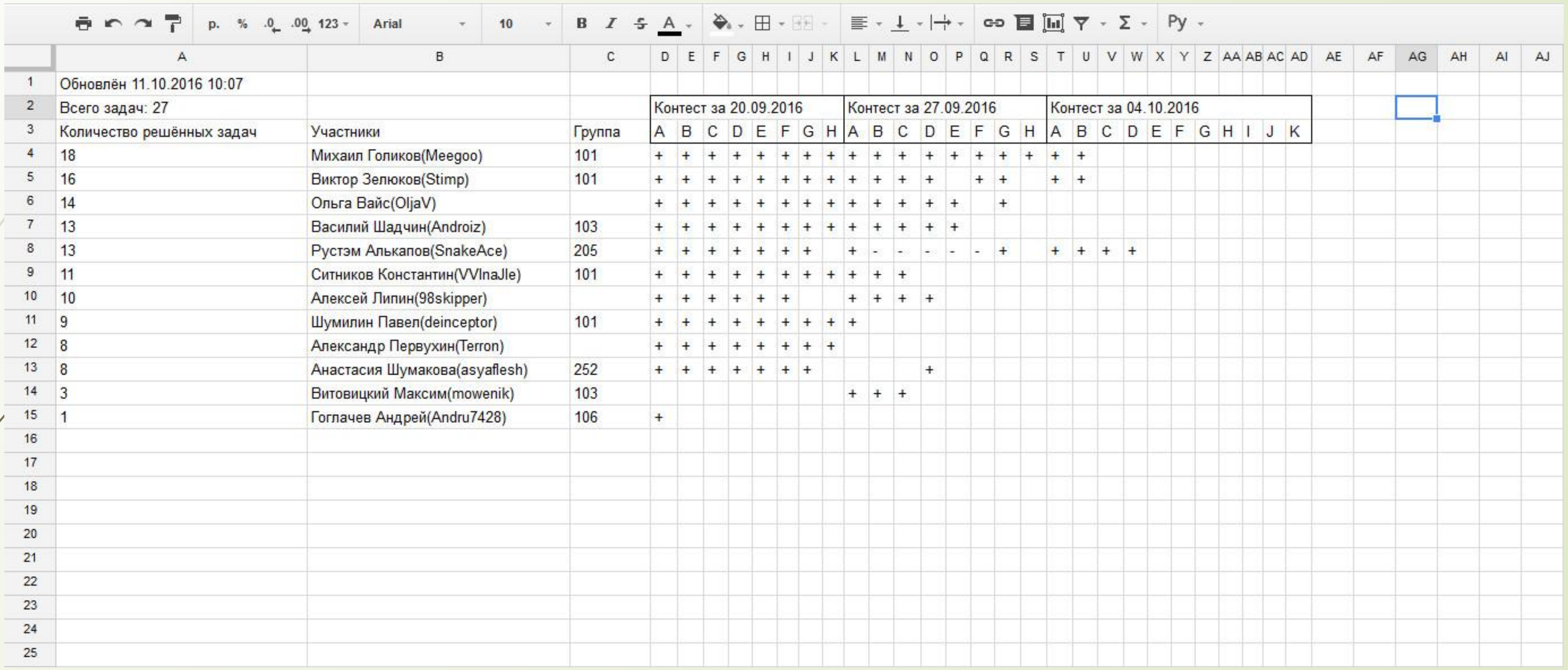


Обновлён 11.10.2016 10:07

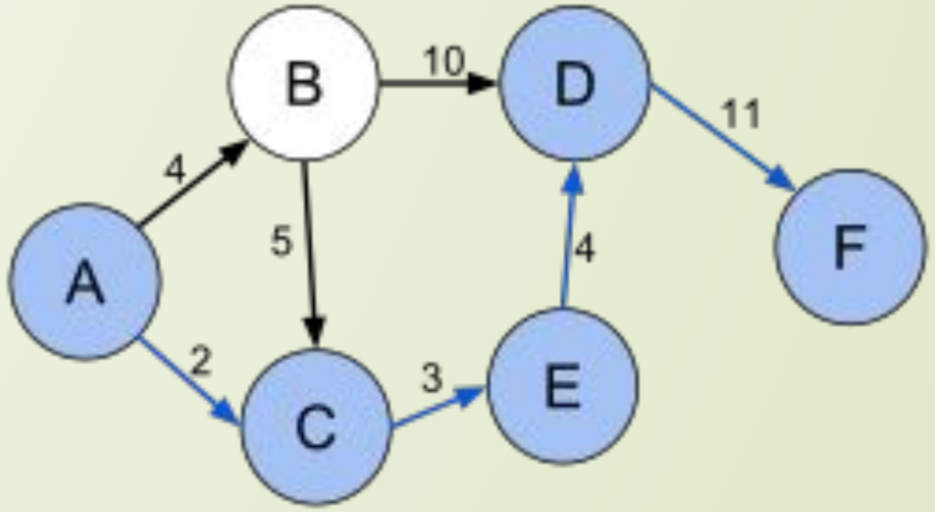
Всего задач: 27			Контест за 20.09.2016								Контест за 27.09.2016								Контест за 04.10.2016										
Количество решённых задач	Участники	Группа	A	B	C	D	E	F	G	H	A	B	C	D	E	F	G	H	A	B	C	D	E	F	G	H	I	J	K
18	Михаил Голиков(Meegoo)	101	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+									
16	Виктор Зелюков(Stimp)	101	+	+	+	+	+	+	+	+	+	+	+	+		+	+		+	+									
14	Ольга Вайс(OljaV)		+	+	+	+	+	+	+	+	+	+	+	+	+		+												
13	Василий Шадчин(Androiz)	103	+	+	+	+	+	+	+	+	+	+	+	+	+														
13	Рустэм Алькапов(SnakeAce)	205	+	+	+	+	+	+	+		+	-	-	-	-	-	+		+	+	+	+							
11	Ситников Константин(VVlnaJle)	101	+	+	+	+	+	+	+	+	+	+	+																
10	Алексей Липин(98skipper)		+	+	+	+	+	+			+	+	+	+															
9	Шумилин Павел(deinceptor)	101	+	+	+	+	+	+	+	+	+																		
8	Александр Первухин(Terron)		+	+	+	+	+	+	+	+																			
8	Анастасия Шумакова(asyaflesh)	252	+	+	+	+	+	+	+					+															
3	Витовицкий Максим(mowenik)	103									+	+	+																
1	Гоглачев Андрей(Andru7428)	106	+																										

Shortest paths and spanning trees in graphs

Lyzhin Ivan, 2015

Shortest path problem

- The problem of finding a path between two vertices such that the sum of the weights of edges in path is minimized.
- Known algorithms:
  - Dijkstra
  - Floyd–Warshall
  - Bellman–Ford
  - and so on...



Dijkstra algorithm

1. There are two sets of vertices – visited and unvisited.
2. For visited vertices we know minimal distance from start. For unvisited vertices we know some distance which can be not minimal.
3. Initially, all vertices are unvisited and distance to each vertex is **INF.** Only distance to start node is equal **0**.
