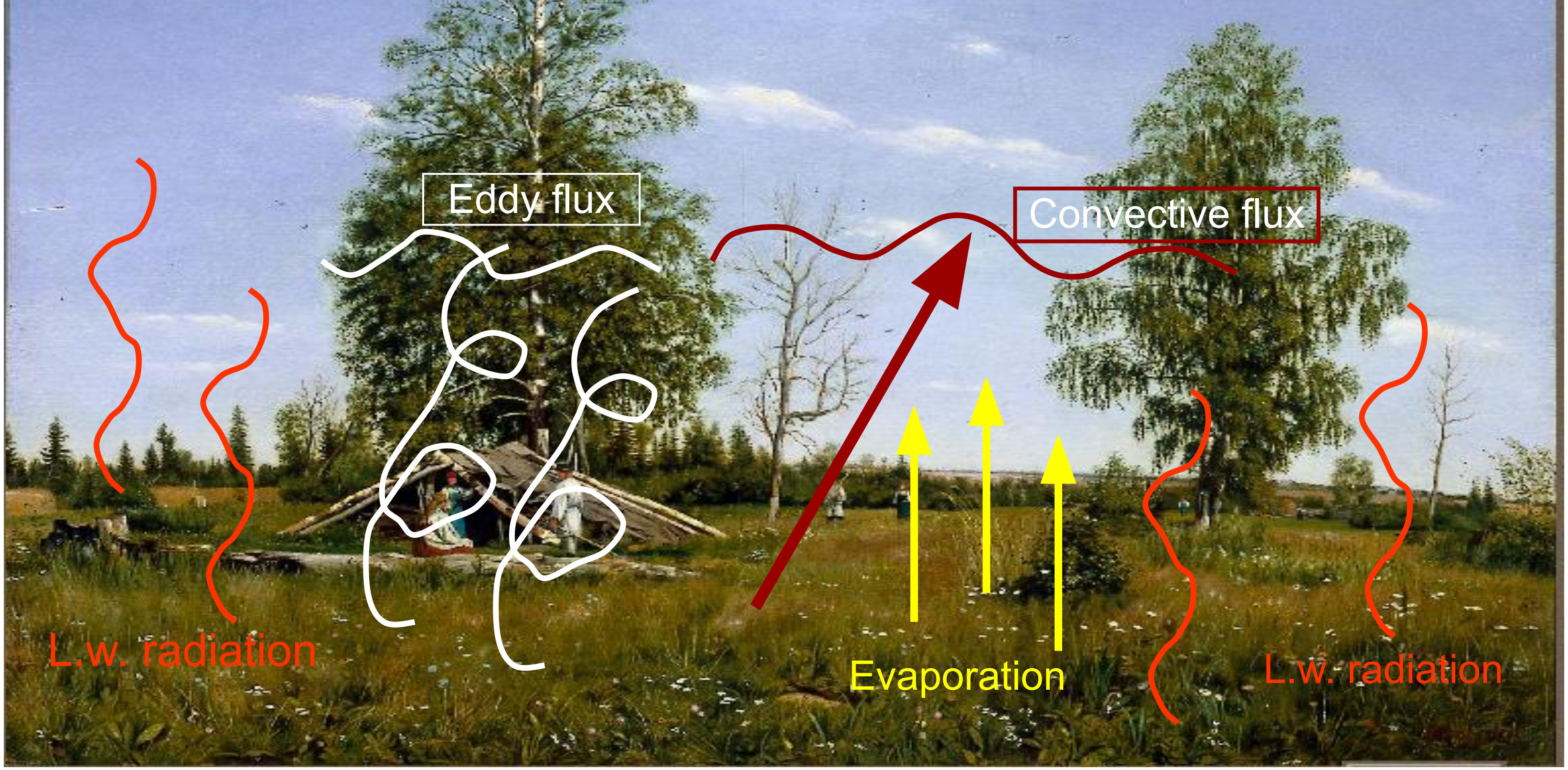
