

# Diurnal temperature variation in the boundary layer

The diurnal temperature variation is defined by the variation of the heat influx to the earth's surface during 24 hour period.

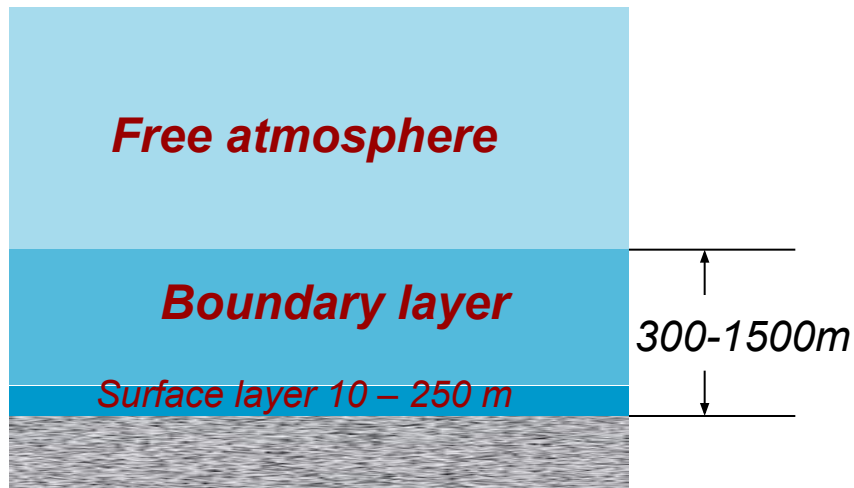
**During daytime the earth's surface is getting heat due to solar radiation influx, and its temperature grows up. During night hours it loses energy due to effective radiation and its temperature grows down.**

Recall, that the atmosphere does not directly absorb the solar radiation. The air's own radiation at night hasn't significant impact on the air temperature.

Therefore, the main reason for temperature variation is the **eddy exchange with the underlying surface**. This process is responsible for the diurnal temperature variation within whole boundary layer up to 1 - 1,5 km altitude.

# Definition of the boundary layer

Directly adjacent to the underlying surface layer of the atmosphere within which diurnal variation of various atmospheric parameters (temperature, humidity, wind speed, etc) is well defined is called ***boundary layer of the atmosphere***



The height of the boundary layer depends upon static stability of the atmosphere.

