Thermodynamics of the atmosphere

The atmosphere is a medium where transition of energy from one form to another constantly occurs.

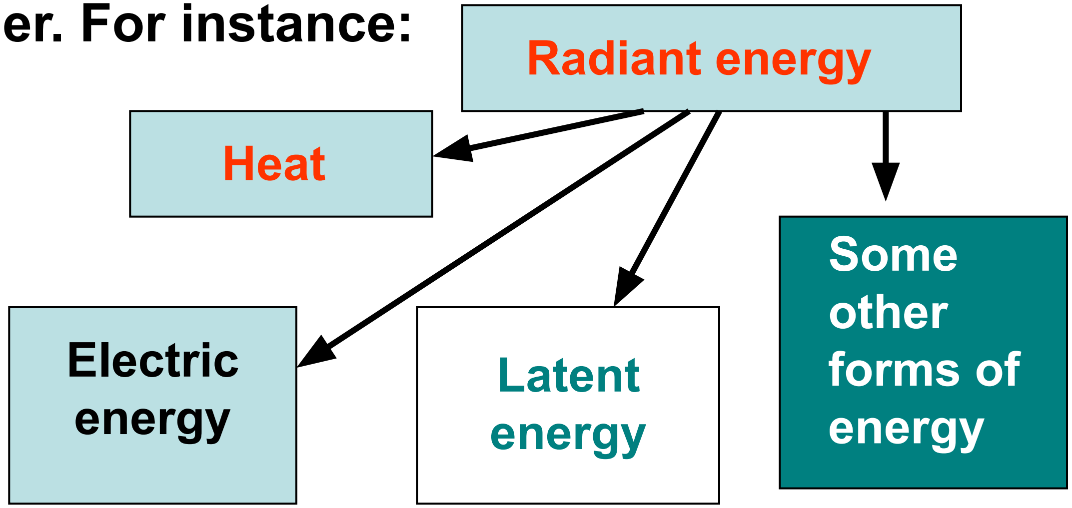
The branch of meteorology dealing with the general regularities of energy transformation and variation of the state of the atmosphere under influence of heat influxes is called

THERMODYNAMICS OF THE ATMOSPHERE

The first law of thermodynamics as applied to the atmosphere

This law is also known as the *law of energy conservation.* It is widely used in the thermodynamics of the atmosphere.

The first law of thermodynamics holds: *the total amount of energy remains constant or is conserved.* It is not possible for the energy to be destroyed or to arise. It can only be transformed from one form to another. For instance:



Radiant energy

Heat

Electric energy

Latent energy

Some other forms of energy

This statement can be expressed in form of energy equation

Energy equation

State of the atmosphere, as well as any parcel of it, can be expressed by three parameters.

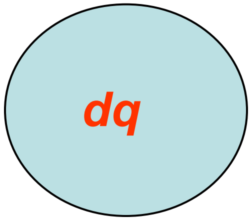
Pressure P_i ***Density*** ρ_i ***Temperature*** T_i

The same parameters for environment we'll denote as P_e, ρ_e, T_e **respectively.**

In general case $\rho_i \neq \rho_e; \quad T_i \neq T_e$ However, $P_i = P_e = P$

Static condition

Suppose a parcel of air has received an amount of heat *dq*



As result its inner energy will increase by du_i

Parcel will expand making work against external pressure force, dw_i.

$$\boxed{dq = du_i + dw_i}$$

If the air is dry (non-saturated), it can be regarded as an ideal gas

Ideal Gas

- An **ideal gas** is a theoretical is a theoretical gas is a theoretical gas composed of a set of randomly-moving, non-interacting point particles is a theoretical gas composed of a set of randomly-moving, non-interacting point particles. The ideal gas concept is useful because it obeys the ideal gas law is a theoretical gas composed of a set of randomly-moving, non-interacting point particles. The ideal gas concept is useful

$$du_i = c_v dT_i$$

Specific heat capacity at the constant volume

$$dw_i = P dv_i$$

Volume increment

$$dq = c_v dT_i + P dv_i$$

The volume can not be measured. Therefore, this formula must be transformed to include those parameters which are measured.

$$\boxed{Pv_i = R_c T_i} \longrightarrow \boxed{P dv_i + v_i dP = R_c dT_i} \longrightarrow \boxed{P dv_i = R_c dT_i - v_i dP}$$

$$\boxed{dq = (c_v + R_c) dT_i - v_i dP} \longrightarrow \boxed{dq = c_p dT_i - R_c T_i \frac{dP}{p}}$$