4. On each step choose unvisited vertex with minimal distance. Now it’s visited vertex. And try to relax distance of neighbors.

Complexity: trivial implementation **O(|V|^2+|E|)**

implementation with set **O(|E|log|V|+|V|log|V|)**

Trivial implementation

```
void dijkstra(int s)
{
 vector<bool> mark(n, false);
 vector<int> d(n, INF);
 d[s] = 0;
 for (int i = 0; i < n; ++i)
 {
  int u = -1;
  for (int j = 0; j < n; ++j)
   if (!mark[j] && (u == -1 || d[j] < d[u]))
    u = j;
  mark[u] = true;
  for (v - сосед u)
   d[v] = min(d[v], d[u] + weight(uv));
 }
}
```

Implementation with set

```
void dijkstra(int s)
{
    set<pair<int, int> > q; //(dist[u], u)
    vector<int> dist(n, INF);
    dist[s] = 0;
    q.insert(mp(0, s));
    while(!q.empty())
    {
        int cur = q.begin()->second;
        q.erase(q.begin());
        for(auto e : g[cur])
            if(dist[e.first] > dist[cur]+e.second)
            {
                q.erase(mp(dist[e.first], e.first));
                dist[e.first] = dist[cur] + e.second;
                q.insert(mp(dist[e.first], e.first));
            }
    }
}
```

Implementation with priority queue

```
void dijkstra(int s)
{
    priority_queue<pair<int, int> > q; //(dist[u], u)
    vector<int> dist(n, INF);
    dist[s] = 0;
    q.push(mp(0, s));
    while(!q.empty())
    {
        int cur = q.top().second;
        int cur_d = -q.top().first; q.pop();
        if(cur_d > dist[cur]) continue;
        for(auto e : g[cur])
            if(dist[e.first] > dist[cur]+e.second)
            {
                dist[e.first] = dist[cur] + e.second;
                q.push(mp(-dist[e.first], e.first));
            }
    }
}
```

Floyd–Warshall algorithm

1. Initially, **dist[u][u]=0** and for each edge (u, v): **dist[u][v]=weight(u, v)**
2. On iteration **k** we let use vertex **k** as intermediate vertex and for each pair of vertices we try to relax distance.