**Very stable atmosphere**

$$h_{B.L.} \approx 300m$$

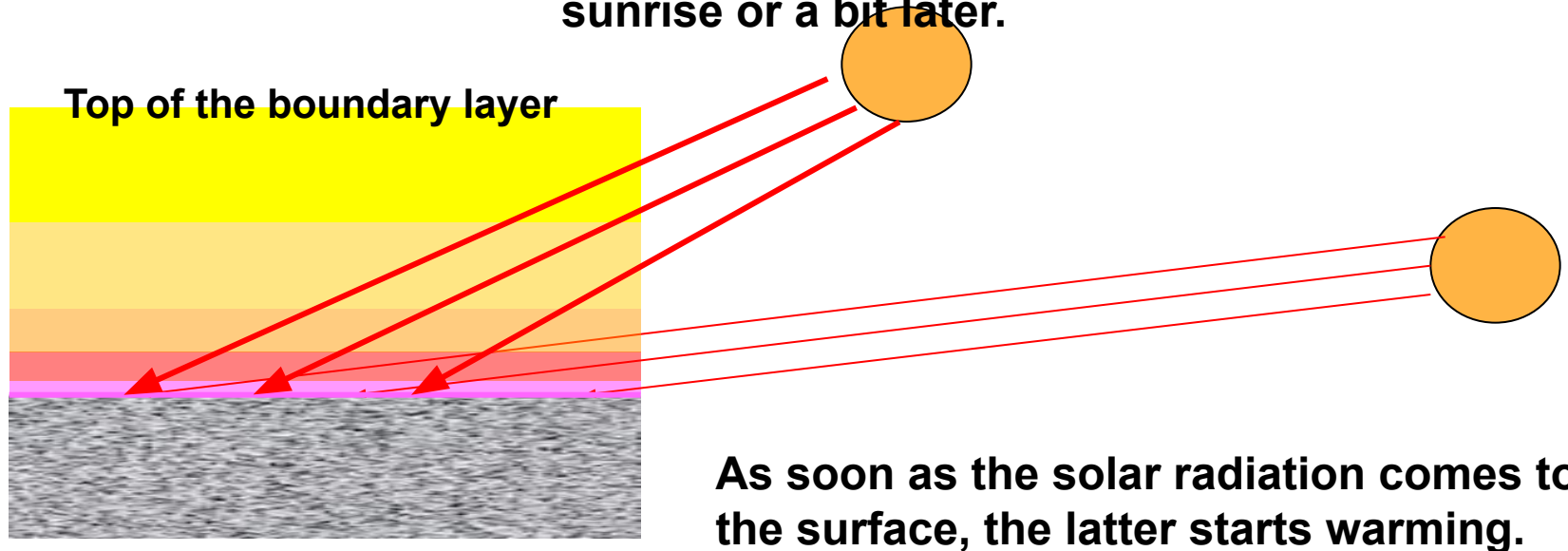
**Very unstable atmosphere**

$$h_{B.L.} \approx 1500m$$

In tropics, where the atmosphere can be... extremely unstable, the top of the boundary layer can reach 3000 m.

# Mechanism of the heat spreading up

According to observation, minimal temperatures are observed near the sunrise or a bit later.



Due to **molecular diffusion**, the heat is transferred to the **thin layer of air**.

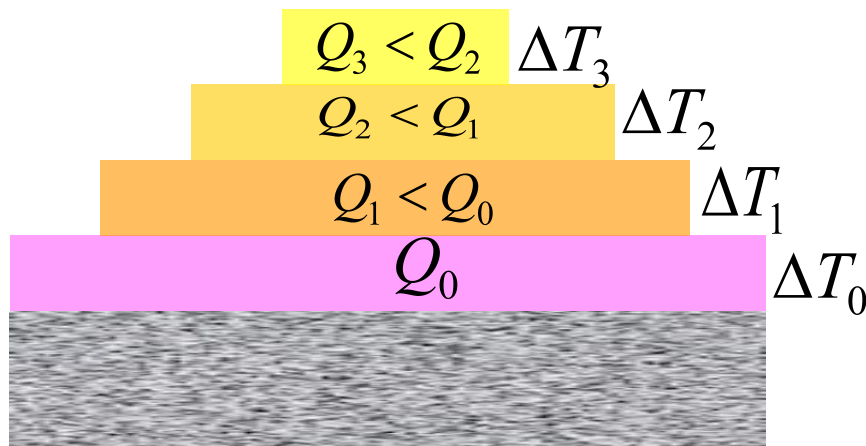
Later the heat is transferred upward due to **eddy mixing** from one layer to that above of it and so on.

Since for the heat transfer up some period of time is needed, there is delay with temperature grow in every following layer

The heat amount, as it spreads up, becomes smaller and smaller because each layer takes some fraction of heat from the ascending heat flux. That makes temperature variation at each following layer smaller and smaller too.

Suppose, there are  $n$  layers, the lowest of them (adjacent to the surface) is the well known “*surface layer*”, where the heat flux is quasi- constant, or well call it zero layer ( $Q_0$ ).

