Lecture 10
part 2

Slice method

1. Particle method (Lect. 8-9):

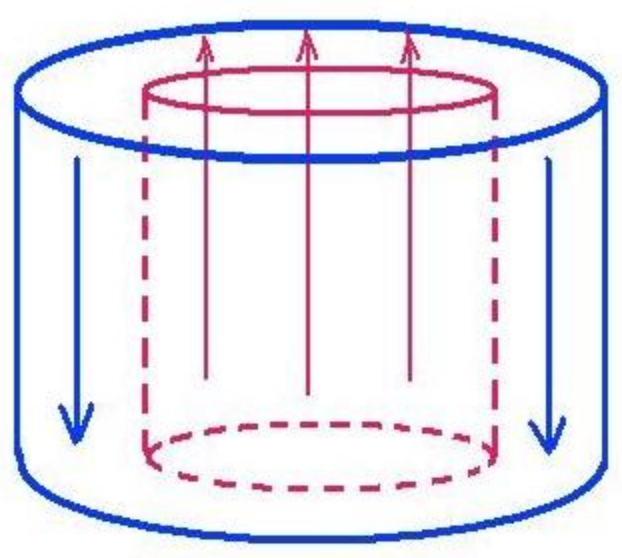
assumption of particle motion within motionless environment.

2. Basic approaches of slice method: 1938-1939 (Bjerknes J.,Petersen S., Shishkin N.)

Let's pick out a sufficiently large layer of air

$S^{'}$ $cross\ section(inner)$

$w^{'} > 0$



z_2 $is\ upper\ layer$

z_1 $is\ lower\ layer$

$S^{''}$ $cross\ section(outer)$

$w^{''} < 0$

Slice method assumptions:

- all variations of the quantities within the picked out layer come out adiabatically;
- no advection (horizontal mass motion) is observed;
- the mass of air above any layer remains unchanged.

The latter means that fluxes of air mass through section S' and S'' are equal:

$$\rho' S' w' = -\rho'' S'' w'' \qquad (10.8)$$

$$\rho \quad is \quad density$$

$$\rho' \approx \rho''$$

$$S' w' = -S'' w'' \qquad (10.9)$$

To derive the stability criterion:

ΔT *is temperature difference,*

$T^{'}$ *is updraft flux temp.,*

$T^{''}$ *is the downdraft one at z level*

$$\Delta T = T^{'} - T^{''}$$

$$\Delta T > 0 \ \ unstable \ \ state(positive \ acceleration)$$

$$\Delta T < 0 \ \ stable \ \ state(negative \ acceleration)$$

Suppose, particles come from lower level to upper one:

$$T' = T_1 - \gamma'(z - z_1) \qquad (10.10)$$

$$T'' = T_2 + \gamma''(z_2 - z) \qquad (10.11)$$

γ' *is adiabatic lapse rate in updraft flux*

γ'' *is adiabatic lapse rate in downdraft flux*

$$z - z_1 = w'\Delta t \qquad (10.12)$$

$$z_2 - z = -w''\Delta t \qquad (10.13)$$

From (10.10)-(10.13) we obtain:

$$\Delta T = T_1 - T_2 - (\gamma^{'} w^{'} - \gamma^{''} w^{''})\Delta t \qquad (10.14)$$

$$T_1 - T_2 = \gamma \ (z_2 - z_1) = \gamma \ (z_2 - z) + \gamma \ (z - z_1) = \gamma \ \Delta t (w^{'} - w^{''})$$

$$\Delta T = \left\lfloor \gamma^{'} (w^{'} - w^{''}) - (\gamma^{'} w^{'} - \gamma^{''} w^{''}) \right\rfloor \Delta t \qquad (10.15)$$

Let’s use (10.9)

$$S' w' = -S'' w'' \qquad (10.9)$$

$$\Delta T = \left\lfloor (\gamma' (w' - w'') - (\gamma' w' - \gamma'' w'') \right\rfloor \Delta t \qquad (10.15)$$

$$\Delta T = \left[(\gamma - \gamma') + \frac{S'}{S''} (\gamma - \gamma'') \right] w' \Delta t \qquad (10.16)$$

Let's consider 3 cases:

1. $\gamma^{'} = \gamma^{''} = \gamma_a$

- Updraft and downdraft air isn't saturated

$$\Delta T = (\gamma - \gamma_a)(1 + \frac{S^{'}}{S^{''}}) w^{'} \Delta t \qquad (10.17)$$

- Criteria of stability of both methods (parcel and slice ones) give the same result.

2nd case:

$$2.\ \gamma' = \gamma'' = \gamma'_a$$

- Updraft and downdraft air fluxes are saturated

$$\Delta T = (\gamma - \gamma_a')(1 + \frac{S'}{S''})w'\Delta t$$

- Criteria of stability of both methods (parcel and slice ones) give the same result.

3^d case:

$$3.\ \gamma' = \gamma'_a,\ \gamma'' = \gamma_a$$

- Updraft air is saturated and downdraft flux is not.

$$\Delta T = \left[(\gamma - \gamma_a') + \frac{S'}{S''}(\gamma - \gamma_a) \right] w' \Delta t \qquad (10.18)$$

- Cu clouds development

- From (10.18) $\Delta T > 0 \quad at \;\; \gamma > \gamma_a$ and

$$\Delta T < 0 \quad at \;\; \gamma < \gamma'_a$$

- Stability criteria of both methods are coincide if the stratification is absolutely unstable and absolutely stable. However, often

$$\gamma'_a < \gamma < \gamma_a$$

$$\Delta T \leftarrow \quad on \;\; \gamma \;\; but \;\; also \;\; S'/S''$$

Let's introduce a *critical lapse rate*

$$\gamma_{cr} \quad at \quad \Delta T = 0$$

$$\left[(\gamma - \gamma_a^{'}) + \frac{S^{'}}{S^{"}} (\gamma - \gamma_a) \right] w^{'} \Delta t = 0$$

$$w^{'} \neq 0, \quad \Delta t \neq 0$$

$$\left[(\gamma - \gamma_a^{'}) + \frac{S^{'}}{S^{"}} (\gamma - \gamma_a) \right] = 0$$

$$\gamma_{cr} = \frac{\gamma^{'}{}_a + \gamma_a \left(S^{'} / S^{"} \right)}{1 + S^{'} / S^{"}} \qquad (10.19)$$

$$\gamma_{cr} \to \gamma'_a \quad at\ small\ \ values\ \ of\ (S'/S'')$$

$$\gamma_{cr} \to \gamma_a \quad at\ large\ \ values\ \ of\ (S'/S'')$$

$$\gamma > \gamma_{cr}\ (\Delta T > 0) \qquad\qquad \gamma < \gamma_{cr}\ (\Delta T < 0)$$

$$\frac{S'}{S''} < \frac{\gamma - \gamma'_a}{\gamma_a - \gamma} \quad at \quad \Delta T > 0 \qquad (10.20)$$

$$\frac{S'}{S''} < \frac{\gamma - \gamma'_a}{\gamma_a - \gamma} \quad at \quad \Delta T < 0 \qquad (10.21)$$

- at conditionally unstable stratification the atmosphere is unstable for **small** size particles and it's stable for LARGE size particles =>>the atmospere is said to be ***selectively unstable***