, T) = \frac{2\pi c k_B T}{\lambda^4}}$$



Blackbody Radiation and Planck's Hypothesis

In 1900, Max Planck developed a theory of blackbody radiation that leads to an equation for $I(\lambda, T)$ that is in complete agreement with experimental results at all wavelengths. Planck assumed the cavity radiation came from atomic oscillators in the cavity walls. Planck made two bold and controversial assumptions concerning the nature of the oscillators in the cavity walls:

- The energy of an oscillator can have only certain *discrete values* E_n:

$$\boxed{E_n = nhf}$$

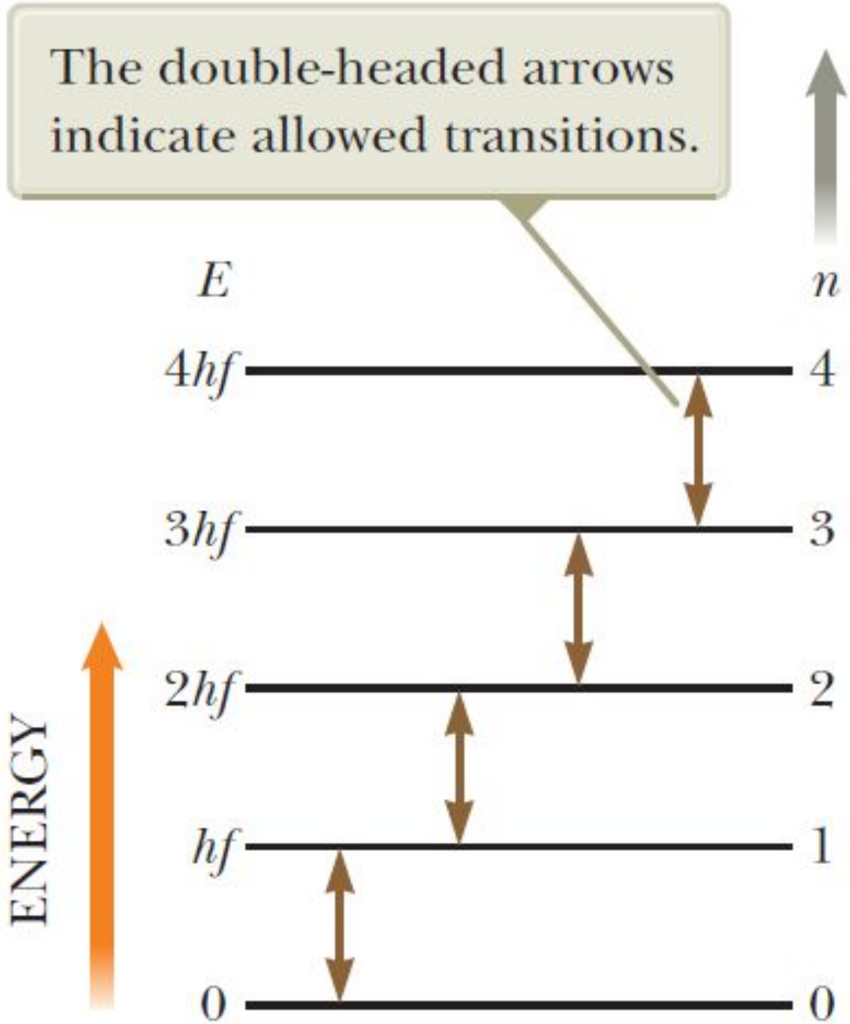
Because the energy of each oscillator can have only discrete values, we say the energy is **quantized**. Each discrete energy value corresponds to a different **quantum state**, represented by the quantum number n. When the oscillator is in the $n=1$ quantum state, its energy is hf; when it is in the $n=2$ quantum state, its energy is $2hf$; and so on.

Blackbody Radiation and Planck's Hypothesis

- The oscillators emit or absorb energy when making a transition from one quantum state to another. The entire energy difference between the initial and final states in the transition is emitted or absorbed as a single quantum of radiation. If the transition is from one state to a lower adjacent state—say, from the *n=3* state to the *n=2* state the amount of energy emitted by the oscillator and carried by the quantum of radiation is

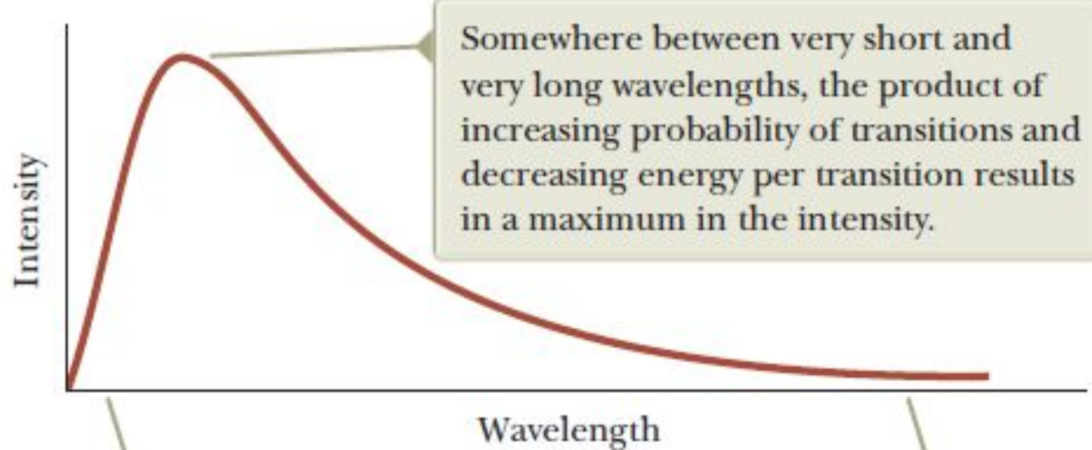
$$\boxed{E = hf}$$

Blackbody Radiation and Planck's Hypothesis



An oscillator emits or absorbs energy only when it changes quantum states. If it remains in one quantum state, no energy is absorbed or emitted. Figure is an **energy-level diagram** showing the quantized energy levels and allowed transitions proposed by Planck.

Blackbody Radiation and Planck's Hypothesis

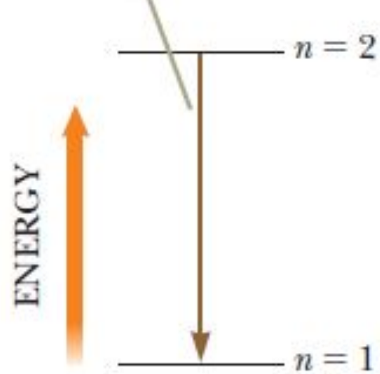


Somewhere between very short and very long wavelengths, the product of increasing probability of transitions and decreasing energy per transition results in a maximum in the intensity.

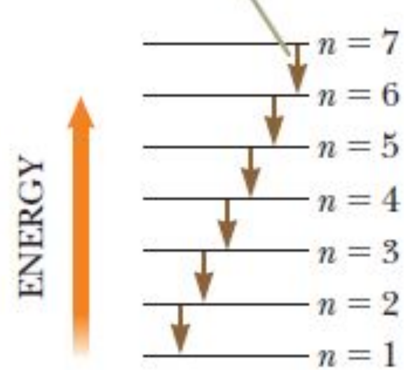
$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda k_B T} - 1\right)}$$

$$h = 6.626 \times 10^{-34}\,\mathrm{J \cdot s}$$

At short wavelengths, there is a large separation between energy levels, leading to a low probability of excited states and few downward transitions. The low probability of transitions leads to low intensity.