In case of an isobaric process *dP=0,* and hence, $dq = (c_v + R_c) dT_i$

On the other hand, at the isobaric process $dq = c_p dT_i$

$$c_v + R_c = c_p$$

$$R_c = c_p - c_v$$

Mayer equation

$$c_v = 718 \, {}^{J}\!/_{kg \cdot K}$$

$$c_p = 1005 \, {}^{J}\!/_{kg \cdot K}$$

$$\frac{c_p}{c_v} = 1{,}4 = \aleph$$

$$c_p - c_v = R_c = 287 \, {}^{J}\!/_{kg \cdot K}$$

Adiabatic process

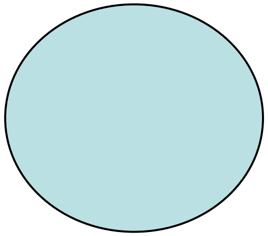
Thermo dynamical process going on without energy income or outflow to a body ($dq = 0$)is called *adiabatic process.*

$$Pdv_i = -c_v dT_i$$

$$R_c T_i \frac{dP}{P} = c_p dT_i$$

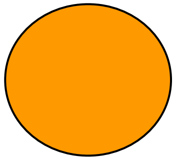
$$dv_i > 0 \rightarrow dT_i < 0$$

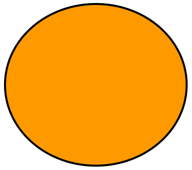
$$dP < 0 \rightarrow dT_i < 0$$



$$dv_i < 0 \rightarrow dT_i > 0$$

$$dP > 0 \rightarrow dT_i > 0$$





For an adiabatic process the equation of the first law of thermodynamics can be also written in integral form.

$$R_c T_i \frac{dP}{P} = c_p dT_i$$

$$\int_{T_{i0}}^{T_i} c_p \frac{dT_i}{T_i} = \int_{p_0}^{p} R_c \frac{dP}{p}$$

Index “0” refers to initial state.

$$c_p \ln \frac{T_i}{T_{i0}} = R_c \ln \frac{P}{P_0}$$

$$\boxed{\frac{T_i}{T_{i0}} = \left(\frac{P}{P_0}\right)^{R_c/C_p}}$$

This equation is called *equation of adiabatic processes in integral form (Poisson’s equation)*

$$\frac{R_c}{c_p} = \frac{287}{1005} \approx 0{,}286$$

$$\boxed{\frac{T_i}{T_{i0}} = \left(\frac{P}{P_0}\right)^{0{,}286}}$$

Dry adiabatic lapse rate

Statics' eq-on

$$\boxed{dP = -g\rho_e dz}$$

State eq-on

$$\boxed{P = \rho_e R_c T_e}$$

$$\rho_e = \frac{P}{R_c T_e}$$

$$dP = -\frac{P}{R_c T_e} gdz$$

$$\boxed{c_p dT_i - R_c T_i \frac{dP}{p} = 0}$$

$$c_p dT_i + \cancel{R_c} T_i \frac{\cancel{P}}{\cancel{P}\cancel{R_c} T_e} gdz = 0$$

$$c_p dT_i + \frac{T_i}{T_e} gdz = 0$$

$$c_p \frac{dT_i}{dz} + \frac{T_i}{T_e} g = 0$$

$$\frac{m \cdot kg \cdot K}{s^2 J} = \frac{m \cdot s^2 K}{s^2 m^2} = \frac{K}{m}$$

$$\boxed{\gamma_a} \longleftarrow$$

$$-\frac{dT_i}{dz} = \frac{T_i}{T_e}\frac{g}{c_p} = \gamma_a = 0{,}0098$$

$$\frac{J}{kg} = \frac{m^2}{s^2}$$

$$\gamma_a = 0{,}0098^0 / m \approx 1^0 / 100m$$

Regarding dry adiabatic lapse rate as a constant value, the expression $\gamma_a = -dT_i/dz$ can be integrated and brought in the following form.

$$T_i = T_{i0} - \gamma_a \left(z - z_0\right) \approx T_{i0} - 0{,}01\left(z - z_0\right)$$

This equation is known as *approximated equation of a dry adiabat.*

The dry adiabat* is also called *curve of the state of a dry air parcel

Polytropic processes

Along with adiabatic processes there are more general ones known as *polytropic processes.*

