



TWO POINTERS METHOD

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Problem

- There is array A with N positive integers.
- *Segment of array* – a sequence of one or more consecutive elements in array.
- *D-good segment* – segment, in which sum of elements not greater than D .
- Count the pairs (L, R) such that segment $[L, R]$ of array A is D -good.

1. Very primitive solution

- Sum elements for each pair (L, R).

```
for (int i = 0; i < n; ++i)
  for (int j = i; j < n; ++j)
  {
    int sum = 0;
    for (int k = i; k <= j; ++k)
      sum += a[k];
    if (sum <= d)
      ans++;
  }
```

$O(N^3)$

2. Primitive solution

- Notice that $\text{sum}(L, R) = \text{sum}(L, R-1) + A[R]$
- If $\text{sum}(L, R1) > D$ then $\text{sum}(L, R2) > D$ for each $R2 > R1$

```
for (int i = 0; i < n; ++i)
{
    int sum = 0;
    for (int j = i; j < n; ++j)
    {
        sum += a[j];
        if (sum <= d) ans++;
        else break;
    }
}
```

$O(N^2)$

3. Good solution

- Notice that it's enough to find $\max R(L) = \max(R)$ such $\text{sum}(L, R) \leq D$ and $\text{sum}(L, R') > D$ for each $R' > R$.
- We can precompute prefix sums and then find $\max R$ by binary search.

3. Good solution

```
prefixSum[0] = a[0];
for (int i = 1; i < n; ++i)
    prefixSum[i] = prefixSum[i - 1] + a[i];
for (int i = 0; i < n; ++i)
{
    if (a[i] > d) continue;
    int l = i, r = n;
    while(r-l>1)
    {
        int mid = (l + r) / 2;
        if (prefixSum[mid] - prefixSum[i] + a[i] <= d)
            l = mid;
        else
            r = mid;
    }
    ans += l - i + 1;
}
```

$O(N \log N)$

4. Best solution

- Notice that $\text{maxR}(L) \geq \text{maxR}(L-1)$. So we can start finding $\text{maxR}(L)$ from $\text{maxR}(L-1)$.
- In this way out pointer R goes only forward.

```
int right = -1;
int sum = 0;
for (int i = 0; i < n; ++i)
{
    while (right + 1 < n && sum + a[right + 1] <= d)
        sum += a[++right];
    ans += right - i + 1;
    sum -= a[i];
}
```