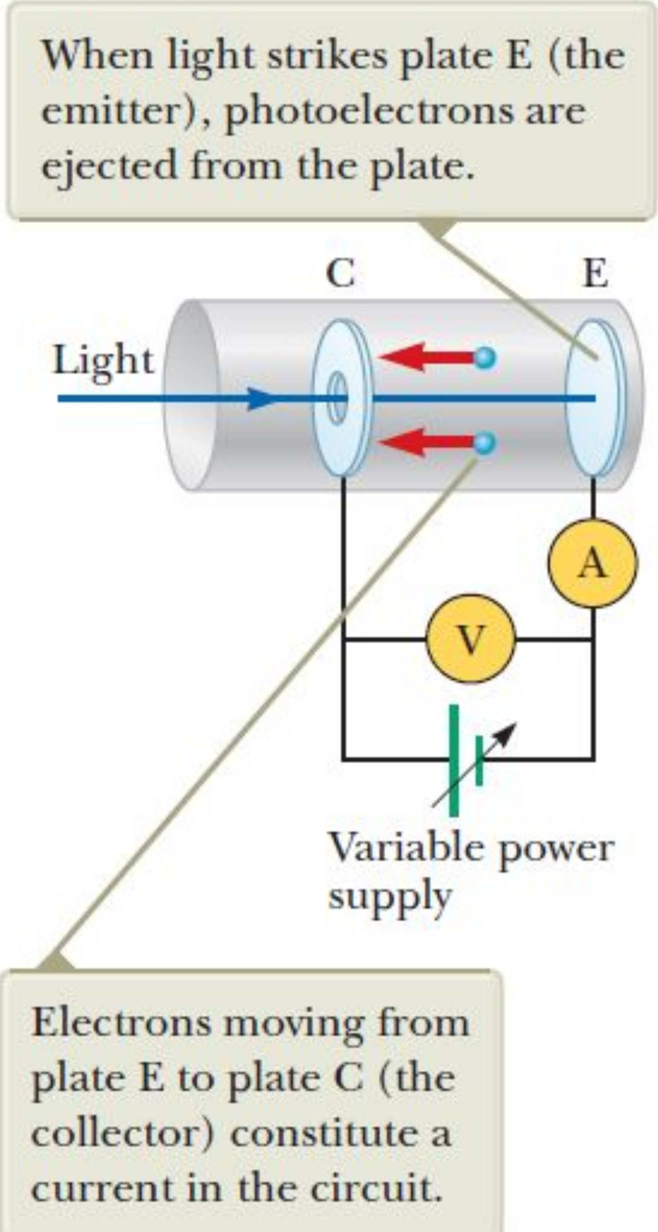


At long wavelengths, there is a small separation between energy levels, leading to a high probability of excited states and many downward transitions. The low energy in each transition leads to low intensity.

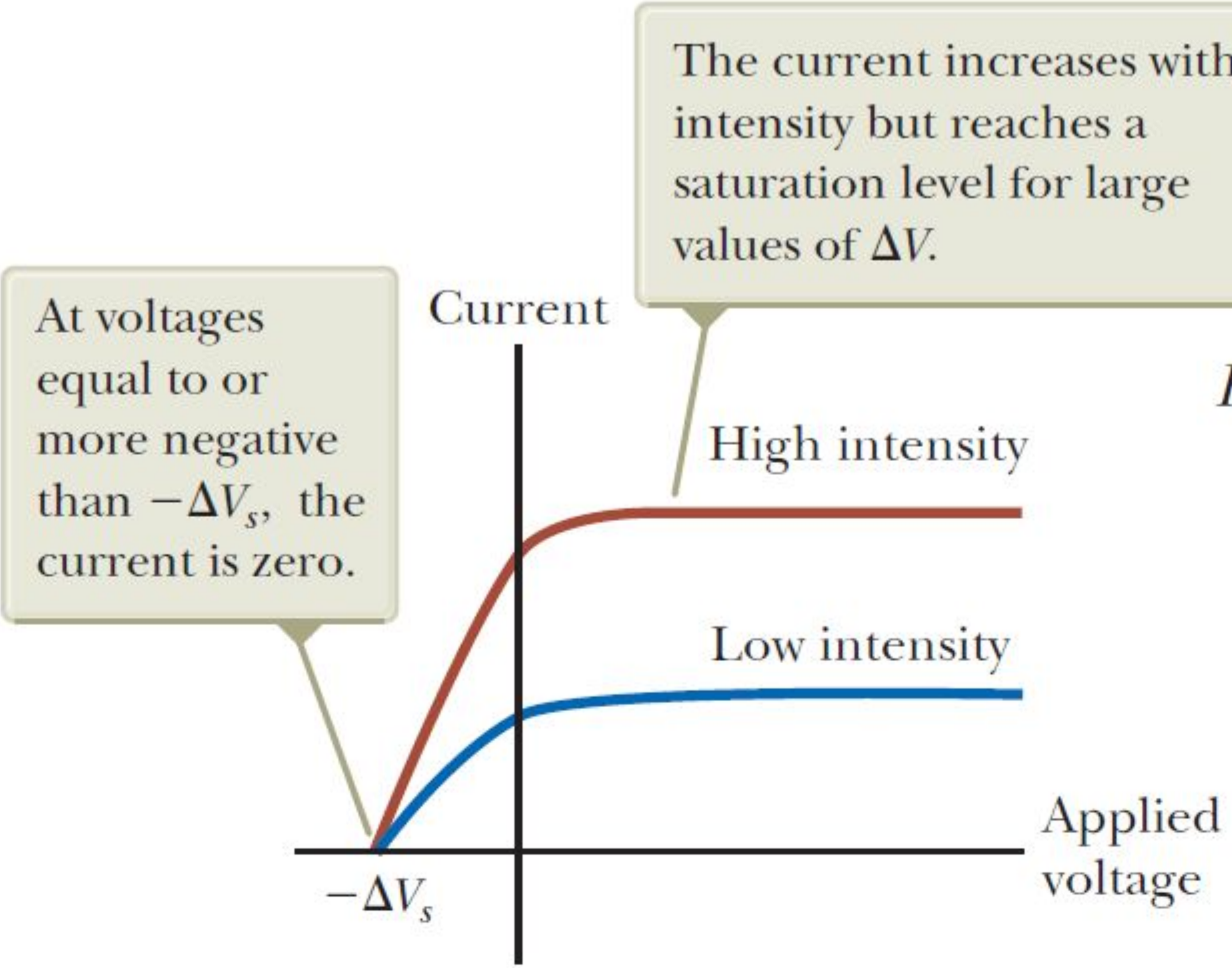


The Photoelectric Effect

Blackbody radiation was the first phenomenon to be explained with a quantum model. In the latter part of the 19th century, at the same time that data were taken on thermal radiation, experiments showed that light incident on certain metallic surfaces causes electrons to be emitted from those surfaces. This phenomenon is known as the **photoelectric effect**, and the emitted electrons are called **photoelectrons.**



The Photoelectric Effect



$$K_i + U_i = K_f + U_f$$

$$K_i + 0 = 0 + (-e)(-\Delta V)$$

$$K_{max} = e\,\Delta V_s$$

ΔV_s is the **stopping potential**

The Photoelectric Effect

In Einstein's model of the photoelectric effect, a photon of the incident light gives all its energy hf to a single electron in the metal. Therefore, the absorption of energy by the electrons is not a continuous absorption process as envisioned in the wave model; rather, it is a discontinuous process in which energy is delivered to the electrons in discrete bundles. The energy transfer is accomplished via a one photon–one electron event. Electrons ejected from the surface of the metal and not making collisions with other metal atoms before escaping possess the maximum kinetic energy K_{max}. According to Einstein, the maximum kinetic energy for these liberated electrons is

$$\boxed{K_{\max} = hf - \phi}$$

where ϕ is called the **work function of the metal.** The work function ϕ represents the minimum energy with which an electron is bound in the metal and is on the order of a few electron volts.

