Reznichenko Valery
Organization of data and knowledge bases

Lecture 7. Relational calculus

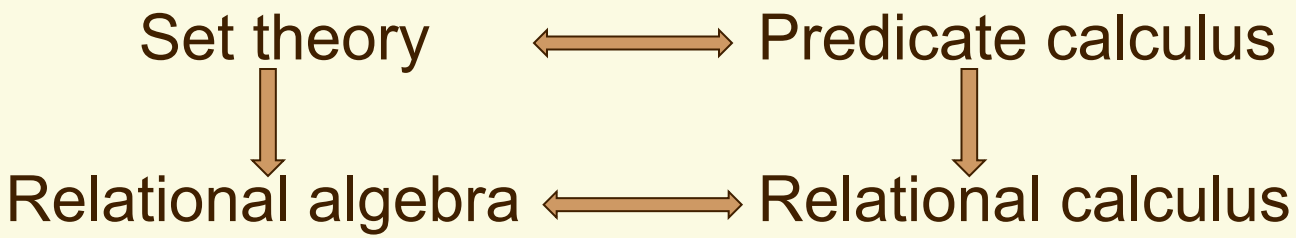
National Aviation University
Computer Science Faculty
Department of Software Engineering

CONTENTS

- Relational calculus (tuple, domain)
- Codd calculus
- ALPHA language
- Equivalence and completeness
- Examples

Set theory and logic

Set theory (relations)	Predicate calculus	Relationships
Set M	P(x); P – predicate symbol x – variable	$x \in M \Leftrightarrow P(x) = t$
M θ N θ = {∪, ∩, □, −}	P(x) θ Q(y) θ = {∨, ∧, ¬, →}	$x \in M\ \theta\ N \Leftrightarrow P(x)\ \theta\ Q(y) = t$



Example of ST and PC relationships:

$T(A) = R(A) \boxtimes S(A) = \{t \mid t \in R \vee t \in S\}$

Relational calculus

- Subset of formulas of the predicate calculus
- Formal description of WHAT it is necessary to select from DB. Example:

  NL - “Output faculties with Fund > 10000"

  RL - **{t | (FAC(t) & t.fund > 10000}**

- It has possibility of query languages but does not have data manipulation possibilities (just the same as relational algebra)

Two variants of relational calculus:

- **Tuple relational calculus** (TRC) – variables represent rows of relations
- **Domain relational calculus** (DRC) - variables represent domains of attributes of relations

Tuple relational calculus (TRC)

Query (in simple case) has the form **{t | (F(t)}**

- t - tuple variable;
- F(t) – formula that contains tuple variable t

Answer. Is the set of all tuples t for which the formula F(t) evaluates to *true*.

Formula. It is recursively defined from simple *atomic formulas* (get tuples from relations or make comparison of values) and build more complex formulas using the logical connectives and quintifiers.

SQL. It is formal foundation of SQL language.

TRC – basic components

- **Constants:** 7, 'john', 3.14159 and so on.
- **Tuple variables:** t1, t2,... –
  They are interpreted by relation tuples.
- **Projection of tuple variables:** t.a,…
  a – attribute name.
- **Predicate symbols:** P, P1,...,Q, Q1,…
  They are interpreted by relations
- **Comparison operators (predicates) :**
  = {=, <, <=, >, >=, <>} и т.д.
- **Logical connectives:** ∨ (or), ∧(&) (and), ¬ (not), → (implies)
- **Quantifiers:** ∃(exists), ∀(for all)

TRC – Well formed formulas

Atomic formulas:

- **P(t)** - **P** – predicate symbol,
  - **t** - tuple variable.
    If **P** interpreted by relation **R** then
    P(t) means **t ∈ R**
- **t.a θ t.b** - **t.a** and **t.b** – tuple projections
- **t.a θ c** - **t.a** – tuple projection, **c** - constant

Well formed formulas (wff):

- Atomic formulas are wff;
- If **F** is wff then **¬F** and **(F)** are wff
- If **F** and **G** are wff then **F ∨ G, F ∧ G, F → G** are wff
- If **F** is wff with free variable t then **∃tF(t)** and **∀tF(t)** are wff with bound variable t**.**

TRC – Free and bound variables. Queries.

