



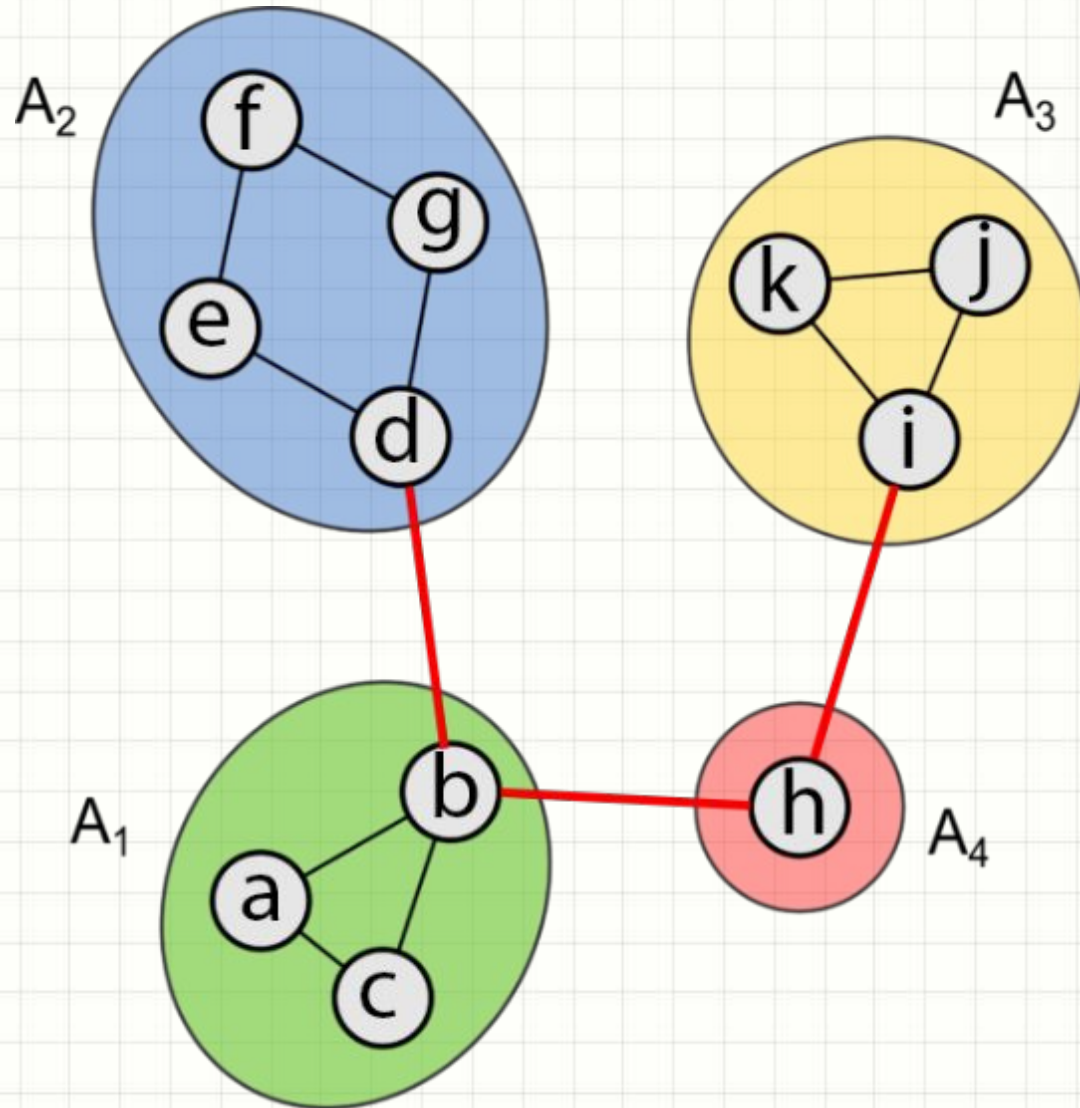
BRIDGES AND CUT VERTICES

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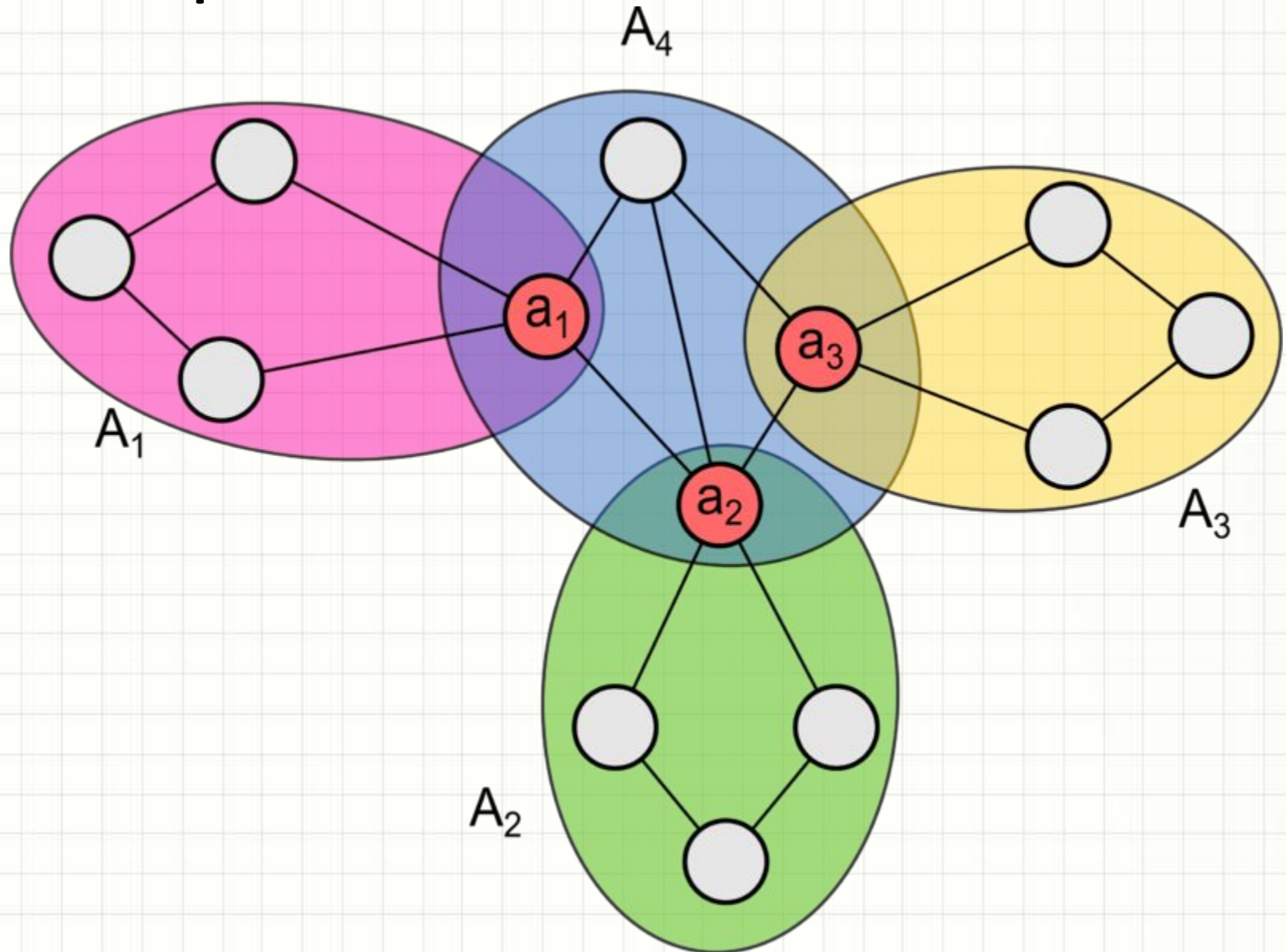
Definitions

- *Bridge* – an edge of a graph whose deletion increases the number of connected components.
- *Cut vertex* – a vertex whose deletion increases the number of connected components.