The Photoelectric Effect

Rearranging the equation:

$$K_{\max} + \phi = hf$$

$$\lambda_c = \frac{c}{f_c} = \frac{c}{\phi/h} = \frac{hc}{\phi}$$

$$f_c = \phi/h$$

cutoff wavelength λ_c

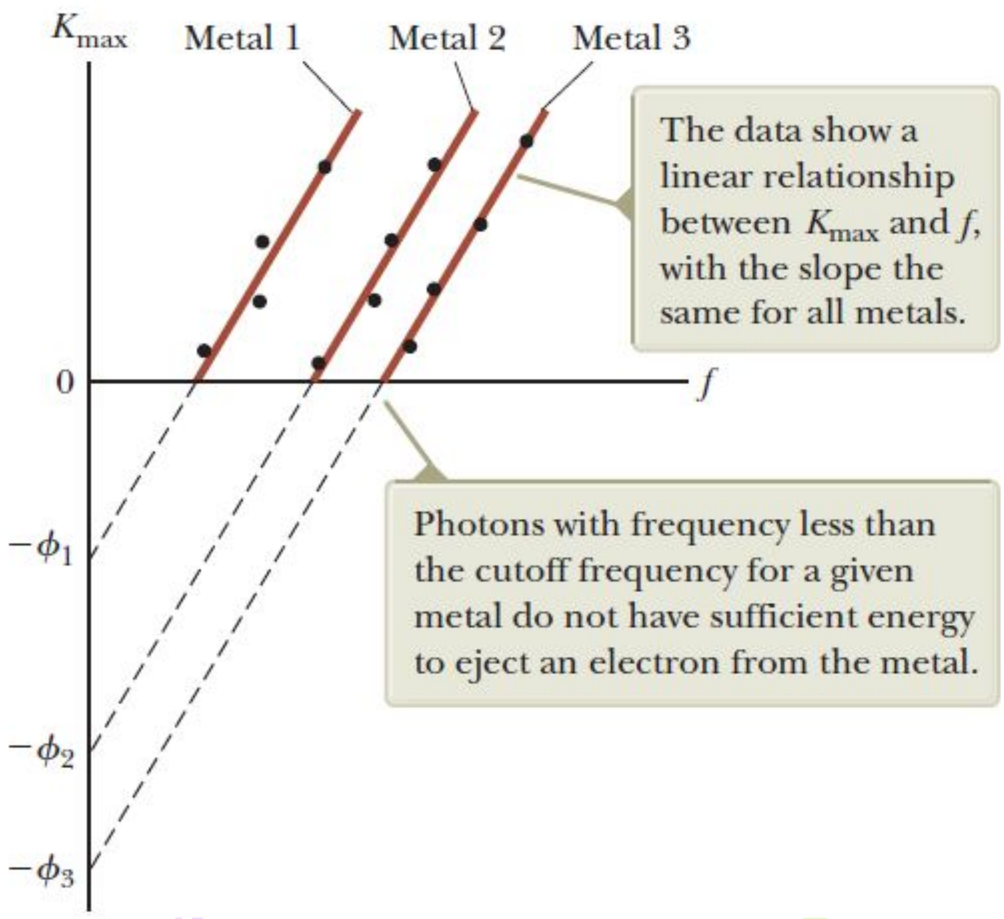
cutoff frequency f_c,

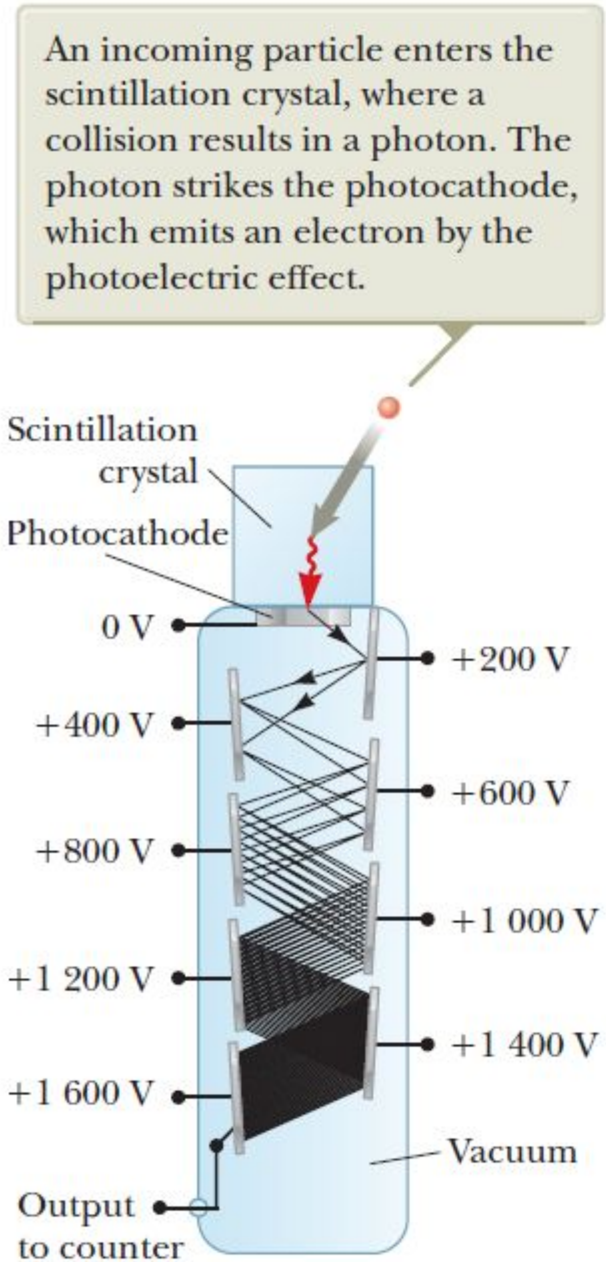
Work Functions of Selected Metals

Metal	ϕ (eV)
Na	2.46
Al	4.08
Fe	4.50
Cu	4.70
Zn	4.31
Ag	4.73
Pt	6.35
Pb	4.14

Einstein assumed light (or any other electromagnetic wave) of frequency f from any source can be considered a stream of quanta. Today we call these quanta **photons.**