The use of quantifiers $\exists tF(t)$ and $\forall tF(t)$ in any formula is said to bind t in the formula F. A variable that is not bound is ***free***.

Query – in general case it is espression:

{(t1,t2,...) | (F(t1,t2,...)}

where - t1,t2,... - tuple variables or their projections;

- F(t1,t2,...) - formula that contains free tuple variables t1,t2,...

IMPOPRTANT RESTRICTION: The variable t1,t2,..., that appears to the left of '|', must be the only free variables in the formula F. In other words all other tuple variables of the formula F must be bound by using quantifiers.

Example of DB for queries in TRC

FAC	FACULTY (FNo, Name, Dean, Bld, Fund)
DEP	DEPARTMENT (DNo, FNo, Name, Head, Bld, Fund)
TCH	TEACHER(TNo, DNo, Name, Post, Tel, Salary, Comm)
GRP	GROUP(GNo, DNo, Course, Num, Quantity, CurNo)
SBJ	SUBJECT(SNo, Name)
ROM	ROOM (RNo, Num, Building, Seats)
LEC	LECTURE (TNo, GNo, SNo, RNo, Type, Day, Week)

Predicate symbols

Relations of database

TRC – Examples of projection, selection and join

1) Projection

Query. Output faculty names and deans

{(f.Name, f.Dean)|FAC(f)}

2) Selection and projection

Query. Output name and post of teachers with salary > 1000 and commission > 500

{(t.Name, t.Post) | TCH(t) & t.Salary > 1000 & t.Comm > 500}

3) Join, selection and projection

Query. Output names of faculties and their departments for faculties in the building5

{(f.Name,d.Name) | FAC(f) & DEP(d) & f.FNo = d.FNo & f.Bld = 5}

TRC – Examples of existential quantifiers

Query. Output names of faculties in the building 5 and their departments

$$\{(f.Name, d.Name) \mid DEP(d) \,\&\, FAC(f) \,\&\, f.FNo = d.FNo \,\&\, f.Bld = 5\}$$

Query. Output department names of faculties in the building 5

$$\{d.Name \mid DEP(d) \,\&\, \exists f(FAC(f) \,\&\, f.FNo = d.FNo \,\&\, f.Buld = 5)\}$$

Query. Outputs names of teacher-professors that have lectures in groups of the 1-st course

$$\{t.Name \mid TCH(t) \,\&\, t.Post='professor' \,\&\, \exists l(LEC(l) \,\&\, l.TNo = t.Tno \,\&\, \exists g(GRP(g) \,\&\, l.GNo = g.GNo \,\&\, g.Course = 1))\}$$

Output names of professors,
for which exist such lectures,
for which (lectures) exist groups of the 1-st course

GRP GROUP(GNo, Course...) TCH TEACHER (TNo, Name, Post...)

LEC LECTURE(GNo,TNo...)

TRC – Examples of universal quantifiers

Query. Output numbers of the teachers that have lectures in all groups

$$\{l.TNo \mid LEC(l) \;\&\; \forall g(GRP(g) \rightarrow g.GNo = l.GNo)\}$$

Query. Output numbers of the teachers that have lectures in all groups of the 1-st course.

$$\{l.TNo \mid LEC(l) \;\&\; \forall g((GRP(g) \;\&\; g.Course = 1) \rightarrow g.GNo = l.GNo)\}$$

Query. Output names of the teachers that have lectures in all groups of the 1-st course

$$\{t.Name \mid TCH(t) \;\&\; \exists l(LEC(l) \;\&\; l.TNo = t.TNo \;\&\; \forall g((GRP(g) \;\&\; g.Course = 1) \rightarrow g.GNo = l.GNo)\}$$