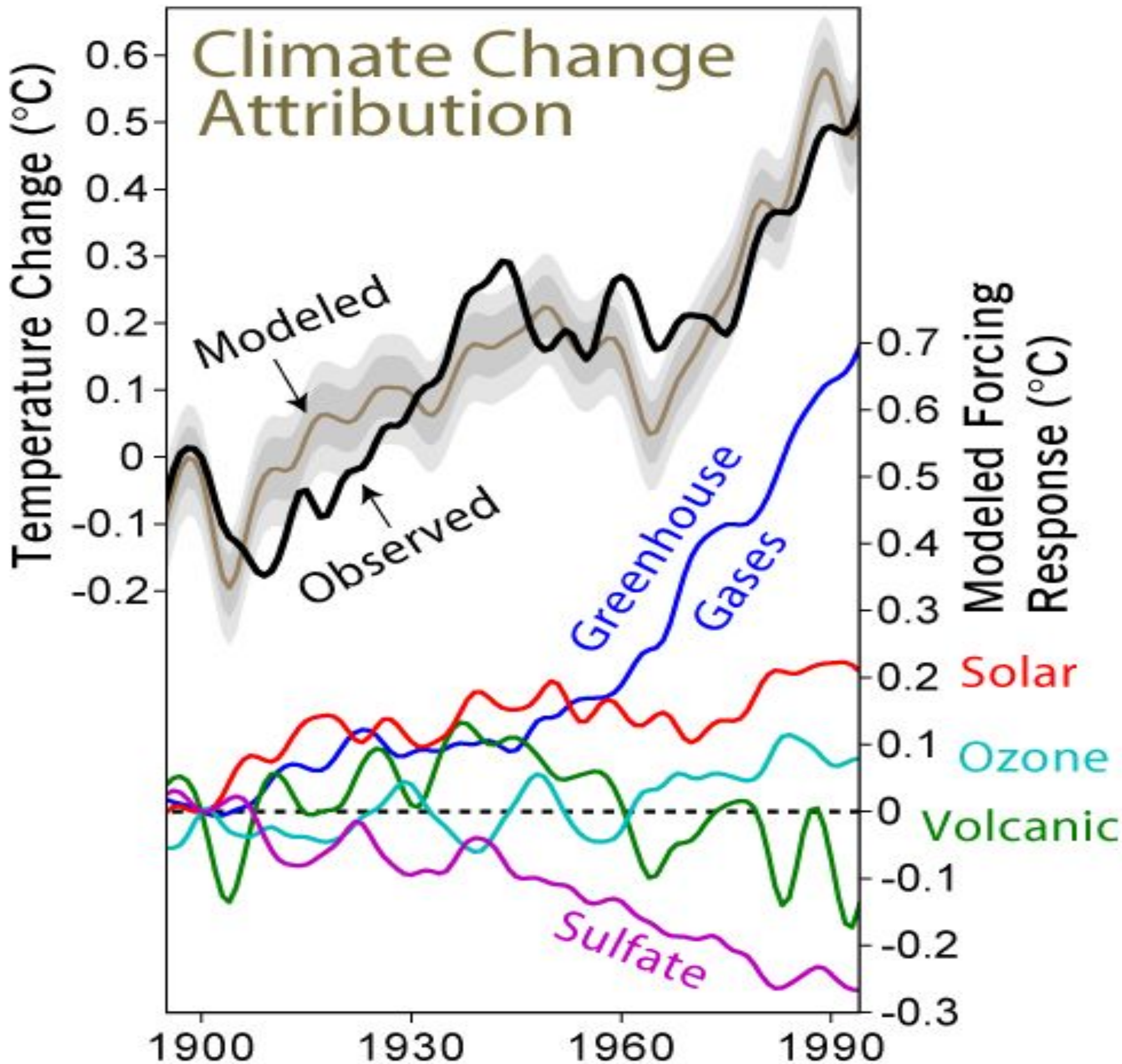
Heat fluxes in the atmosphere