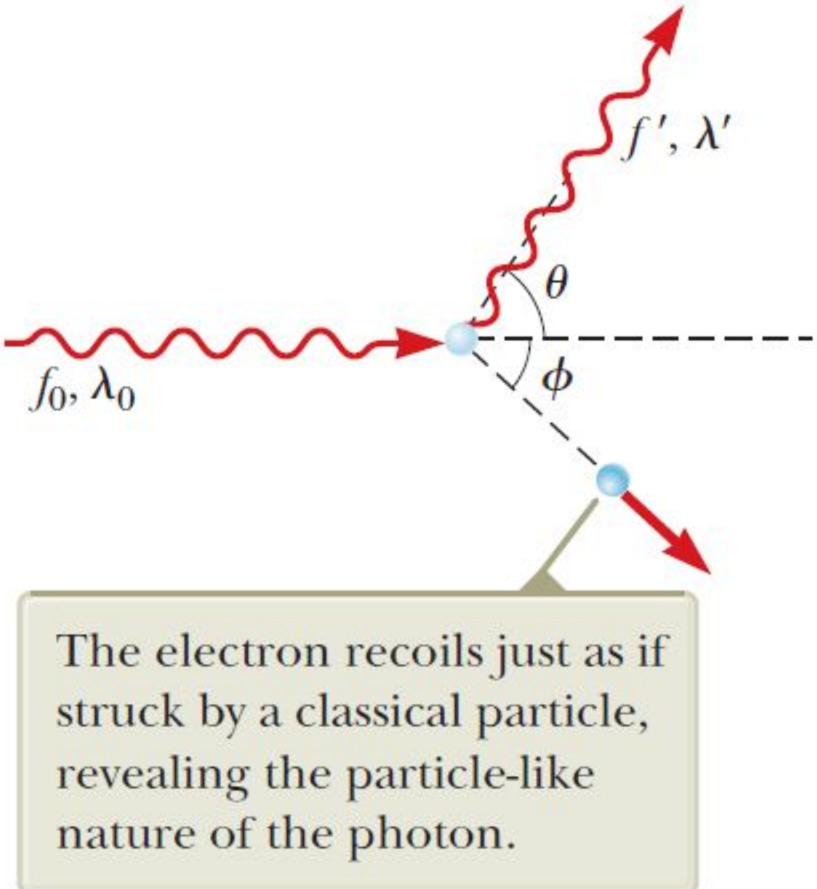
The Photoelectric Effect





$$hc = 1\,240 \text{ eV} \cdot \text{nm}$$

The Compton Effect

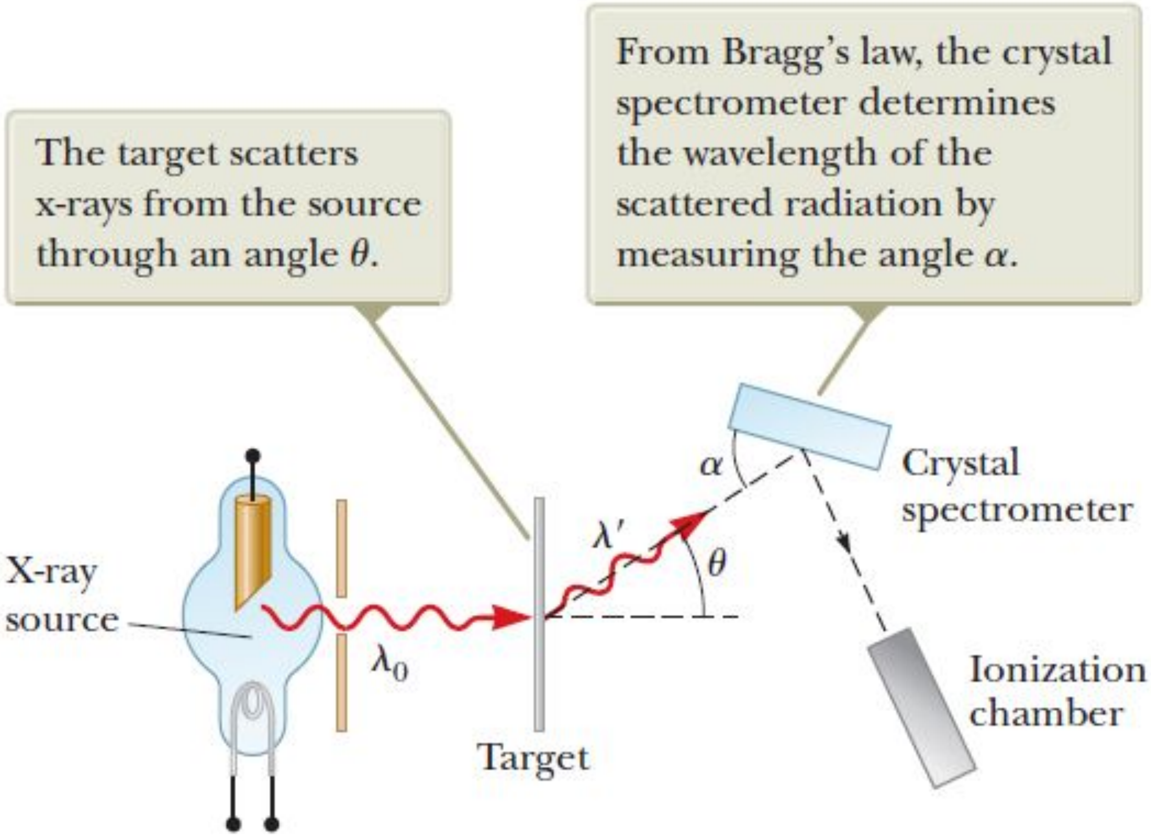


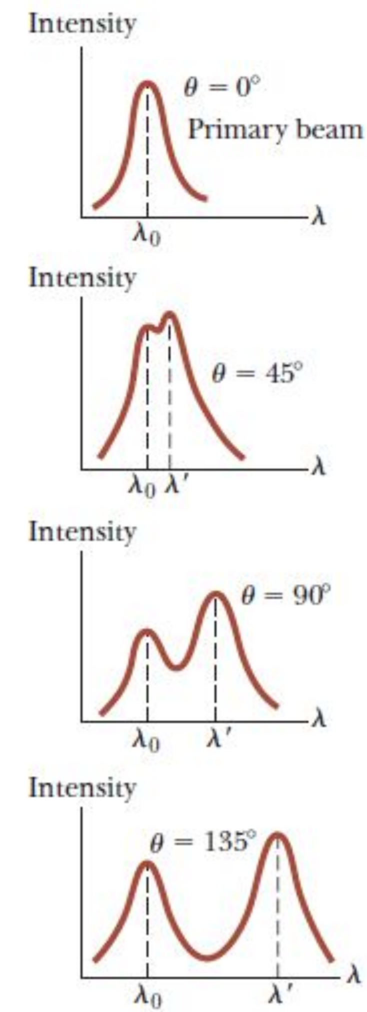
$$E = hf \quad p = E/c$$

$$p = hf/c \quad p_e = \gamma m_e u$$

Because different electrons move at different speeds after the interaction, depending on the amount of energy absorbed from the electromagnetic waves, the scattered wave frequency at a given angle to the incoming radiation should show a distribution of Doppler-shifted values. Contrary to this prediction, Compton's experiments showed that at a given angle only one frequency of radiation *is* observed. Compton and his coworkers explained these experiments by treating photons not as waves but rather as point-like particles having energy *hf* and momentum *hf/c* and by assuming the energy and momentum of the isolated system of the colliding photon–electron pair are conserved. Compton adopted a particle model for something that was well known as a wave, and today this scattering phenomenon is known as the **Compton effect.**

The Compton Effect





$$\boxed{\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)}$$

the Compton shift equation

$$\boxed{\lambda_C = \frac{h}{m_e c} = 0.002\ 43\ \text{nm}}$$

the **Compton wavelength of the electron**

The Compton Effect

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + K_e \qquad K_e = (\gamma - 1)m_e c^2$$

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + (\gamma - 1)m_e c^2$$

$$\gamma = 1/\sqrt{1 - (u^2/c^2)}$$

$$E = hf.$$

$$p = hf/c$$

$$p_e = \gamma m_e u$$

$$\boxed{x \text{ component:} \quad \frac{h}{\lambda_0} = \frac{h}{\lambda'} \cos\theta + \gamma m_e u \cos\phi}$$

$$\boxed{y \text{ component:} \quad 0 = \frac{h}{\lambda'} \sin\theta - \gamma m_e u \sin\phi}$$

The Nature of Electromagnetic Waves

Phenomena such as the photoelectric effect and the Compton effect offer ironclad evidence that when light (or other forms of electromagnetic radiation) and matter interact, the light behaves as if it were composed of particles having energy hf and momentum h/λ. How can light be considered a photon (in other words, a particle) when we know it is a wave? On the one hand, we describe light in terms of photons having energy and momentum. On the other hand, light and other electromagnetic waves exhibit interference and diffraction effects, which are consistent only with a wave interpretation.

The Nature of Electromagnetic Waves

Some experiments can be explained either better or solely with the photon model, whereas others are explained either better or solely with the wave model. We must accept both models and admit that the true nature of light is not describable in terms of any single classical picture. The same light beam that can eject photoelectrons from a metal (meaning that the beam consists of photons) can also be diffracted by a grating (meaning that the beam is a wave). In other words, the particle model and the wave model of light complement each other.

The Wave Properties of Particles

Every large-scale observation can be interpreted by considering either a wave explanation or a particle explanation, but in the world of photons and electrons, such distinctions are not as sharply drawn.