The process is called *polytropic* in the case as a heat influx to an air parcel is proportional to the parcel temperature variation.

$$dq = cdT$$

Name of process	C	dq	dT
Adiabatic	0	0	-
Isobaric	1005	$c_p dT$	$dT \neq 0$
Isosteric	718	$c_v dT$	$dT \neq 0$
Isothermal	∞	-	$dT = 0$

Potential temperature

Initial state: T_i, P

$$\boxed{\frac{T_i}{T_{i0}} = \left(\frac{P}{P_0}\right)^{0,286}}$$

Terminal state:
θ. 1000 hPa

$$\frac{T_i}{\theta} = \left(\frac{P}{1000}\right)^{0,286} \xrightarrow[0,286 = {}^{R_c}\!/_{c_p}]{} \boxed{\theta = T_i\left(\frac{1000}{P}\right)^{0,286}}$$

Properties of the potential temperature

1. It remains unchanged at adiabatic displacement (up or down)

$$\ln\theta = \ln T_i + \frac{R_c}{c_p}\ln 1000 - \frac{R_c}{c_p}\ln P$$

$$\ln\theta = \ln T_i + \underbrace{\frac{R_c}{c_p}\ln 1000}_{\text{Constant}} - \frac{R_c}{c_p}\ln P$$

$$\frac{d\theta}{\theta} = \frac{dT_i}{T_i} - \frac{R_c}{c_p}\frac{dP}{P} \longrightarrow \frac{R_c T_i \frac{dP}{P} = c_p dT_i}{c_p T_i}$$

$$\frac{dT_i}{T_i} - \frac{R_c}{c_p}\frac{dP}{P} = 0$$

$$\boxed{\frac{d\theta}{\theta} = 0}$$

$$\theta \neq 0$$

$$d\theta = 0$$

If an air particle moves without heat exchange with environment, its potential temperature remains unchanged, while its molecular temperature changes.

Any variation of the potential temperature means that the air gains or looses energy.

$$\boxed{dq = c_p T_i \frac{d\theta}{\theta}}$$

2. Potential temperature determines total amount of energy.

$$\frac{d\theta}{\theta} = \frac{dT_i}{T_i} - \frac{R_c}{c_p}\frac{dP}{P} \quad \times c_p T_i$$

$$T_i c_p \frac{d\theta}{\theta} = \boxed{c_p dT_i - R_c T_i \frac{dP}{P}} = dq$$

$$\boxed{dq = c_p T_i \frac{d\theta}{\theta}}$$

$$-R_c T_i \frac{dP}{P} = R_c T_i g \frac{Pdz}{T_e R_c P} = \frac{gT_i}{T_e} dz$$

$$T_i c_p \frac{d\theta}{\theta} = c_p dT_i + \boxed{\frac{gT_i}{T_e} dz}$$

$$g\frac{T_i}{T_e} dz - gdz + gdz = gdz + g\left(\frac{T_i}{T_e} - 1\right) dz = gdz + g\left(\frac{T_i - T_e}{T_e}\right) dz$$

$$\boxed{c_p T_i \frac{d\theta}{\theta} = c_p dT_i + gdz + g\left(\frac{T_i - T_e}{T_e}\right) dz}$$

$$\boxed{c_p T_i \frac{d\theta}{\theta} = \underbrace{c_p dT_i}_{d\text{Э}_i} + \underbrace{gdz}_{d\Phi} + \underbrace{g\left(\frac{T_i - T_e}{T_e}\right)dz}_{dE_i}}$$

$$\boxed{\text{Э}_i = c_p T_i + const}$$

Entalpy or total heat

Φ is potential energy (Geopotential)

E_i is instability energy

(CAPE=convective available potential energy)

$$\boxed{c_p T_i \frac{d\theta}{\theta} = d\text{Э}_i + d\Phi + dE_i}$$

$$\boxed{c_p T_i \frac{d\theta}{\theta} = d\Pi_i}$$

Total energy of a particle of a unit mass ←

$$\boxed{d\Pi_i = d\text{Э}_i + d\Phi + dE_i}$$

$$\boxed{d\theta = \frac{\theta}{c_p T_i} d\Pi_i}$$

Potential temperature variation is uniquely determined by the total energy variation of the air particle.

