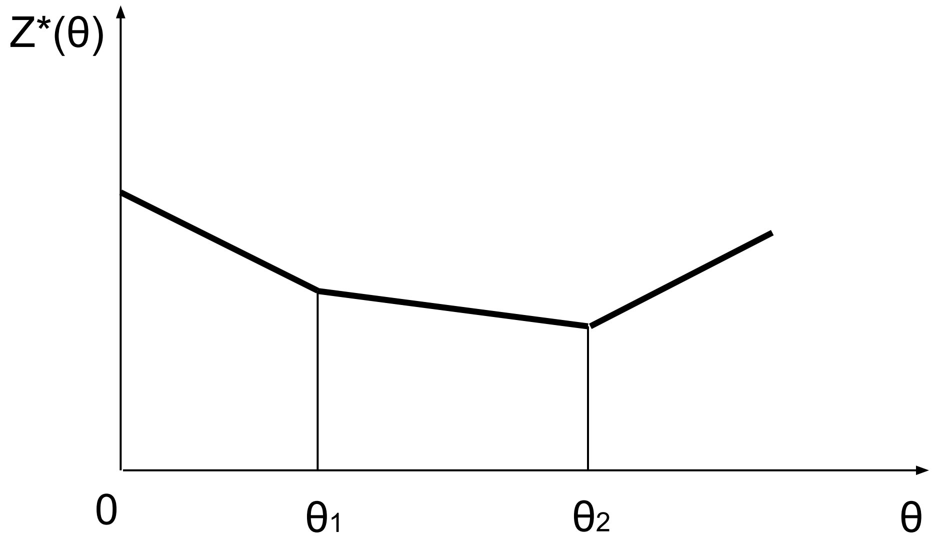
# Parametric Linear Programming

# Systematic Changes in $c_j$

- Objective function $Z = \sum_{j=1}^{n} c_j x_j$ is replaced by

$$Z(\theta) = \sum_{j=1}^{n} \left(c_j + \alpha_j \theta\right) x_j$$

- Find the optimal solution as a function of θ



# Example: Wyndor Glass Problem

- $Z(\theta) = (3 + 2\theta)\, x_1 + (5 - \theta)\, x_2$

# Example: Wyndor Glass Problem

| Range of θ | Basic Var. | Z | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | RHS |
|---|---|---|---|---|---|---|---|---|
| $0 \le \theta \le 9/7$ | $Z(\theta)$ | 1 | 0 | 0 | 0 | $(9-7\theta)/6$ | $(3+2\theta)/3$ | $36-2\theta$ |
| | $x_3$ | 0 | 0 | 0 | 1 | 1/3 | -1/3 | 2 |
| | $x_2$ | 0 | 0 | 1 | 0 | 1/2 | 0 | 6 |
| | $x_1$ | 0 | 1 | 0 | 0 | -1/3 | 1/3 | 2 |

# Example: Wyndor Glass Problem

| Range of θ | Basic Var. | Z | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | RHS |
|---|---|---|---|---|---|---|---|---|
| $9/7 \le \theta \le 5$ | $Z(\theta)$ | 1 | 0 | 0 | $(-9+7\theta)/2$ | 0 | $(5-\theta)/2$ | $27+5\theta$ |
| | $x_4$ | 0 | 0 | 0 | 3 | 1 | -1 | 6 |
| | $x_2$ | 0 | 0 | 1 | -3/2 | 0 | 1/2 | 3 |
| | $x_1$ | 0 | 1 | 0 | 1 | 0 | 0 | 4 |

# Example: Wyndor Glass Problem

| Range of θ | Basic Var. | Z | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | RHS |
|---|---|---|---|---|---|---|---|---|
| $\theta \geq 5$ | $Z(\theta)$ | 1 | 0 | $-5+\theta$ | $3+2\theta$ | 0 | 0 | $12+8\theta$ |
| | $x_4$ | 0 | 0 | 2 | 0 | 1 | 0 | 12 |
| | $x_5$ | 0 | 0 | 2 | -3 | 0 | 1 | 6 |
| | $x_1$ | 0 | 1 | 0 | 1 | 0 | 0 | 4 |