Output names of teachers,

that have such lectures,

that are taught in all groups of the 1-st course

TRC - Save formulas and queries

Exist such syntactically correct queries that do not have correct interpretation in DB. OR, other words, such queries have an infinite number of answers (answer is infinite relation). Such queries are called ***unsafe*** Examples:

{t | t.Post = 'professor'}
{t | ¬TCH(t)}
{(t,d) | TCH(t) ∨ DEP(d)}

Query is ***safe*** if it is interpreted by a finite relation.

Query is safe if:

- All variables in the formula are *restricted*;
- All logical connectives in the formula are *restricted*;
- All quantifiers in the formula are *restricted*.

Restricted variables

Tuple variable is restricted if it belongs to any predicate that is interpreted by DB relation, or it is restricted by other restricted variable or constant.

Variable t is restricted if :

- It belongs to a predicate P(t), where P is interpreted by DB relation;
- It appears in the formula t.a1 = c1 & ... & t.an = cn, where a1,..., an are all attributes of the tuple t, and c1, ..., cn are constants;
- It appears in the formula t = s, where s is restricted variable.

Examples:

P(t) - variable t is restricted by the predicate P
P(t) & t = x - variable x is restricted by variable t that is restricted itself
t.Name='john' & t.Post='prof' & t.Salary = '1200'

Restricted logical connectives

If two formulas F and G have restricted variables then:

- F ∨ G is a formula with restricted variables if F and G have the same free variables;
- F & G will always be the formula with restricted variables in spite of the list of free variables in both formulas;
- F & ¬G is the formula with restricted variables if F and G have the same free variables;
- F → G will never be a formula with restricted varibles.

Examples:

P(t) ∨ Q(t) – it is formula with restricted variable
P(t) ∨ Q(q) – variables in this formula are not restricted
P(t) & Q(q) – it is formula with restricted variable
¬Q(t) – it is not formula with restricted variable
P(t) & ¬Q(q) – it is not formula with restricted variables
P(t) & ¬Q(t) – it is formula with restricted variable

Restricted quantifiers

If F(t) is restricted formula and G(t) is arbitrary formula then the formula **∃t(F(t)& G(t))** is a formula with ***restricted existential quantifier.*** This formula is interpreted by restricted (finite) relation.

If F(t) is restricted formula and G(t) is arbitrary formula then the formula ∀t(F(t) → G(t)) is a formula with ***restricted universal quantifier.*** This formula is interpreted by restricted (finite) relation.

Примеры:

∃x(x.Fund < 5) – existential quantifier is not restricted

∃x(P(x) & x.Fund < 5) – existential quantifier is restricted

∀x(P(x)) → Q(x, y)) – universal quantifier is restricted

Domain Relational Calculus (DRC)

Query has the form {x1,x2,...,xn | (F(x1,x2,...,xn)}

- x1,x2,...,xn - attributes domain variables;
- F(x1,x2,...,xn) – a formula with the only free variables x1,x2,...,xn

Answer. Set of such tuples <c1,c2,...,cn>, that evaluates the formula F to *true* value.

Formula. It is recursively defined by using atomic formulas and more complex formulas with the help of logical connectives and quantifiers **just the same as in tuple relational calculus**.
Safe formulas are defined **just the same as in tuple relational calculus.**

QBE. Is a formal base of QBE language.