   dist[u][v] = min(dist[u][v], dist[u][k]+dist[k][v])

Complexity: **O(|V|^3)**

Implementation

```cpp
void floyd_warshall()
{
    vector<vector<int> > dist(n, vector<int>(n, INF));
    for (int i = 0; i < n; ++i)
        dist[i][i] = 0;
    for (int i = 0; i < n; ++i)
        for (auto e : g[i])
            dist[i][e.first] = e.second;
    for (int k = 0; k < n; ++k)
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j)
                dist[i][j] = min(dist[i][j], dist[i][k] + dist[k][j]);
}
```

Bellman–Ford algorithm

- |V|-1 iterations, on each we try relax distance with all edges.
- If we can relax distance on |V| iteration then negative cycle exists in graph
- Why |V|-1 iterations? Because the longest way without cycles from one node to another one contains no more |V|-1 edges.
- Complexity **O(|V||E|)**

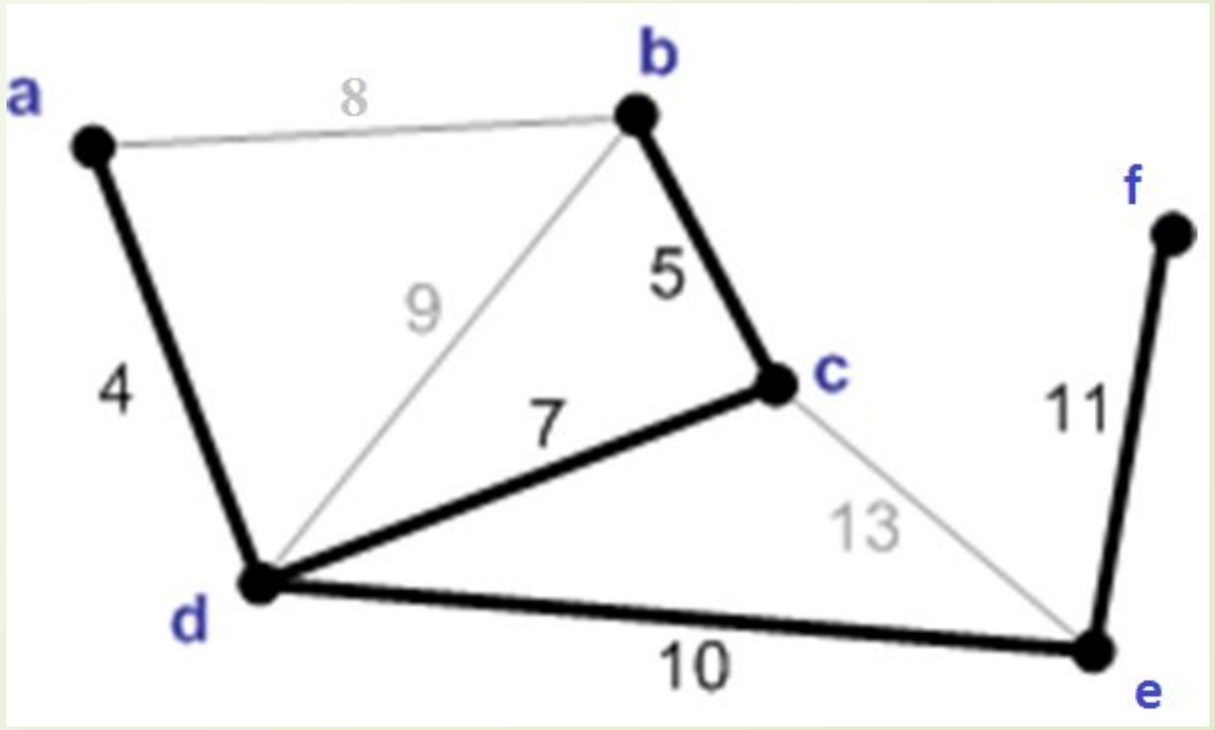
Implementation

```
void bellman_ford(int s)
{
    vector<int> dist(n, INF);
    dist[s] = 0;
    for (int i = 0; i < n - 1; ++i)
        for (auto e : edges)
        dist[e.v] = min(dist[e.v], dist[e.u] + e.weight);

    for (auto e : edges)
        if (dist[e.v] > dist[e.u] + e.weight)
            cout << "Negative cycle!" << endl;
}
```

Minimal spanning tree

- A spanning tree T of an undirected graph G is a subgraph that includes all of the vertices of G that is a tree.
- A minimal spanning tree is a spanning tree and sum of weights is minimized.



Prim’s algorithm

1. Initialize a tree with a single vertex, chosen arbitrarily from the graph.
2. Grow the tree by one edge: of the edges that connect the tree to vertices not yet in the tree, find the minimum-weight edge, transfer it to the tree and try to relax distance for neighbors.
3. Repeat step 2 (until all vertices are in the tree).