$$Q_n \rightarrow 0 \quad \Delta T_n \rightarrow 0$$

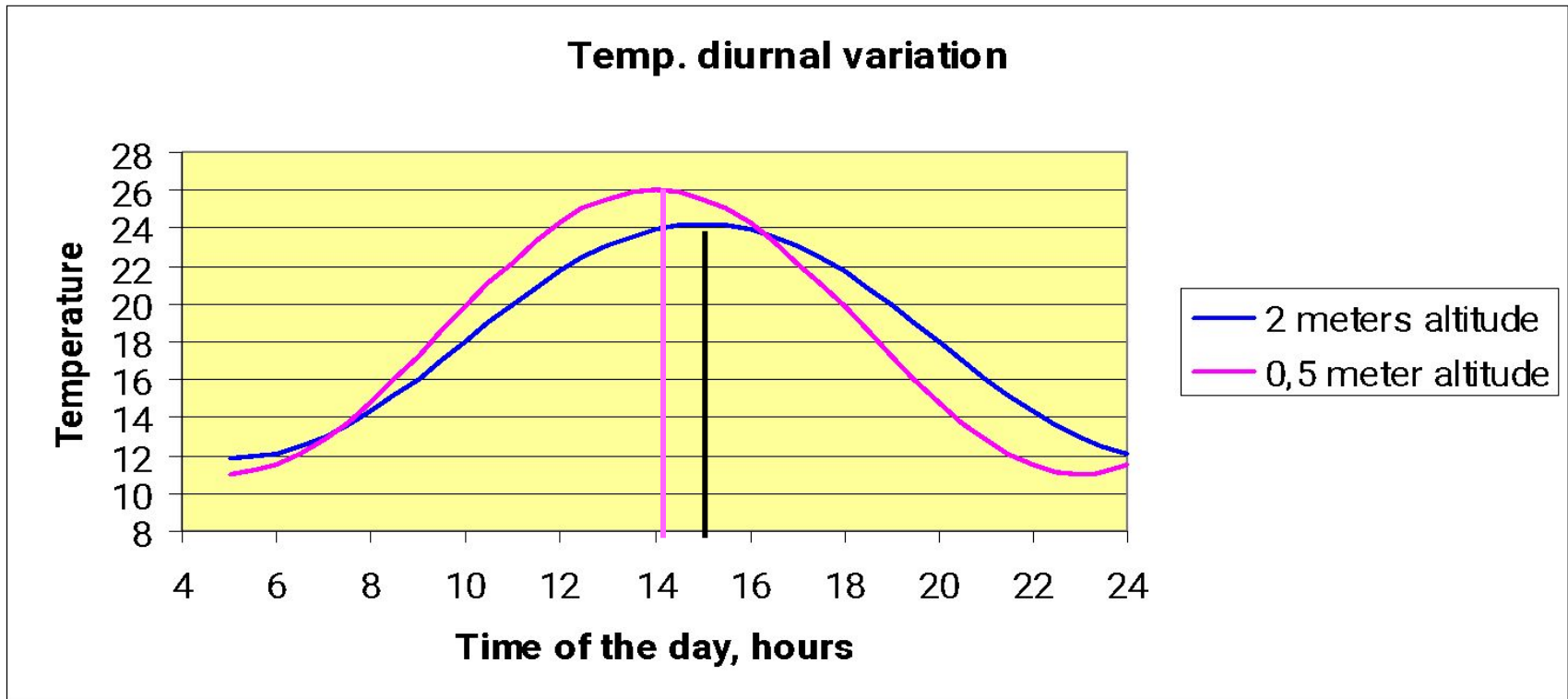


The following layers are 1, 2, ...  $n$ . In every of these layers the heat influx is smaller than in the lower one.

It means that  $\Delta T$  values will decrease with height.

$$\Delta T_0 > \Delta T_1 > \Delta T_2 \dots > \Delta T_n$$

# An example of the diurnal temperature variation



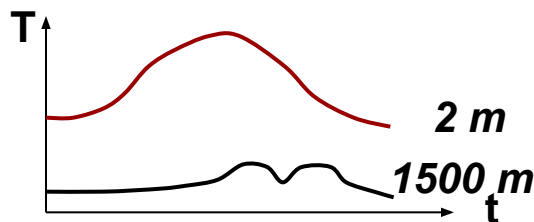
$$A_{0.5} = 15^{\circ} C$$

$$A_2 = 12,2^{\circ} C$$

If we go up, the amplitude will become smaller, and, at the boundary layer top, it will be close to zero.

# Diurnal temperature variation at different altitudes

1. The amplitude of the variation decreases with height. At the altitude of about 1.5 km it is 6 – 7 times less than near the surface.
2. Near the top of the boundary layer the amplitude can be very complicated with 2 or 3 maxima.