Example of queries in DRC

1) Projection

Query. Output faculty names and deans

{(y, z)| ∃x ∃u ∃vFAC(x,y,z,u,v)}

2) Selection and projection

Query. Output faculty names of the building 5 and dean names

{(y, z) | ∃x ∃u ∃vFAC(x,y,z,u,v) & u = 5}

3) Join, selection and projection

Query. Output faculty names of the building 5 and their department names

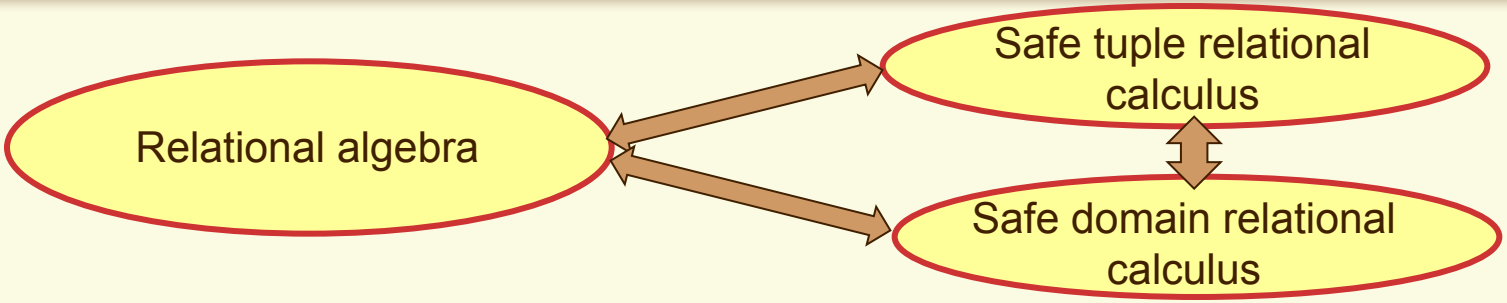
{(y,n)| ∃x ∃f(∃z ∃u ∃v(FAC(x,y,z,u,v) & u=5) & ∃d ∃h ∃b ∃uDEP(d,f,n,h,b,u) & x=f)}

FAC (FNo,Name,Dean,Bld,Fund) **DEP(DNo,FNo,Name,Head,Bld,Fund)**

x y z u v **d f n h b u**

Equivalence of RA, TRC, DRC and relational completeness.

Assertion: Relational algebra, safe TRC and safe DRC are equivalent in their expressive power. It means that every query that can be expressed in relational algebra can also be expressed in TRC and DRC. The converse is also true.



Thesis (relational completeness): Any relational language is relationally complete if it has expressive power (selective possibilities) of relational algebra.

Assertion: SQL language is relationally complete.

Codd relational calculus (CRC)

- CRC is subset of the predicate calculus (of the 1-st order)
- It is tuple oriented
- It solve differences between wff and safe formulas
- CRC queries are safe.
- It is equivalent to the relational algebra; it means that CRC is relationally complete

CRC – basic components

- **Constants:** 7, 'john', 3.14159 and so on.
- **Tuple variables:** t1, t2,... –
  They are interpreted by relation tuples.
- **Projections of tuple variables:** t.[i],…
  i is a component of tuple variable.
- **Predicate symbols:** P, P1,...,Q, Q1,…
  They are interpreted by relations
- **Comparison operators (predicates):**
  = {=, <, <=, >, >=, <>} и т.д.
- **Logical connectives:** ∨ (or), ∧(&) (and), ¬ (not), → (implies)
- **Quantifiers:** ∃(exist), ∀(for all)

CRC – well formed formulas

- Terms:
  - **P.t** – ***value term***: P - predicate, t - tuple variable.
    If P is interpreted by relation R, then P.t means t ∈ R
  - **t[i] θ s[j], t[i] = c** - ***join term***

    Examples: $t_1[3] = t_2[1]$; $t_4[7] = 15$
- Formula well defined over tuple variable:
  - All predicate symbols is interpreted by relations that are union compatible .
  - This formula contains only value terms with the only tuple variable.
  - Value terms are connected by logical operands ∨, &, ¬. More over, operand ¬ directly preceded by operand &.

    Examples:

    $P_1.t \vee P_2.t \vee (P_3.t \,\&\, P_4.t)$; $(P_1.t \vee P_2.t) \,\&\, \neg P_3.t$.