Complexity: trivial implementation **O(|V|^2+|E|)**

implementation with set **O(|E|log|V|+|E|)**

Implementation

```
void prima()
{
    set<pair<int, int> > q; //(dist[u], u)
    vector<int> dist(n, INF);
    dist[0] = 0;
    q.insert(mp(0, 0));
    while (!q.empty())
    {
        int cur = q.begin()->second;
        q.erase(q.begin());
        for (auto e : g[cur])
            if (dist[e.first] > e.second)
            {
                q.erase(mp(dist[e.first], e.first));
                dist[e.first] = e.second;
                q.insert(mp(dist[e.first], e.first));
            }
    }
}
```

Kruskal's algorithm

- Create a forest F (a set of trees), where each vertex in the graph is a separate tree
- Create a set S containing all the edges in the graph
- While S is nonempty and F is not yet spanning:
  - remove an edge with minimum weight from S
  - if the removed edge connects two different trees then add it to the forest F, combining two trees into a single tree

Complexity: trivial **O(|V|^2+|E|log|E|)**

with DSU **O(|E|log|E|)**

Trivial implementation

```
void trivial_kruskal()
{
    vector<int> color(n);
    for (int i = 0; i < n; ++i)
        color[i] = i;
    sort(edges.begin(), edges.end());
    for(auto e : edges)
        if(color[e.u]!=color[e.v])
        {
            add_to_spanning_tree(e);
            int c1 = color[e.u];
            int c2 = color[e.v];
            for (int i = 0; i < n; ++i)
                if (color[i] == c1)
                    color[i] = c2;
        }
}
```

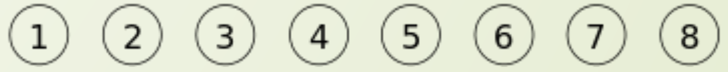
Implementation with DSU

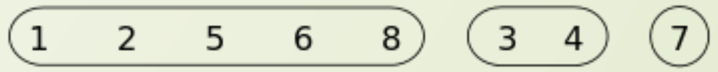
```
void kruskal()
{
    DSU dsu(n);
    sort(edges.begin(), edges.end());
    for(auto e : edges)
        if(dsu.findSet(e.u)!=dsu.findSet(e.v))
        {
            add_to_spanning_tree(e);
            dsu.unionSets(e.u, e.v);
        }
}
```

Disjoint-set-union (DSU)

Two main operations:

- **Find(U)** – return root of set, which contains U, complexity **O(1)**
- **Union(U, V)** – join sets, which contain U and V, complexity **O(1)**

After creating DSU:


After some operations:


Implementation

```
struct DSU
{
    vector<int> p;

    DSU(int n) {
        p.resize(n);
        for (int i = 0; i < n; ++i)
            p[i] = i;
    }

    int find(int u) {
        return u == p[u] ? u : find(p[u]);
    }

    void merge(int u, int v) {
        int pu = find(u);
        int pv = find(v);
        p[pv] = pu;
    }
};
```

Path compression

- When we go up, we can remember root of set for each vertex in path

```
int findSet(int u)
{
    return u == p[u] ? u : p[u] = findSet(p[u]);
}
```

Union by size

```
DSU(int size)
{
    p.resize(size);
    sizes.resize(size, 1);
    for (int i = 0; i < size; ++i)
        p[i] = i;
}

int unionSets(int u, int v)
{
    int pu = findSet(u);
    int pv = findSet(v);
    if (pu == pv) return;
    if (sizes[pu] < sizes[pv])
        swap(pu, pv);
    p[pv] = pu;
    sizes[pu] += sizes[pv];
}
```

Links

- https://en.wikipedia.org/wiki/Dijkstra%27s_algorithm
- https://en.wikipedia.org/wiki/Floyd–Warshall_algorithm
- https://en.wikipedia.org/wiki/Bellman–Ford_algorithm
- https://en.wikipedia.org/wiki/Kruskal%27s_algorithm
- https://en.wikipedia.org/wiki/Prim%27s_algorithm
- https://en.wikipedia.org/wiki/Disjoint-set_data_structure
- http://e-maxx.ru/algo/topological_sort

Home task

- http://ipc.susu.ac.ru/210-2.html?problem=1903
- http://ipc.susu.ac.ru/210-2.html?problem=186
- http://acm.timus.ru/problem.aspx?space=1&num=1982
- http://acm.timus.ru/problem.aspx?space=1&num=1119
- http://acm.timus.ru/problem.aspx?space=1&num=1210
- http://acm.timus.ru/problem.aspx?space=1&num=1272