Even more disconcerting is that, under certain conditions, the things we unambiguously call "particles" exhibit wave characteristics. In 1923 Louis de Broglie postulated that because photons have both wave and particle characteristics, perhaps all forms of matter have both properties. This highly revolutionary idea had no experimental confirmation at the time. According to de Broglie, electrons, just like light, have a dual particle–wave nature.

the **de Broglie wavelength of that particle**

$$p = \frac{h}{\lambda} \qquad \lambda = h/p \qquad p = mu \qquad \boxed{\lambda = \frac{h}{p} = \frac{h}{mu}}$$

The Wave Properties of Particles

Furthermore, in analogy with photons, de Broglie postulated that particles obey the Einstein relation $E=hf$, where E is the total energy of the particle. The frequency of a particle is then

$$f = \frac{E}{h}$$

The **principle of complementarity** states that

the wave and particle models of either matter or radiation complement each other.

Neither model can be used exclusively to describe matter or radiation adequately. Because humans tend to generate mental images based on their experiences from the everyday world (baseballs, water waves, and so forth), we use both descriptions in a complementary manner to explain any given set of data from the quantum world.

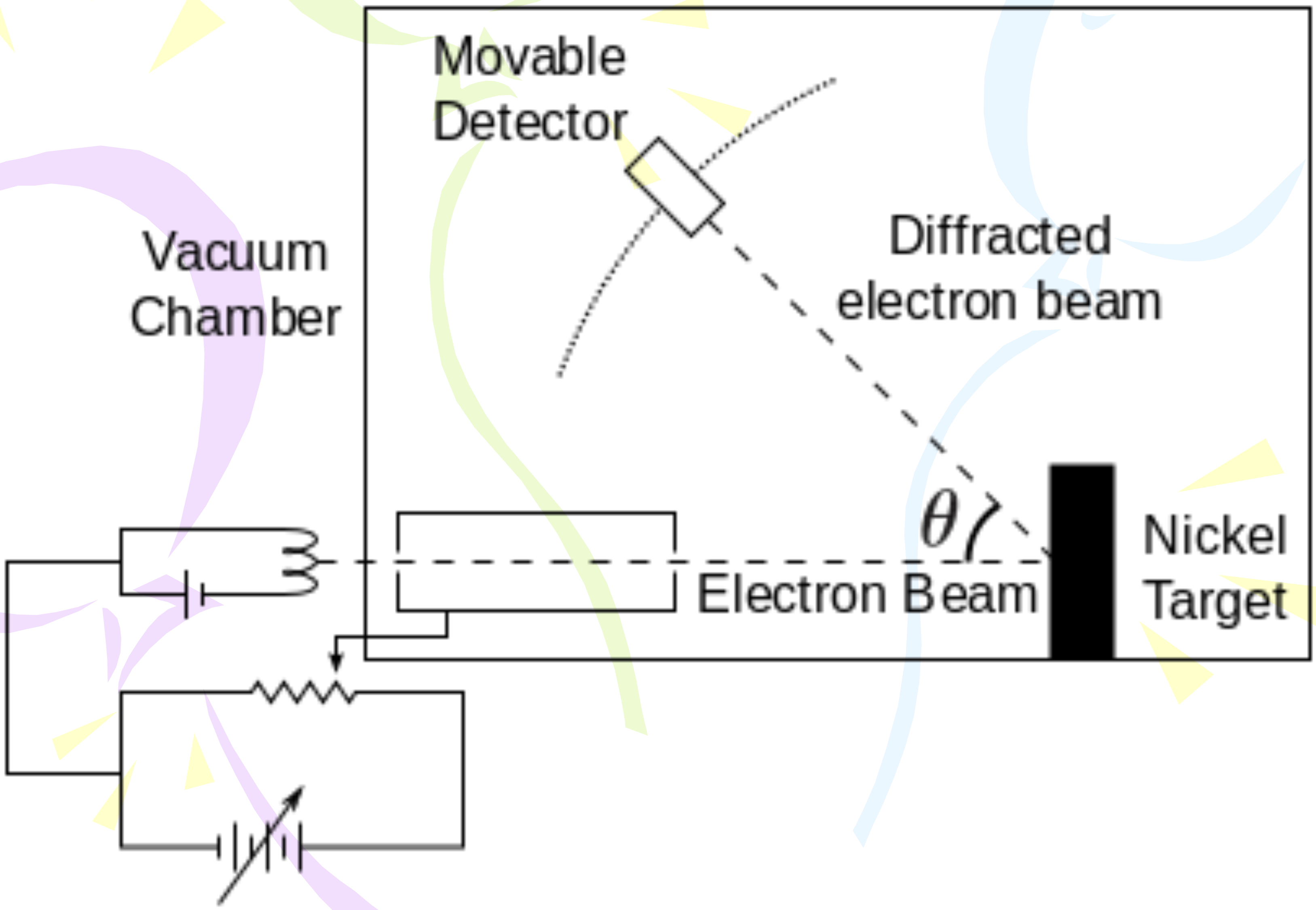
The Wave Properties of Particles

The Davisson–Germer Experiment

The experiment involved the scattering of low-energy electrons (approximately 54 eV) from a nickel target in a vacuum. During one experiment, the nickel surface was badly oxidized because of an accidental break in the vacuum system. After the target was heated in a flowing stream of hydrogen to remove the oxide coating, electrons scattered by it exhibited intensity maxima and minima at specific angles. The experimenters finally realized that the nickel had formed large crystalline regions upon heating and that the regularly spaced planes of atoms in these regions served as a diffraction grating for electrons.

The Wave Properties of Particles

The Davisson–Germer Experiment



The Wave Properties of Particles

The Davisson–Germer Experiment

Shortly thereafter, Davisson and Germer performed more extensive diffraction measurements on electrons scattered from single-crystal targets. Their results showed conclusively the wave nature of electrons and confirmed the de Broglie relationship $p=h/l$. In the same year, G. P. Thomson (1892–1975) of Scotland also observed electron diffraction patterns by passing electrons through very thin gold foils. Diffraction patterns have since been observed in the scattering of helium atoms, hydrogen atoms, and neutrons. Hence, the wave nature of particles has been established in various ways.