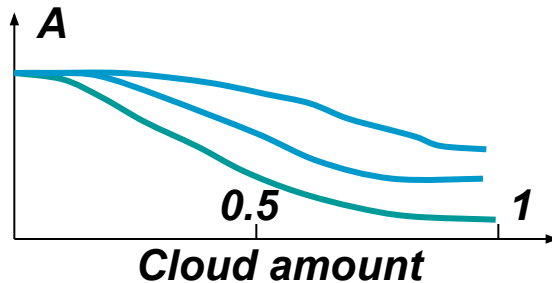
## *Reasons*

1. Observations are not accurate enough.
2. Fluctuation of eddy intensity.

Near the surface, variation is rather significant. Therefore, comparably small fluctuation of eddy intensity do not seriously distort the diurnal temperature variation rate. At the top of the boundary layer, situation is different. Here, the amplitude is much smaller than at the surface, and even not significant eddy activity fluctuation may result in distortion of the diurnal variation rate.

# Cloudiness and wind impact on the diurnal temperature variation

## Cloudiness

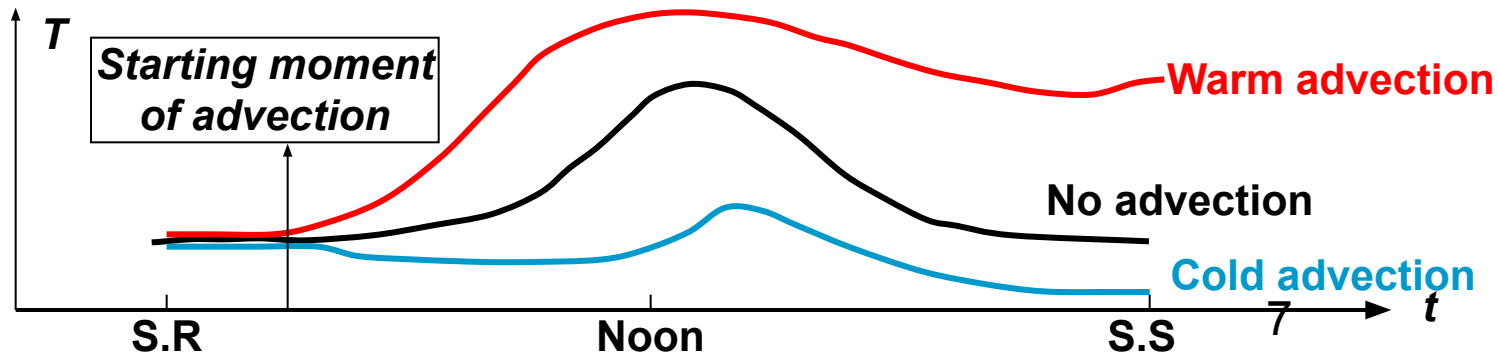


- Low clouds
- Middle clouds
- Upper clouds

An increase of the cloud amount always results in decrease of the diurnal temperature variation amplitude

## Wind

Wind makes the eddy mixing more intensive, and by this way it diminishes the variation amplitude. Along with this, it is associated with temperature advection that also distorts the rate of the variation.



# Simplified theory of the diurnal temperature variation

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z} K \frac{\partial T}{\partial z} + \varepsilon'(z, t); \quad K = \text{const}; \quad \frac{\partial}{\partial z} K \frac{\partial T}{\partial z} \gg \varepsilon'(z, t)$$

$$T(z, t) = \bar{T}(z) + \tau(z, t)$$

$$\frac{\partial \tau}{\partial t} = \frac{\partial}{\partial z} K \frac{\partial \tau}{\partial z} \quad \text{Partial differential equation of the second order}$$