At adiabatic displacement, the total energy of the air particle remains unchanged.

$$E_i = \int_{Z_1}^{Z_2} dE_i = \int_{Z_1}^{Z_2} g\left(\frac{T_i - T_e}{T_e}\right) dZ$$

$$E_i = g\left(\frac{T_i - T_e}{T_e}\right) \Delta Z$$

For a unit of mass
m=1 kg

$$F = a \cdot m = a$$

$$E = F \cdot L = aL$$

In our case *L*=Δ*Z*

$$E_i = g\left(\frac{T_i - T_e}{T_e}\right) \Delta Z = \frac{dW}{dt} \Delta Z$$

$$a = \frac{dW}{dt}$$

$$\boxed{W = W(E_i)}$$

$$\boxed{g\left(\frac{T_i - T_e}{T_e}\right) = \frac{dW}{dt}}$$

$$\boxed{W_{\max} = \sqrt{2E_i}}$$

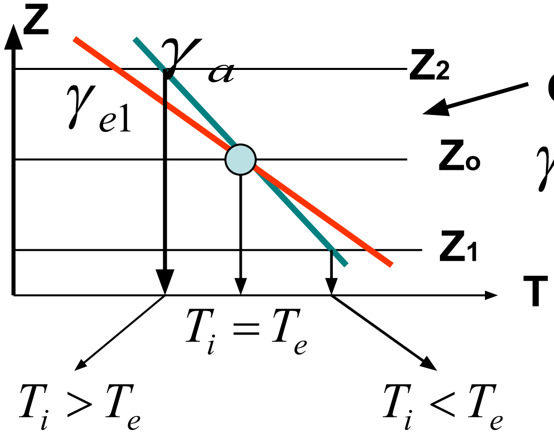
Criterion of the atmosphere stability

$$\gamma = -\frac{\partial T_e}{\partial Z}$$

Environmental vertical temperature gradient or *lapse rate*

$$T_i = T_{i0} - \gamma_a (z - z_0) \approx T_{i0} - 0{,}01(z - z_0)$$

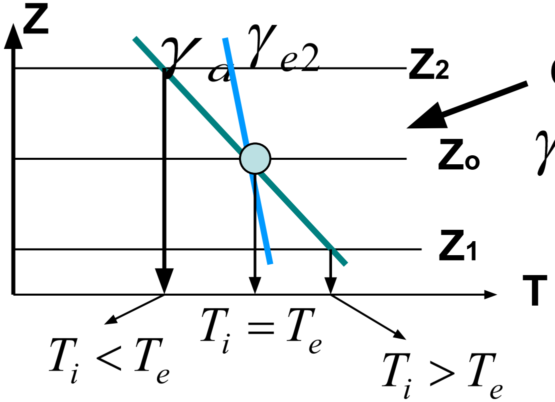
Temperature and other meteorological parameters distribution with height is known as *stratification of the atmosphere.*



Case 1

$$\gamma_{e1} > \gamma_a$$

The atmosphere is unstable (unstable stratification)

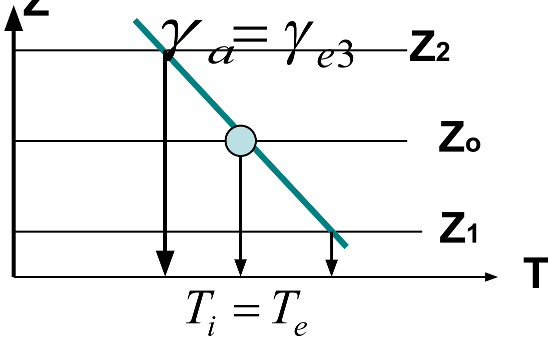


Case 2

$$\gamma_{e1} < \gamma_a$$

The atmosphere is stable (stable stratification)

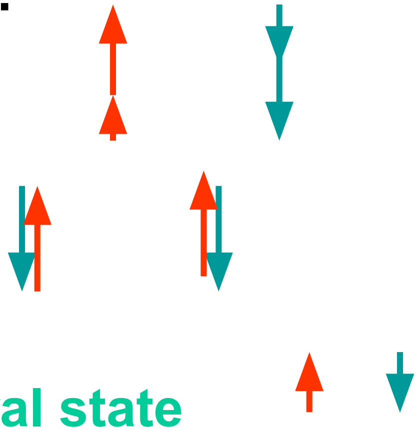
Case 3



Neutral state

$$\boxed{\frac{dW}{dt} = g\left(\frac{T_i - T_e}{T_e}\right)}$$

The reasoning above suggests:



$\gamma_a < \gamma_e$ **The atmosphere is unstable**

$\gamma_a > \gamma_e$ **The atmosphere is stable**

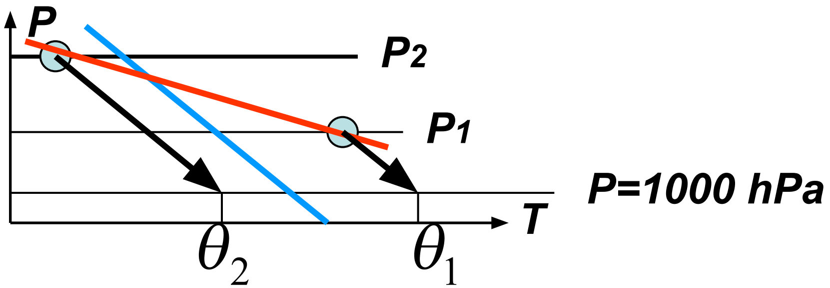
$\gamma_a = \gamma_e$ **The atmosphere is in the neutral state**

Our reasoning and conclusion are valid assuming the process goes without any exchange with environment. In the real atmosphere at leastways some exchange takes place. However, in spite this exchange, conclusion on stability conditions and corresponding vertical displacement are valid.

Potential temperature variation at different types of the stratification

Case 1.
Unstable stratification

$\gamma > \gamma_a$

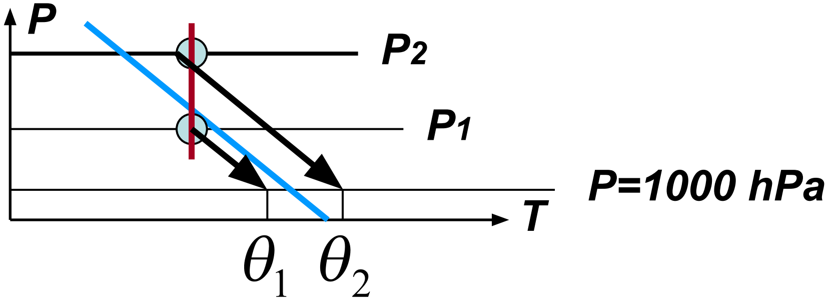


$$\frac{\partial \theta}{\partial Z} < 0$$

Potential temperature decreases with height in unstable atmosphere or *if potential temperature decreases with height*, it means that the *atmosphere is unstable*

Case 2. Stable stratification

$$\gamma < \gamma_a$$



$$\frac{\partial \theta}{\partial Z} > 0$$

Potential temperature increases with height in the stable atmosphere or *if potential temperature increases with height*, it means that the *atmosphere is stable*

From the similar reasoning it goes without saying that at the neutral condition potential temperature does not vary with height.

$$\frac{\partial \theta}{\partial Z} = 0$$

Theoretical support of the reasoning above

$$\boxed{\theta = T\left(\frac{1000}{P}\right)^{R/c_p}} \longrightarrow \ln\theta = \ln T + R/c_p \ln 1000 - \frac{R}{c_p}\ln P$$

$$\frac{1}{\theta}\frac{\partial\theta}{\partial z} = \frac{1}{T}\frac{\partial T}{\partial z} - \frac{R}{c_p P}\frac{\partial P}{\partial z} \longrightarrow -\frac{R}{Pc_p}\frac{\partial P}{\partial z} = \frac{R}{Pc_p}g\rho = g\frac{R}{c_p P}\cancel{\frac{P}{RT}}$$

$$\boxed{\begin{array}{l} \frac{1}{\theta}\frac{\partial\theta}{\partial z} = \frac{1}{T}\frac{\partial T}{\partial z} + \frac{1}{T}\frac{g}{c_p} \\ \frac{g}{c_p} = \gamma_a;\ -\frac{\partial T}{\partial z} = \gamma \end{array}} \longrightarrow \frac{1}{\theta}\frac{\partial\theta}{\partial z} = \frac{1}{T}\gamma_a - \frac{1}{T}\gamma = \frac{1}{T}(\gamma_a - \gamma)$$

$$\boxed{\frac{\partial\theta}{\partial z} = \frac{\theta}{T}(\gamma_a - \gamma)}$$