The Wave Properties of Particles

The Electron Microscope

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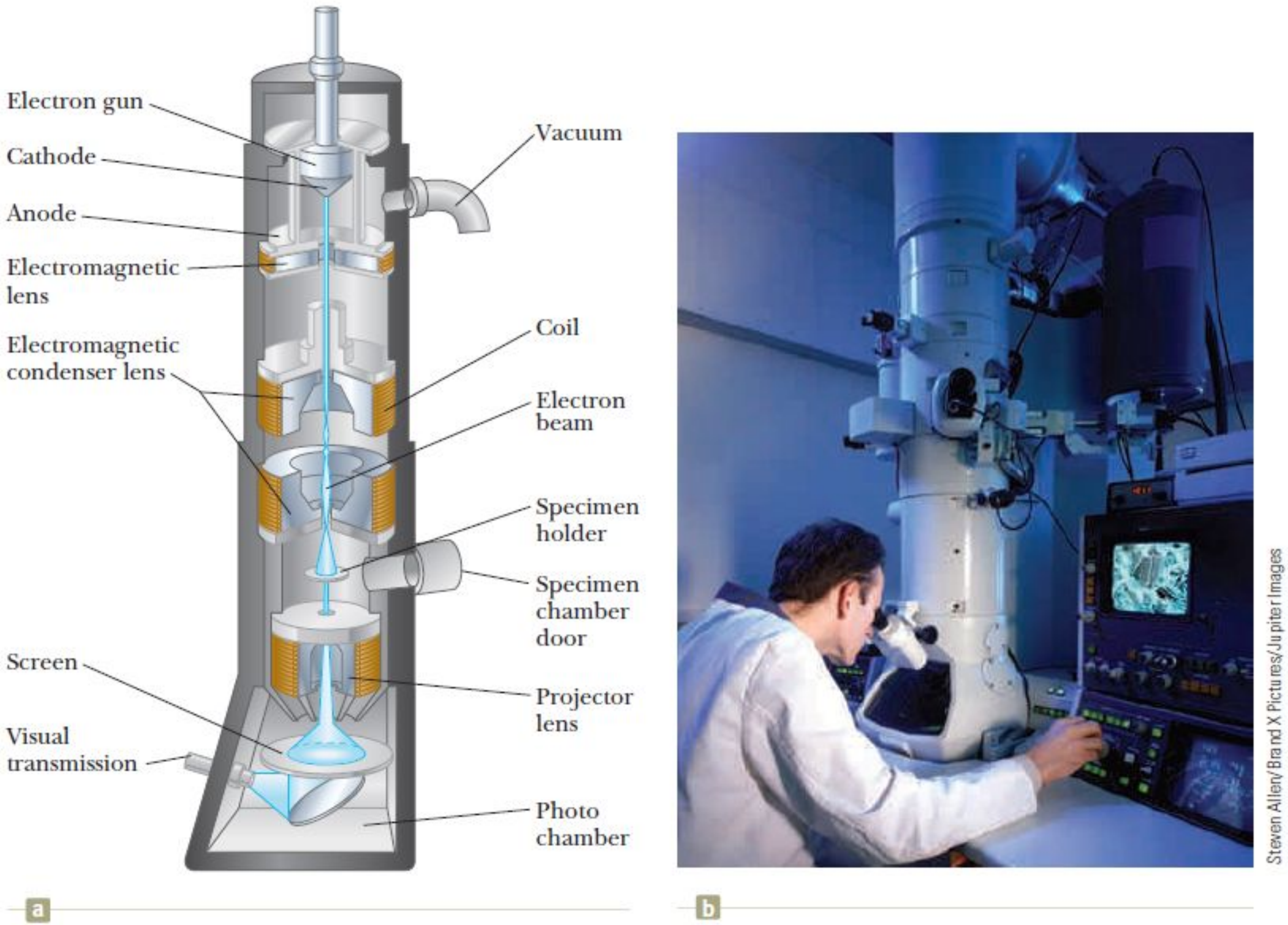
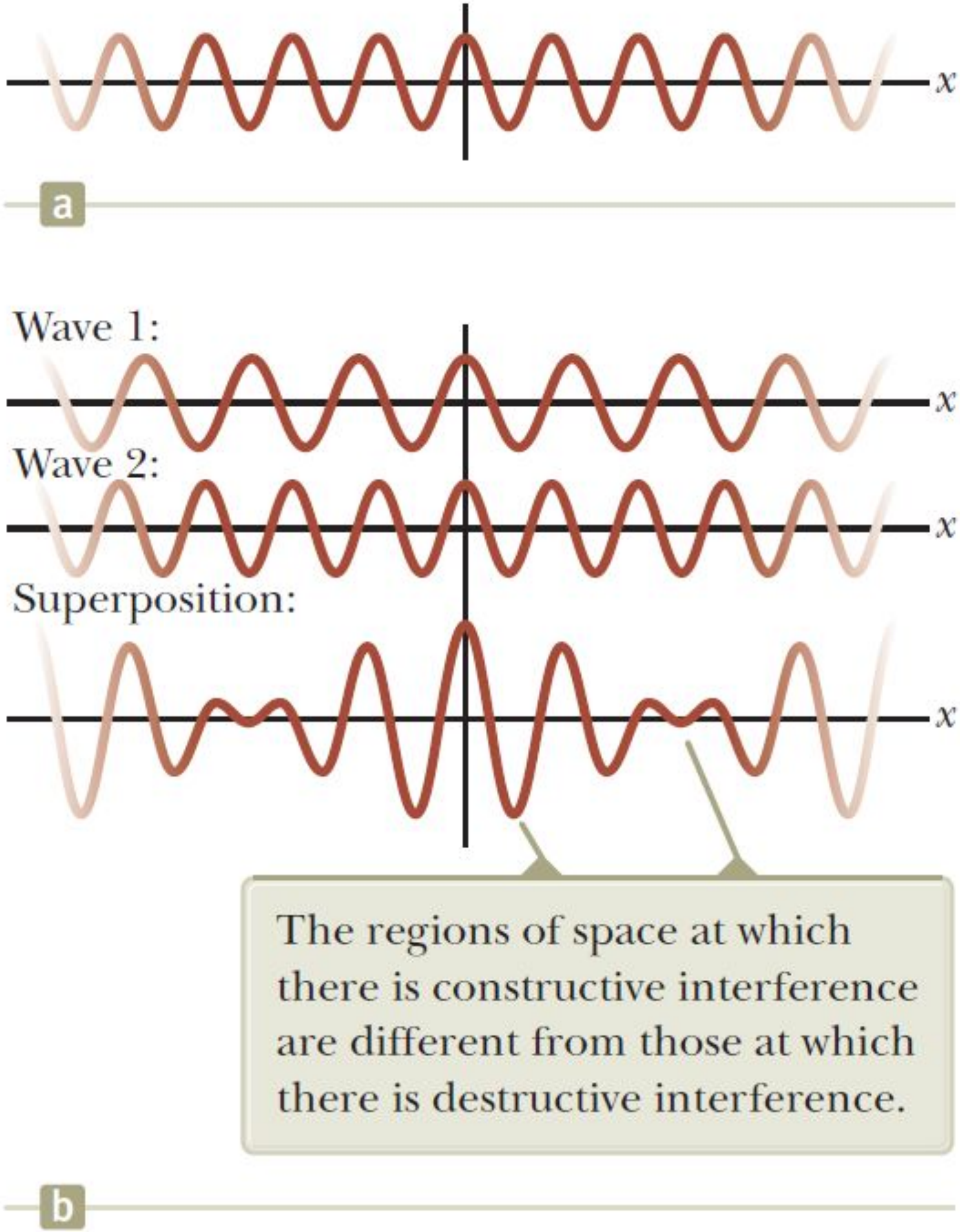


A New Model: The Quantum Particle

Because in the past we considered the particle and wave models to be distinct, the discussions presented in previous sections may be quite disturbing. The notion that both light and material particles have both particle and wave properties does not fit with this distinction. Experimental evidence shows, however, that this conclusion is exactly what we must accept. The recognition of this dual nature leads to a new model, the **quantum particle**, which is a combination of the particle model and the wave model. In this new model, entities have both particle and wave characteristics, and we must choose one appropriate behavior—particle or wave—to understand a particular phenomenon.

We shall explore this model in a way that might make you more comfortable with this idea. We shall do so by demonstrating that an entity that exhibits properties of a particle can be constructed from waves.

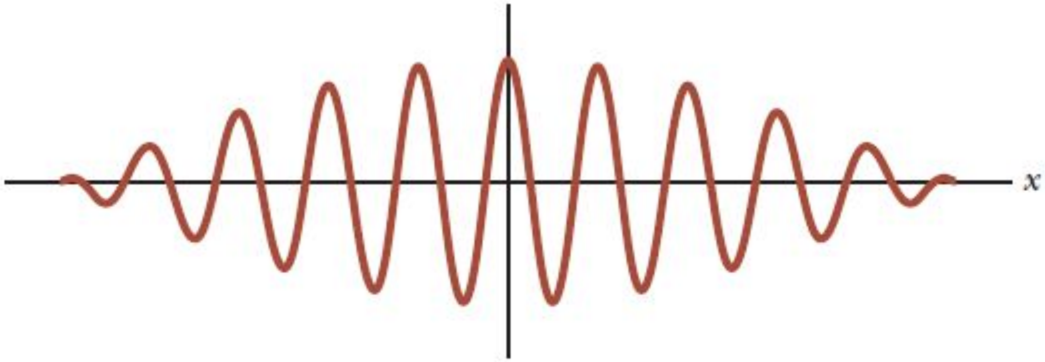
A New Model: The Quantum Particle



Let's first recall some characteristics of ideal particles and ideal waves. An ideal particle has zero size. Therefore, an essential feature of a particle is that it is *localized* in space.

An ideal wave has a single frequency and is infinitely long. Therefore, an ideal wave is *unlocalized in space.*

A New Model: The Quantum Particle



If a large number of waves are combined, the result is a **wave packet**, which represents a particle.