## Boundary conditions

$$z \rightarrow \infty \Rightarrow \tau \rightarrow 0$$

$$z = z_0 \Rightarrow \tau(t) = A_0 \text{COS}(\omega t - \varphi)$$

At the given boundary conditions, the solution of the equation will be:

$$\tau = A_0 \exp(-az) \cdot \text{COS}(\omega t - az - \varphi)$$

$$a = \sqrt{\frac{\omega}{2K}} = \sqrt{\frac{\pi}{K\Pi}}$$

$$\omega = \frac{2\pi}{\Pi} = 7.29 \times 10^{-5} \text{ s}^{-1}$$



$$\tau = \overset{A(z)}{A_0 \exp(-az)} \cdot \text{COS}(\omega t - az - \varphi)$$

$$A(z) = A_0 \exp(-az) \quad \text{Amplitude variation with height}$$

Suppose, we took two altitudes  $z_1$  and  $z_2$ , where *the amplitudes* are of the same value.

$$A(z_1) = A_0 \exp\left(-z_1 \sqrt{\frac{\omega}{2K_1}}\right) \quad A(z_2) = A_0 \exp\left(-z_2 \sqrt{\frac{\omega}{2K_2}}\right)$$

$$A(z_1) = A(z_2) \quad z_1 \sqrt{\frac{\omega}{2K_1}} = z_2 \sqrt{\frac{\omega}{2K_2}} \quad \frac{z_1}{z_2} = \sqrt{\frac{K_1}{K_2}}$$

Adopting as a boundary layer altitude  $z^*$ , where the amplitude 100 times less than that at the surface, we obtain:

$$\frac{A(z^*)}{A_0} = 0.01 \quad \text{If } A_0 = 10^\circ\text{C}, \quad A(z^*) = 0.1^\circ\text{C}$$

Thus, the key role of the amplitude variation with height belongs to K value. On the base of that value, one can calculate the boundary layer altitude

<b><math>K \text{ m}^2/\text{s}</math></b>	$0,18 \cdot 10^{-4}$	<b>0.1</b>	<b>1</b>	<b>5</b>	<b>10</b>	<b>20</b>	<b>100</b>
<b><math>Z^* \text{ m}</math></b>	<b>3</b>	<b>240</b>	<b>761</b>	<b>1702</b>	<b>2408</b>	<b>3405</b>	<b>7613</b>

Here, in the first column  $K = K_m \approx 0.18 \cdot 10^{-4} \text{ m}^2/\text{s}$  is the **molecular heat conductivity coefficient**. It shows clearly the role of the K value in determination of the  $z^*$  value.

The normal K value is known to be from 1 to 5  $\text{m}^2/\text{s}$ . However, some cases were reported with  $K > 100 \text{ m}^2/\text{s}$ . This kind of a situation is associated with extremely well developed convection, for instance, in tropics. At these cases the diurnal temperature variation can be observed through the whole troposphere.

# Annual temperature variation

In the Northern Hemisphere maximal temperature is usually observed in July, and minimal temperature in January or February.

The annual temperature variation amplitude decreases with height in the same manner as the diurnal one does.

$\Pi_1=24$  hours is the period of one Earth's spin.

$\Pi_2=24 \cdot 365.25$  hours is the period of annual Earth's rotation around the Sun.

$$\frac{z'}{z''} = \sqrt{\frac{\Pi_1}{\Pi_2}} = \frac{24}{24 \cdot 365.25} = \frac{1}{365.25}$$

Assuming normal condition ( $K=5 \text{ m}^2/\text{s}$ ), the annual temperature variation spreads over a layer of about 32 km (whole troposphere and significant part of the stratosphere). We know a little about K value variation with height except the fact that it increases within the surface layer. There are some evidences that it remains almost constant in the boundary layer. As to the free atmosphere, we can judge about it from indirect evidences such as comparison observed and calculated parameters.

# Rate of heat wave propagation and lag time

We have known:

- Air temperature variation, above else, depends on Earth's surface temperature variation.
- The rate of the heat propagation is a finite value.
- The extreme temperatures are to occur the later, the higher altitude is.