Heat to the atmosphere comes mainly from the underlying surface. Processes responsible for the heat transfer are:





Climate_Change_Attribution.

Heat flux notion

The quantity $c_p T$ being transferred by the air parcels in a unit of time through a unit of area facing the transfer direction is called HEAT FLUX.

There are *convective heat flux* and *eddy heat flux.*

***Convective heat flux,* in turn, is divided into *advective one* (horizontal heat transfer) and *real convective* (vertical heat transfer).**

In meteorology, the horizontal heat flux is called *advective flux (Q_a),* and the vertical one is called *convective flux (Q_c).*

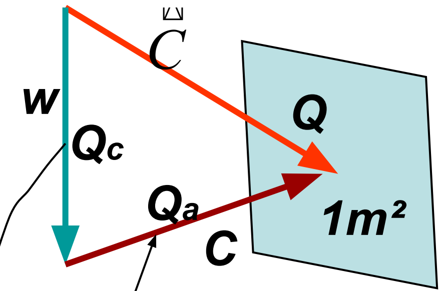
Convective and advective heat fluxes

$$Q = c_p \rho \vec{C} T$$

$$c_p \rightarrow \frac{J}{kg \cdot K}$$

$$\{\rho C\} \rightarrow \frac{kg}{m^3}\frac{m}{s} = \frac{kg}{m^2 \cdot s}$$

$$\{Q\} \rightarrow \frac{J}{kg \cdot K}\frac{kg}{m^2 s}K = \frac{J}{m^2 s}$$



$$Q_c = c_p \rho w T$$

$$Q_a = c_p \rho C T$$

Any air particle contains some amount of heat. When moving, it carries this heat along. By this way the heat is distributed in the atmosphere. However, that is not the only way for the heat distribution. Not less effective way is *EDDY MIXING (EXCHANGE)*

Incoming heat flux is *positive*, outgoing flux is *negative*.

Eddy heat flux

Eddy heat flux **is caused by wind velocity pulsation**

General conditions for eddy exchange

- **Permanency**
- **Conservation**
- **Passivity**

Quantity $c_p T$ does not satisfy this conditions. Air temperature changes as the air ascending or descending. However, potential temperature satisfy.

$$Q_E = -c_p A \frac{\partial \theta}{\partial z} = -c_p \rho K \frac{\partial \theta}{\partial z}$$

$$\frac{\partial \theta}{\partial z} = \gamma_a - \gamma$$

$$Q_E = -c_p \rho K \left(\gamma_a - \gamma \right)$$

$$c^* = \bar{c} + c' \qquad Q_a = c_p \rho \cdot c \cdot T \qquad 10^3 \cdot 10^1 \cdot 10^0 = 10^4$$

$$c^* = c + c' \qquad Q_E = c_p \rho \cdot k(\gamma_a - \gamma) \qquad 10^3 \cdot 10^1 \cdot 10^{-1} = 10^3$$

$$w^* = w + w' \qquad Q_c = c_p \rho \cdot w \cdot T \qquad 10^3 \cdot 10^{-2} \cdot 10^0 = 10^1$$

c_p	10^3	$\gamma_a - \gamma$	10^{-1}
ρ	10^0	w	10^{-2}
c	10^1	Q_a	10^4
T	$10^0 C$	Q_E	10^3
k	10^1	Q_c	10^1

$$\langle c_p \rho \rangle \sim 10^3 \cdot 10^0 = 10^3$$

→ **The strongest flux** (Q_a)

→ **The weakest flux** (Q_c)

Along with vertical eddy flux there are horizontal fluxes.

$$Q_{Ex} = -c_p \rho K' \frac{\partial \theta}{\partial x}$$

$$Q_{Ey} = -c_p \rho K' \frac{\partial \theta}{\partial y}$$

At the same level

$$\frac{\partial \theta}{\partial x} = \frac{\partial T}{\partial x} \qquad \frac{\partial \theta}{\partial y} = \frac{\partial T}{\partial y}$$

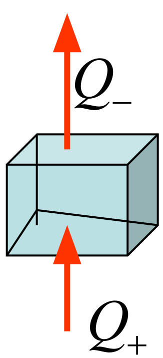
Eddy heat exchange differs from that of other substances. Coefficients *A* and *K* (for eddy heat exchange) are called coefficient of eddy heat conductivity and temperature conductivity respectively.

As the stratification is stable ($\gamma_a > \gamma$) eddy heat flux directed downward $Q_E < 0 \downarrow$

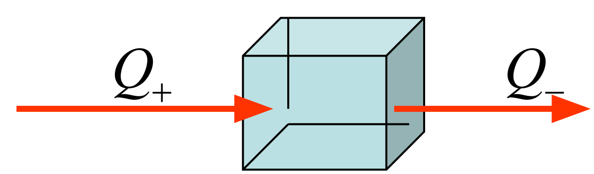
As the stratification is unstable ($\gamma_a < \gamma$) eddy heat flux directed upward $Q_E > 0 \uparrow$

Heat influx (outflow) notion

Heat influx = incoming heat flux – outgoing heat flux $HI = Q_+ - Q_-$



Heat outflow



Heat influx

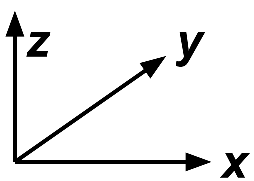
Individual and local (partial) derivatives

When an air parcel moves, its state parameters are not necessarily constant; they are function of coordinates and time.

$$F = F(x, y, z, t)$$

For the moving parcel, the coordinates, in turn, are functions of time.

$$x = x(t);\ y = y(t);\ z = z(t) \qquad \frac{dx}{dt} = u;\ \frac{dy}{dt} = v;\ \frac{dz}{dt} = w$$



$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \frac{\partial F}{\partial x}\frac{dx}{dt} + \frac{\partial F}{\partial y}\frac{dy}{dt} + \frac{\partial F}{\partial z}\frac{dz}{dt}$$

Local derivative

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + u\frac{\partial F}{\partial x} + v\frac{\partial F}{\partial y} + w\frac{\partial F}{\partial z}$$

Convective derivative

Advective derivative

Individual derivative

Energy equation

Temperature variation is of prime interest for meteorologists. It depends on heat influx. It can be determined on the base of the energy conservation equation.