Example - Bridges

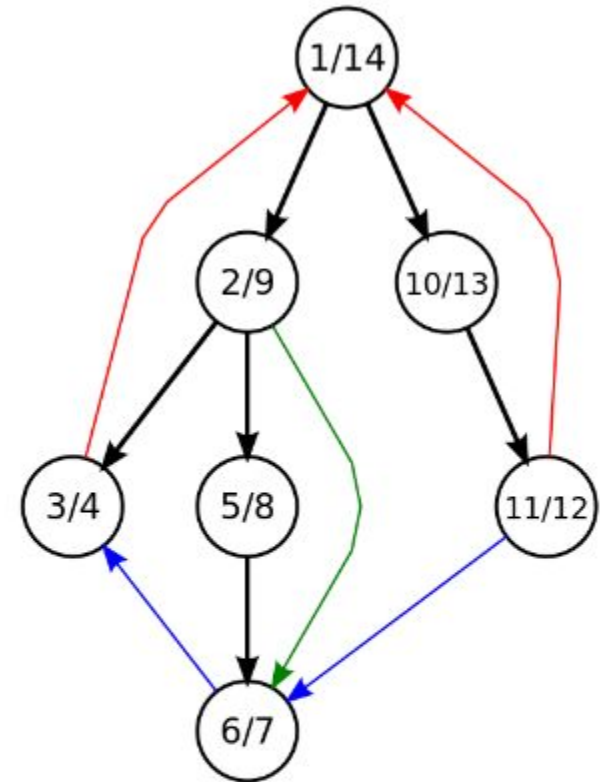


Example – Cut Vertices



Definitions for Depth-First-Search in undirected graph

- *Tree edge* – edge to unvisited vertex.
- *Back edge* – edge to visited vertex.
- *Parent edge* – edge to parent vertex.



- 3/4 метки времени (вход/выход)
- дуги дерева обхода
 - прямые дуги
 - обратные дуги
 - перекрёстные дуги

Algorithm - Bridges

- Start DFS from unvisited vertex.
- $tin[v]$ – entry time for vertex.

- $fup[v] = \min \begin{cases} tin[v] \\ tin[p], \text{ for all } (v, p) - \text{back edge} \\ fup[to], \text{ for all } (v, to) - \text{tree edge} \end{cases}$

- If $fup[to] > tin[v]$, then (v, to) is bridge.

Implementation - Bridges

```
void dfs(int v, int p = -1) {
    used[v] = true;
    tin[v] = fup[v] = timer++;
    for (size_t i = 0; i < g[v].size(); ++i) {
        int to = g[v][i];
        if (to == p) continue; // Parent edge
        if (used[to]) // Back edge
            fup[v] = min(fup[v], tin[to]);
        else { // Tree edge
            dfs(to, v);
            fup[v] = min(fup[v], fup[to]);
            if (fup[to] > tin[v])
                IS_BRIDGE(v, to);
        }
    }
}
```

Algorithm – Cut Vertices

- Start DFS from unvisited vertex.
- $tin[v]$ – entry time for vertex.
- $fup[v] = \min \begin{cases} tin[v] \\ tin[p], \text{ for all } (v, p) - \text{back edge} \\ fup[to], \text{ for all } (v, to) - \text{tree edge} \end{cases}$
- If $fup[to] \geq tin[v]$ and $v \neq root$, then v is cut vertex.
- If $v = root$ and $children > 1$, then v is cut vertex.

Implementation – Cut Vertices

```
void dfs(int v, int p = -1) {
    used[v] = true;
    tin[v] = fup[v] = timer++;
    int children = 0; // Number of children
    for (size_t i = 0; i < g[v].size(); ++i) {
        int to = g[v][i];
        if (to == p) continue; // Parent edge
        if (used[to]) // Back edge
            fup[v] = min(fup[v], tin[to]);
        else { // Tree edge
            dfs(to, v);
            fup[v] = min(fup[v], fup[to]);
            if (fup[to] >= tin[v] && p != -1)
                IS_CUTPOINT(v);
            ++children;
        }
    }
    if (p == -1 && children > 1) // If root
        IS_CUTPOINT(v);
}
```

Additional links and home task

- Bridge searching

http://e-maxx.ru/algo/bridge_searching

- Cut vertex searching

<http://e-maxx.ru/algo/cutpoints>

- Tasks

<http://codeforces.com/group/Hq4vrJcA4s/contest/100703/problem/E>