Suppose  $t_1$  is the time the earth's surface temperature reaches its maximum;  $t_2$  is the respective time for the altitude  $z$ .

$$\omega t_1 - \varphi = 0; \quad \omega t_2 - z\sqrt{\frac{\omega}{2K}} - \varphi = 0; \quad \omega t_2 - z\sqrt{\frac{\omega}{2K}} - \cancel{\varphi} = \omega t_1 - \cancel{\varphi}$$

$$t_2 - t_1 = \frac{z}{\omega} \sqrt{\frac{\omega}{2K}} = \frac{z}{\sqrt{2\omega K}}$$

Lag time

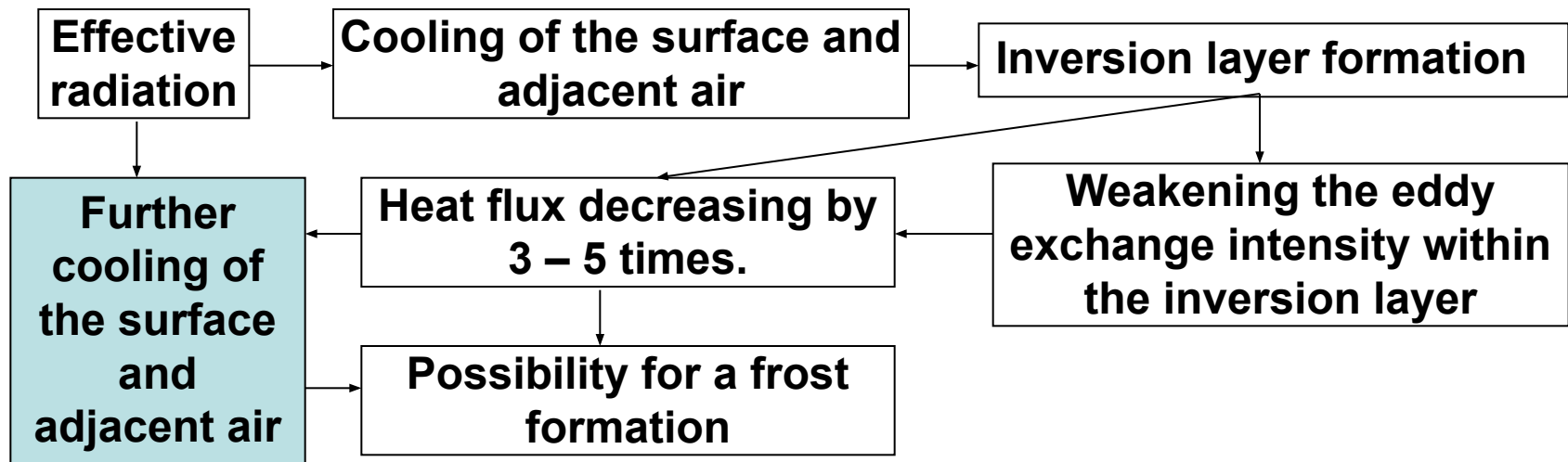
Phase velocity:

$$V_{ph} = \frac{z}{t_2 - t_1} = \sqrt{2\omega K} = 1,21 \cdot 10^{-2} \sqrt{K}$$

In case  $K=5 \text{ m}^2/\text{s}$        $V_{ph} = 1.21 \cdot 10^{-2} \sqrt{5} \approx 2.7 \text{ cm} / \text{s}$

# Nocturnal temperature decrease

The main reason for nocturnal temperature decrease is effective radiation. The intensity of the effective radiation, in turn, depends on the properties of the soil and state of the sky. Significant fall of the temperature occurs at cloudless sky condition.



Brent's formula

$$T(0, t) = T(0, 0) - \frac{2B^*}{\sqrt{\pi} \cdot c^* \rho^* \sqrt{K_m}} \sqrt{t}$$

# Frosts

Temperature fall below  $0^{\circ}\text{C}$  on a positive temperature background is called **FROST**.

There are two types of the frosts;

1. Radiative frosts
2. Advective frosts

**Favorable conditions for radiative frosts**

- *Low air humidity*
- *Weak wind (1 – 2 m/s)*
  - *Cloud free sky*

**Favorable conditions for advective frost**

- *Low air humidity*
- *Strong cold wind*
- *Small cloud amount*