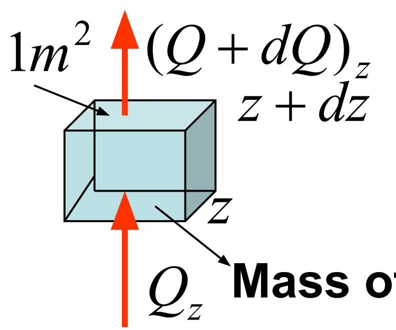
$$\frac{dq}{dt} = c_p \frac{dT}{dt} - \frac{RT}{P}\frac{dP}{dt}$$

Heat influx:

$$\frac{dq}{dt} = \varepsilon_E + \varepsilon_r + \varepsilon_{ph} + \varepsilon_k$$

$$\{\varepsilon\}\frac{J}{kg \cdot s}$$

Heat influx unit



$$Q_z - (Q+dQ)_z = -dQ_z$$

$$dQ_z = \frac{\partial Q_z}{\partial z} dz$$

$$\varepsilon_E + \varepsilon_r + \varepsilon_{ph} + \varepsilon_k = \varepsilon$$

Mass of the volume is $1\ m^2 \times \rho \times dz = dm$

$$\varepsilon = -\frac{dQ_z}{dm} = -\frac{dQ_z}{\rho dz} = -\frac{1}{\rho}\frac{\partial Q_z}{\partial z}\frac{dz}{dz}$$

$$\varepsilon = -\frac{dQ_z}{dm} = -\frac{dQ_z}{\rho dz} = -\frac{1}{\rho}\frac{\partial Q_z}{\partial z}$$

The same reasoning can be applied for horizontal heat fluxes

$$\varepsilon = -\frac{1}{\rho}\left(\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{\partial Q_z}{\partial z}\right) = div\vec{Q}$$

$$(Q_E)_x = -c_p A'\frac{\partial Q}{\partial x} \qquad (Q_E)_y = -c_p A'\frac{\partial Q}{\partial y} \qquad (Q_E)_z = -c_p A\frac{\partial Q}{\partial z}$$

$$\varepsilon = \frac{c_p}{\rho}\left(\frac{\partial}{\partial x}A'\frac{\partial Q_x}{\partial x} + \frac{\partial}{\partial y}A'\frac{\partial Q_y}{\partial y} + \frac{\partial}{\partial z}A\frac{\partial Q_z}{\partial z}\right)$$

Eddy heat influx

Since the horizontal exchange is much smaller the vertical one, for practical purposes this formula can be simplified

$$\varepsilon \approx \frac{c_p}{\rho}\left(\frac{\partial}{\partial z}A\frac{\partial Q_z}{\partial z}\right) = c_p\frac{\partial}{\partial z}K\frac{\partial Q_z}{\partial z}$$

$$\frac{dq}{dt} = c_p \frac{dT}{dt} - \frac{RT}{P}\frac{dP}{dt}$$

Δt	10^4	s	V	10^1	m/s
ΔP	10^2	Pa	W	10^{-2}	m/s
ΔP_z	10^4	Pa	Δx	10^5	m
ΔT	10^0	K	Δy	10^5	m
ΔT_z	10^1	K	Δz	10^3	m

$$\frac{dP}{dt} = \frac{\partial P}{\partial t} + u\frac{\partial P}{\partial x} + v\frac{\partial P}{\partial y} + w\frac{\partial P}{\partial z}$$

$$\frac{\partial P}{\partial t} = \frac{10^2}{10^4} = 10^{-2}$$

$$u\frac{\partial P}{\partial x} = v\frac{\partial P}{\partial y} = 10^1\frac{10^2}{10^5} = 10^{-2}$$

$$w\frac{\partial P}{\partial z} = 10^{-2}\frac{10^4}{10^3} = 10^{-1}$$

$$\frac{dP}{dt} \approx w\frac{\partial P}{\partial z}$$

All members of this formula have the same order of magnitude

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}$$

$$\frac{\partial T}{\partial t} = \frac{10^0}{10^4} = 10^{-4}$$

$$u\frac{\partial T}{\partial x} = v\frac{\partial T}{\partial y} = 10^1\frac{10^0}{10^5} = 10^{-4}$$