$\theta > 0$

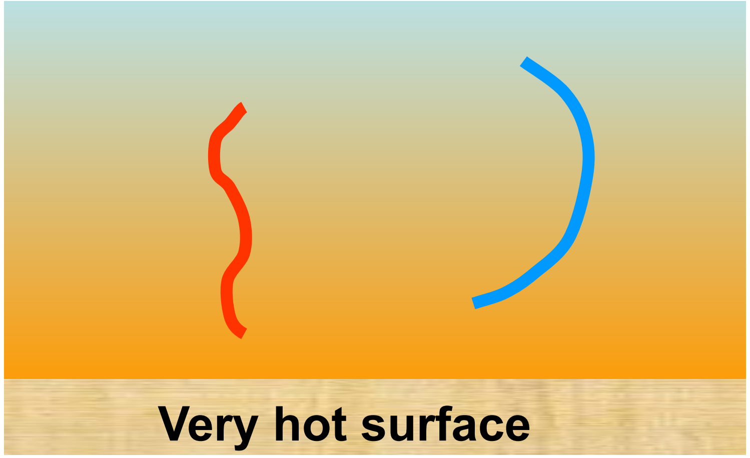
$T > 0$

Unstable state $\gamma_a - \gamma < 0 \longrightarrow \frac{\partial\theta}{\partial z} < 0$

Stable state $\gamma_a - \gamma > 0 \longrightarrow \frac{\partial\theta}{\partial z} > 0$

Neutral state $\gamma_a - \gamma = 0 \longrightarrow \frac{\partial\theta}{\partial z} = 0$

All above refers to the dry air (non-saturated air)



In case of very dry air, instead of cloud formation, overturning of layer occurs. Cold air moves down, and the overheated air ascents up in convective flows.

This process results in formation of eddies similar to tornado, but these vortexes are very small. In our latitudes, this process leads to gusty winds and squalls.

Adiabatic processes in moist but non-saturated air

Suppose, a unit of non-saturated mass of air get some amount of energy *dq*.

$$dq = du_{d.a.} + du_{w.a.} + dw$$

$$du_{d.a.} = (1-s)c_{v.d.}dT_i \qquad du_{w.a.} = sc_{v.w.}dT_i$$

***S* denotes a fraction of water vapor**

$c_{v.a.}$ **and** $c_{v.a.}$ **are specific heat capacities of the dry air and water vapor respectively.**

$$\boxed{dq = [(1-s)c_{v.d.} + sc_{v.w.}]dT_i + Pdv_i} \quad Pv_i = [R_c(1-s) + R_w s]T_i = RT_i$$

Equation of state for moist, non-saturated air

$$Pdv_i = [R_c(1-s) + R_w s]dT_i - v_i dP$$

$$c_v + R = c_p$$

$$dq = [(1-s)c_{v.d.} + sc_{v.w.}]dT_i + [R_c(1-s) + R_w s]dT_i - v_i dP - RT_i\frac{dP}{P}$$

$$v_i = \frac{RT_i}{P}$$

$$\boxed{dq = [(1-s)c_{p.} + sc_p]dT_i - RT_i\frac{dP}{P}}$$

$$R_c = 287 \, {}^{J}\!/_{kg \cdot K} \qquad c_{p.d} = 1005 \, {}^{J}\!/_{kg \cdot K} \qquad c_{p.w} = 1864 \, {}^{J}\!/_{kg \cdot K}$$

$$R_w = 461 \, {}^{J}\!/_{kg \cdot K} \qquad c_{v.d.} = 718 \, {}^{J}\!/_{kg \cdot K} \qquad c_{v.w.} = 1409 \, {}^{J}\!/_{kg \cdot K}$$

$$\boxed{s_{\max} = 0{,}04} \qquad \left[(1 - 0{,}04)1005 + 0{,}04 \cdot 1864\right] = 1039 \, {}^{J}\!/_{kgK} = c_{p.m.}$$

For practical purposes we may adopt

$$\gamma_a = \frac{g}{c_{p.m}} = \frac{9{,}81}{1039} \approx 0{,}95 \, {}^{K}\!/_{100m} \approx 1 \, {}^{K}\!/_{100m}$$

$$\boxed{R = R_c = 287 \, {}^{J}\!/_{kgK}}$$

$$R = \left[(1 - 0{,}04)287 + 0{,}04 \cdot 461\right] = 294 \, {}^{J}\!/_{kgK}$$

$$\boxed{c_p = c_{p.d} = 1005 \, {}^{J}\!/_{kgK}}$$

Conclusion

At adiabatic ascent, the temperature of a moist, non-saturated parcel of air varies, practically, in the same way as a dry parcel does.

The dry adiabat is a curve of state for the moist, non-saturated air too.