ω_1 ω_2

We can identify the wave packet as a particle because it has the localized nature of a particle!

$$y_1 = A\cos(k_1x - \omega_1 t) \quad \text{and} \quad y_2 = A\cos(k_2x - \omega_2 t)$$

$$k = 2\pi/\lambda \text{ and } \omega = 2\pi f.$$

$$y = y_1 + y_2 = A\cos(k_1x - \omega_1 t) + A\cos(k_2x - \omega_2 t)$$

$$\cos a + \cos b = 2\cos\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$$

A New Model: The Quantum Particle

$$a = k_1 x - \omega_1 t \text{ and } b = k_2 x - \omega_2 t$$

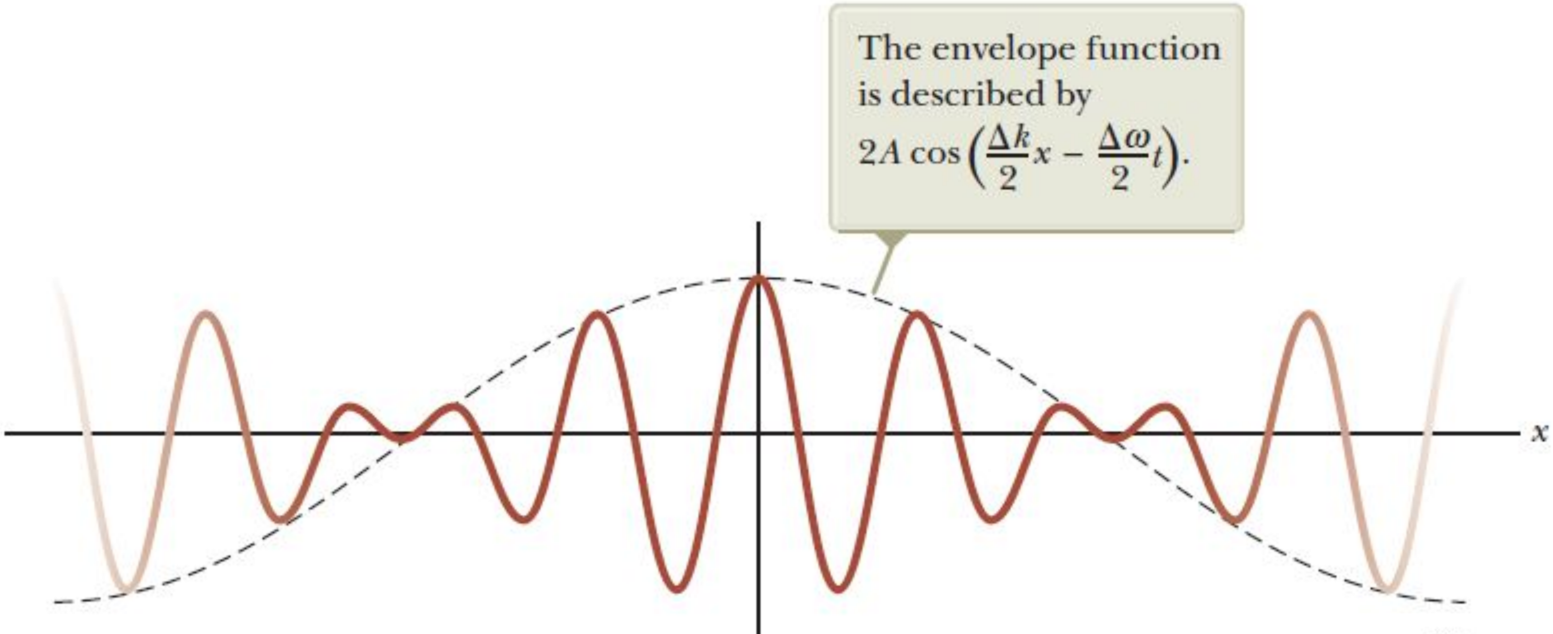
$$y = 2A \cos\left[\frac{(k_1 x - \omega_1 t) - (k_2 x - \omega_2 t)}{2}\right] \cos\left[\frac{(k_1 x - \omega_1 t) + (k_2 x - \omega_2 t)}{2}\right]$$

$$\boxed{y = \left[2A \cos\left(\frac{\Delta k}{2} x - \frac{\Delta \omega}{2} t\right)\right] \cos\left(\frac{k_1 + k_2}{2} x - \frac{\omega_1 + \omega_2}{2} t\right)}$$

where $\Delta k = k_1 - k_2$ and $\Delta\omega = \omega_1 - \omega_2$.

The second cosine factor represents a wave with a wave number and frequency that are equal to the averages of the values for the individual waves.

A New Model: The Quantum Particle



For an individual wave, the speed is given by $v_{\text{phase}} = \dfrac{\omega}{k}$

This speed is called the **phase speed** because it is the rate of advance of a crest on a single wave, which is a point of fixed phase. This equation can be interpreted as follows: the phase speed of a wave is the ratio of the coefficient of the time variable *t* to the coefficient of the space variable x in the equation representing the wave, *y=Acos(kx-vt)*.

A New Model: The Quantum Particle

$$v_g = \frac{\text{coefficient of time variable } t}{\text{coefficient of space variable } x} = \frac{(\Delta\omega/2)}{(\Delta k/2)} = \frac{\Delta\omega}{\Delta k}$$

The **group speed,** or the speed of the wave packet (the *group* of waves) we have built.

When a large number of waves are superposed to form a wave packet, this ratio becomes a derivative:

$$v_g = \frac{d\omega}{dk}$$

$$\hbar = h/2\pi$$

$$v_g = \frac{\hbar\, d\omega}{\hbar\, dk} = \frac{d(\hbar\,\omega)}{d(\hbar\, k)}$$

A New Model: The Quantum Particle

$$\hbar\,\omega = \frac{h}{2\pi}(2\pi f) = hf = E$$

$$\hbar\,k = \frac{h}{2\pi}\left(\frac{2\pi}{\lambda}\right) = \frac{h}{\lambda} = p$$

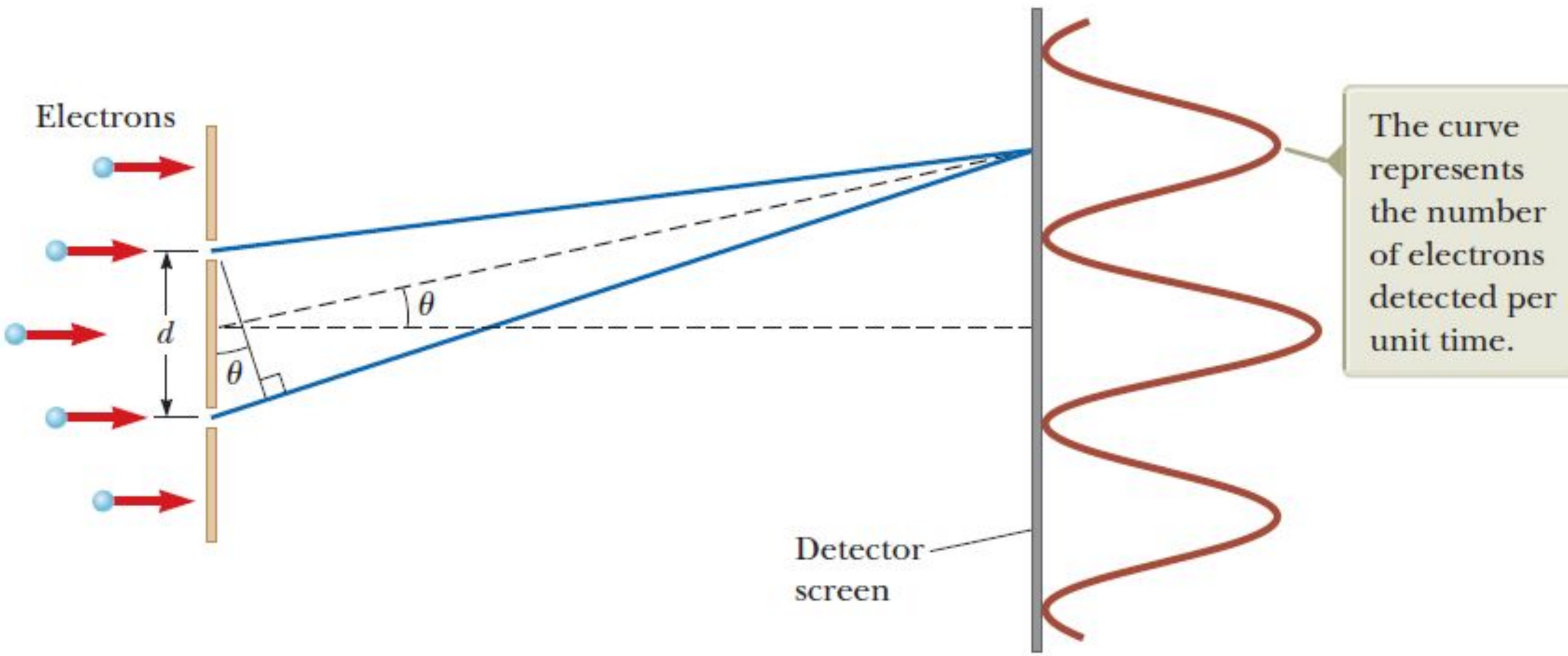
$$v_g = \frac{d(\hbar\,\omega)}{d(\hbar\,k)} = \frac{dE}{dp}$$