$O(N)$

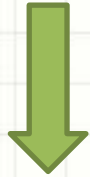
Tracing, step 0

D=6

ANS=0

SUM=0

Left=0



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=-1

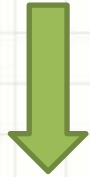
Tracing, step 1

D=6

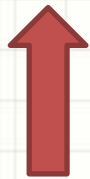
ANS=0

SUM=1

Left=0



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=0

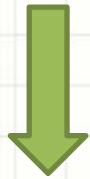
Tracing, step 2

D=6

ANS=0

SUM=3

Left=0



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=1

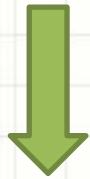
Tracing, step 3

D=6

ANS=0

SUM=6

Left=0



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=2

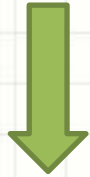
Tracing, step 4

D=6

ANS=3

SUM=6

Left=0



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=2

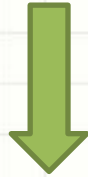
Tracing, step 5

D=6

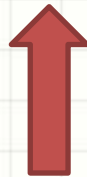
ANS=3

SUM=5

Left=1



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=2

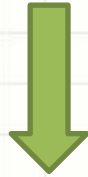
Tracing, step 6

D=6

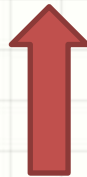
ANS=5

SUM=5

Left=1



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=2

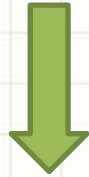
Tracing, step 7

D=6

ANS=5

SUM=3

Left=2



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=2

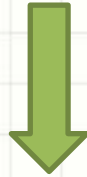
Tracing, step 8

D=6

ANS=5

SUM=6

Left=2



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=3

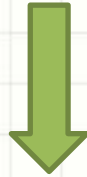
Tracing, step 9

D=6

ANS=7

SUM=6

Left=2



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=3

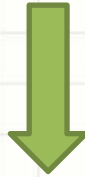
Tracing, step 10

D=6

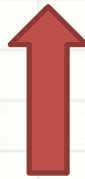
ANS=7

SUM=3

Left=3



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=3

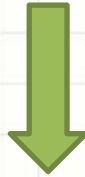
Tracing, step 11

D=6

ANS=8

SUM=3

Left=3



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=3

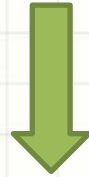
Tracing, step 12

D=6

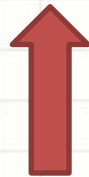
ANS=8

SUM=0

Left=4



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=3

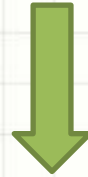
Tracing, step 13

D=6

ANS=8

SUM=-7

Left=5



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=3

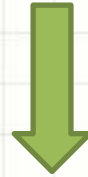
Tracing, step 14

D=6

ANS=8

SUM=0

Left=5



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=4

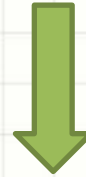
Tracing, step 15

D=6

ANS=8

SUM=-8

Left=6



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=4

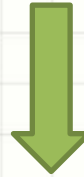
Tracing, step 16

D=6

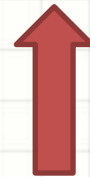
ANS=8

SUM=0

Left=6



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=5

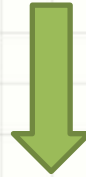
Tracing, step 17

D=6

ANS=8

SUM=6

Left=6



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=6

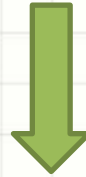
Tracing, step 18

D=6

ANS=9

SUM=6

Left=6



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=6

Tracing, step 19

D=6

ANS=9

SUM=0

Left=7



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=6

Tracing, step 20

D=6

ANS=9

SUM=4

Left=7



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=7

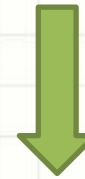
Tracing, step 21

D=6

ANS=9

SUM=6

Left=7



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=8

Tracing, step 22

D=6

ANS=11

SUM=6

Left=7



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=8

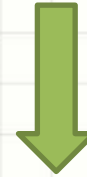
Tracing, step 23

D=6

ANS=11

SUM=2

Left=8



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=8

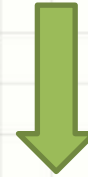
Tracing, step 24

D=6

ANS=11

SUM=5

Left=8



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=9

Tracing, step 25

D=6

ANS=13

SUM=5

Left=8



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=9

Tracing, step 26

D=6

ANS=13

SUM=3

Left=9



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=9

Tracing, step 27

D=6

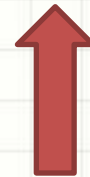
ANS=14

SUM=3

Left=9



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=9

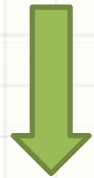
Tracing, step 28

D=6

ANS=14

SUM=0

Left=9



1	2	3	3	7	8	6	4	2	3
---	---	---	---	---	---	---	---	---	---



Right=9

Other examples

- There are 2 sorted arrays of integers: A and B. Count pairs (i, j) such that $A[i]=B[j]$.
- There are N points on circle. Find two points such that distance between them is maximal.
- There are N points on circle. Find three points such that area of triangle is maximal.

Additional links and home task

- Article about two pointers method

<http://informatics.mccme.ru/mod/resource/view.php?id=12716>

- Discussion on codeforces

<http://codeforces.com/blog/entry/4136?locale=ru>

- Contest

<http://codeforces.com/group/Hq4vrJcA4s/contest/206340>