Formula well defined over quantifiers

Lets F is a formula well defined over tuple variable t, a G is a formula that contains t as a free variable, but does not contains value terms of the form P.t. Then formulas:

$$\exists t(F \,\&\, G); \quad \forall t(F \rightarrow G)$$

are called ***well defined over existence and universal quantifies***.

As a matter of fact, it is a special case of the restricted existence and universal quantifiers. Formula F determines interpretation domain of the variable that is used quantifies. We also will use the following notation on these formulas:

$$\exists F(G) \text{ и } \forall F(G)$$

Examples:

$$\exists t((P.t \vee Q.t) \,\&\, \neg S.t) \,\&\, t[2] = \text{‘john’} \,\&\, t[5] = 1000)$$

$$\forall t((P.t \vee Q.t) \,\&\, \neg S.t) \rightarrow (t[2] = \text{‘john’} \,\&\, t[5] = 1000))$$

Formula with domain of definition

Formula W refers to as ***formula with domain of definition*** if it has the following expression :

W = U1 & U2 &…& Un & V, n ≥ 1

and has the following properties:

- Each formula Ui is well defined over tuple variable ti.
- Formula V is empty or has the following properties:
  - If the are quantifies then they are well defined .
  - Each free variable from V belongs to one of the variables of formulas U1, U2,…, Un
  - B Formula V does not have value terms with free variables.

Examples: **P.r & Q.s & R.t** - Cartesian product

P.r & (r[3] = b) - selection

P.r & Q.s & (r [2] = s[1]) - join

Alpha expression

Expression (t1, t2,…, tk) : W is calles ***simple alpha expression*** if it has the following priperties:

- W is a formula with domain of definition.
- t1, t2,…, tk – list of tuple variables and their projections. These variables must be free in the formula W.

Arbitrary ***alpha expression*** is defined recursively in such a way:

- Every simple alpha expression is alpha expression .
- Let us t = (t1, t2,…, tk). If t : W1 и t : W2 is alpha expression then expressions t : W1 ∨ W2, t : W1 & W2 and t : W1 & ¬W2 are alpha expressions.

ALPHA language

Simplified syntax:

RANGE <relation name> <variable name> [SOME | ALL]

…

GET <workspace> (<target list>) : <wff>

range – it shows name of the the tuple variable **<variable name>** of the relation **<relation name>.**

SOME | ALL – is it necessary to interpret variable with existential or universal quantifier?

workspace – it is name of temporary work relation that contains result of executing of GET command.

target list – target list of tuple variables and their projections; they show columns that are projected into the resulting relation.

wff – well wormed formula of the tuple relational calculus

Example of queries in ALPHA and CRC

Query. Output names of faculties and their deans

CRC: {f[2], f[3] | FAC.f}
ALPHA: RANGE FACULTY f
GET W(f.Name, f.Dean)

Query. Output dean name of the CSF faculty

CRC: {f[3] | FAC.f & f[2] = 'CSF'}
ALPHA: RANGE FACULTY f
GET W(f.Dean) : f.Name = 'CSF'

Query. Output department names of the CSF faculty

CRC: {d[3] | DEP.d & ∃f(FAC.f & f[2] = 'CSF' & f[1] = d[2])}
ALPHA: RANGE FACULTY f SOME
RANGE DEPARTMENT d
GET W(d.Name) = f.Name = 'CSF' & f.FNo = d.DNo

Summary

- Relational calculus is declarative (not procedural) language that formulates WHAT it is necessary to receive but not HOW the result should be calculated.
- Safe relational calculus (TRC, DRC, CRC) and relational algebra have the same expressive power and they are used to formulate thesis about relational completeness.
- It is possible to formulate correct but not effective queries. So it is necessary to use optimizers.
- There are languages that implement relational calculus in straight a way (ALPHA, QUEL) and are also based on the tuple calculus (SQL) and the domain calculus (QBE).