$$E = \tfrac{1}{2}mu^2 = \frac{p^2}{2m}$$

The group speed of the wave packet is identical to the speed of the particle that it is modeled to represent, giving us further confidence that the wave packet is a reasonable way to build a particle.

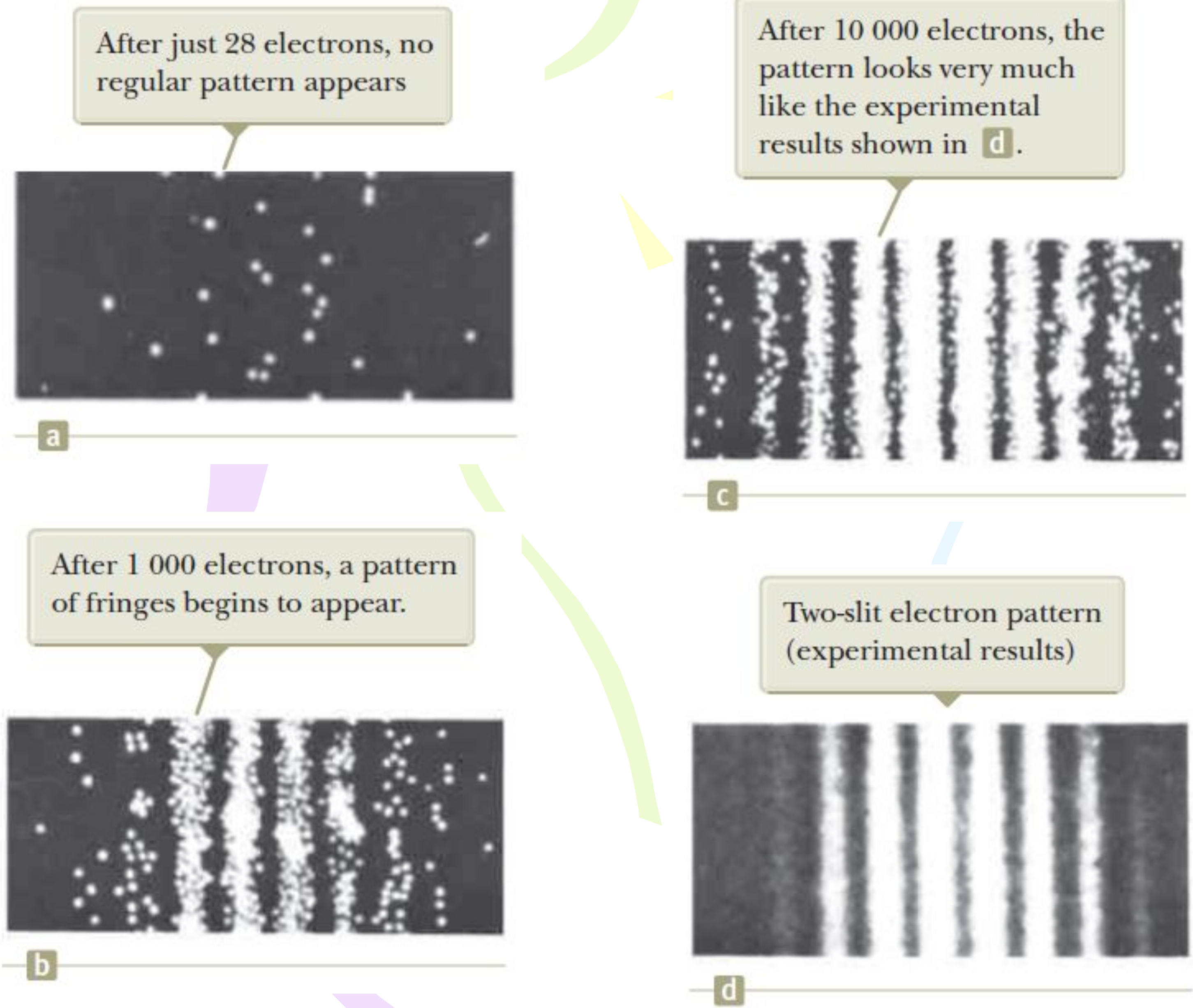
$$v_g = \frac{dE}{dp} = \frac{d}{dp}\left(\frac{p^2}{2m}\right) = \frac{1}{2m}(2p) = u$$

The Double-Slit Experiment Revisited



The dual nature of the electron is clearly shown in this experiment: the electrons are detected as particles at a localized spot on the detector screen at some instant of time, but the probability of arrival at that spot is determined by finding the intensity of two interfering waves.

The Double-Slit Experiment Revisited



The Uncertainty Principle

Whenever one measures the position or velocity of a particle at any instant, experimental uncertainties are built into the measurements. According to classical mechanics, there is no fundamental barrier to an ultimate refinement of the apparatus or experimental procedures. In other words, it is possible, in principle, to make such measurements with arbitrarily small uncertainty. Quantum theory predicts, however, that it is fundamentally impossible to make simultaneous measurements of a particle's position and momentum with infinite accuracy.

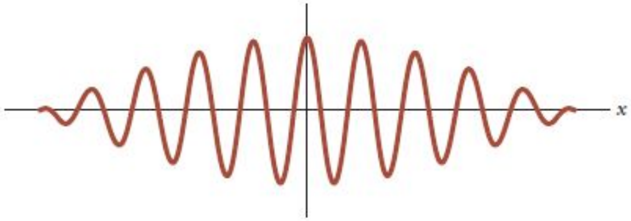
The Uncertainty Principle

In 1927, Werner Heisenberg (1901–1976) introduced this notion, which is now known as the **Heisenberg uncertainty principle:**

If a measurement of the position of a particle is made with uncertainty *Δx* and a simultaneous measurement of its *x* component of momentum is made with uncertainty *Δpx*, the product of the two uncertainties can never be smaller than ħ/2:

$$\boxed{\Delta x \, \Delta p_x \geq \frac{\hbar}{2}}$$

The Uncertainty Principle



Imagine that the horizontal axis is time rather than spatial position *x*. We can then make the same arguments that were made about knowledge of wavelength and position in the time domain. The corresponding variables would be frequency and time. Because frequency is related to the energy of the particle by $E=hf$, the uncertainty principle in this form is

$$\boxed{\Delta E\,\Delta t \geq \frac{\hbar}{2}}$$

The form of the uncertainty principle given in this equation suggests that energy conservation can appear to be violated by an amount ΔE as long as it is only for a short time interval Δt consistent with that equation.