$$w\frac{\partial T}{\partial z} = 10^{-2}\frac{10^1}{10^3} = 10^{-4}$$

Substituting and

into energy equation, we obtain

See the next slide

$$\varepsilon_E + \varepsilon_r + \varepsilon_{ph} + \varepsilon_k = c_p \left[\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} \right] - \frac{RT}{P} w \frac{\partial P}{\partial z}$$

$$(\varepsilon_E + \varepsilon_r + \varepsilon_{ph} + \varepsilon_k)\frac{1}{c_p} = \left[\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} \right] - \frac{RT}{c_p P} w \frac{\partial P}{\partial z}$$

This quantity is small and can be neglected

$$-\frac{RT}{c_p P} w \frac{\partial P}{\partial z} = w\frac{RT}{c_p P} g\rho = w\frac{gRT}{c_p P}\frac{P}{RT}$$

$$\frac{g}{c_p} = \gamma_a \qquad \varepsilon_E = \frac{\partial}{\partial z} K \frac{\partial \theta}{\partial z} \qquad -w\frac{\partial T}{\partial z} + w\gamma_a = w(\gamma_a - \gamma)$$

After substituting into the basic equation

and solution with respect to $\frac{\partial T}{\partial t}$**, we obtain**

$$\frac{\partial T}{\partial t} = -\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) - w(\gamma_a - \gamma) + \frac{\partial}{\partial z} K \frac{\partial \theta}{\partial z} + \frac{\varepsilon_r + \varepsilon_{ph}}{c_p}$$

$$\frac{\partial T}{\partial t} = -\left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) - w(\gamma_a - \gamma) + \frac{\partial}{\partial z} K \frac{\partial \theta}{\partial z} + \frac{\varepsilon_r + \varepsilon_{ph}}{c_p}$$

$$\left(\frac{\partial T}{\partial t} \right)_{ad} = -\left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right)$$

Air temperature variation due to advection

$$\left(\frac{\partial T}{\partial t} \right)_{w} = -w(\gamma_a - \gamma)$$

Air temperature variation due to vertical motion

$$\left(\frac{\partial T}{\partial t} \right)_{E} = \frac{\partial}{\partial z} K \frac{\partial \theta}{\partial z}$$

Air temperature variation due to eddy mixing

$$\left(\frac{\partial T}{\partial t} \right)_{r,ph} = \frac{\varepsilon_r + \varepsilon_{ph}}{c_p}$$

Air temperature variation due to radiation and water phase transfer

1. Non-periodical T variations

- Above boundary layer (in the free atm.)
- Small time intervals (about 24 h)
- □ no heat influx □ adiab. process

Energy equation:

$$\frac{\partial T}{\partial t} = -\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) - w(\gamma_a - \gamma)$$

$$\frac{\Delta T_a}{\Delta t} = -\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) \quad or \quad \Delta T_a = -\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)\Delta t$$

$$\Delta T_w = w(\gamma - \gamma_a)$$

$$within\ \ a\ \ cloud\quad \Delta T_w = w(\gamma - \gamma'_a)$$

2. Periodical T variations

- Within the boundary layer (diurnal T variations)
- long time intervals
- □ only vertical eddy heat influx

Energy equation (or equation of the conductivity of the atm.):

$$\left(\frac{\partial T}{\partial t}\right)_E = \frac{\partial}{\partial z} K \frac{\partial \theta}{\partial z}$$

3. Air mass moving over non-homogeneous surface

- Advection and eddy exchange are important
- Taking steady state process
- The process is called

air mass transformation

$$\frac{\partial T}{\partial t} = 0$$

Energy equation

$$u\left(\frac{\partial T}{\partial x}\right) = \frac{\partial}{\partial z} K \frac{\partial \theta}{\partial z}$$

4. Annual T variation

- Most important role to $\varepsilon_e, \varepsilon_r \quad and \quad \varepsilon_{ph}$
- Local, advective and convective derivatives □ 0 over a long period of time

Energy equation

$$c_p\left[\frac{\partial}{\partial z}k\frac{\partial\theta}{\partial z}+\frac{\partial}{\partial x}k'\frac{\partial\theta}{\partial x}+\frac{\partial}{\partial y}k'\frac{\partial\theta}{\partial y}\right]+\varepsilon_r+\varepsilon_{ph}=0$$