# Procedure Summary for Systematic Changes in $c_j$

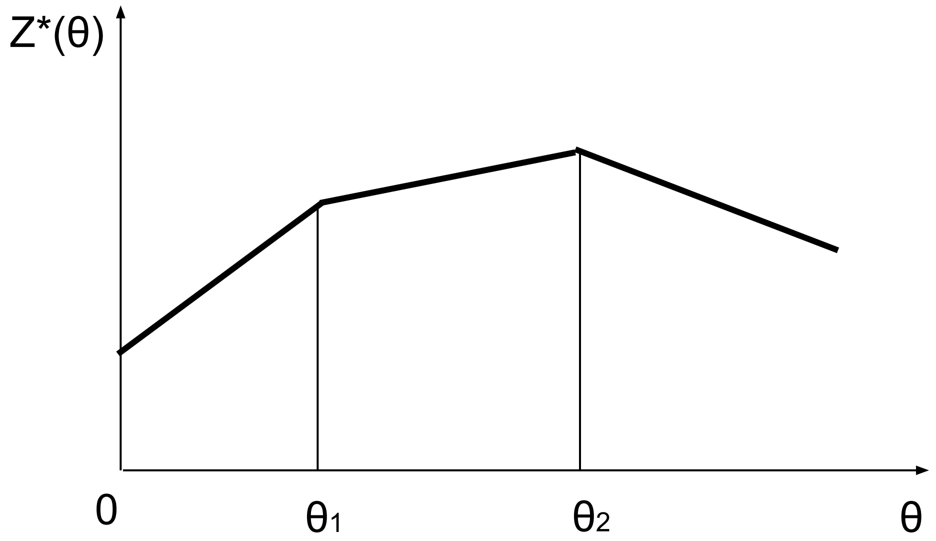
1. Solve the problem with $\theta = 0$ by the simplex method.
2. Use the sensitivity analysis procedure to introduce the $\Delta c_j = \alpha_j \theta$ changes into Eq.(0).
3. Increase $\theta$ until one of the **nonbasic** variables has its coefficient in Eq.(0) go negative (or until $\theta$ has been increased as far as desired).
4. Use this variable as the entering basic variable for an iteration of the **simplex** method to find the new optimal solution. Return to Step 3.

# Systematic Changes in $b_i$

- Constraints $\sum_{j=1}^{n} a_{ij} x_j \le b_i$ for $i = 1, 2, \ldots, m$ are replaced by

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i + \alpha_i \theta \quad \text{for } i = 1, 2, \ldots, m$$

- Find the optimal solution as a function of θ



# Example: Wyndor Glass Problem

- $y_1 \qquad\quad + 3y_3 \geq 3 + 2\theta$
  $2y_2 + 2y_3 \geq 5 - \theta$

# Example: Wyndor Glass Problem

| Range of θ | Basic Var. | Z | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ | RHS |
|---|---|---|---|---|---|---|---|---|
| | $Z(\theta)$ | 1 | 2 | 0 | 0 | 2 | 6 | $-36+2\theta$ |
| $0 \leq \theta \leq 9/7$ | $y_3$ | 0 | 1/3 | 0 | 1 | -1/3 | 0 | $(3+2\theta)/3$ |
| | $y_2$ | 0 | -1/3 | 1 | 0 | 1/3 | -1/2 | $(9-7\theta)/6$ |

# Example: Wyndor Glass Problem

| Range of $\theta$ | Basic Var. | Z | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ | RHS |
|---|---|---|---|---|---|---|---|---|
| | $Z(\theta)$ | 1 | 0 | 6 | 0 | 4 | 3 | $-27-5\theta$ |
| $9/7 \le \theta \le 5$ | $y_3$ | 0 | 0 | 1 | 1 | 0 | -1/2 | $(5-\theta)/2$ |
| | $y_1$ | 0 | 1 | -3 | 0 | -1 | 3/2 | $(-9+7\theta)/2$ |

# Example: Wyndor Glass Problem

| Range of $\theta$ | Basic Var. | Z | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ | RHS |
|---|---|---|---|---|---|---|---|---|
| | $Z(\theta)$ | 1 | 0 | 12 | 6 | 4 | 0 | $-12-8\theta$ |
| $\theta \geq 5$ | $y_5$ | 0 | 0 | -2 | -2 | 0 | 1 | $-5+\theta$ |
| | $y_1$ | 0 | 1 | 0 | 3 | -1 | 0 | $3+2\theta$ |

# Procedure Summary for Systematic Changes in $b_i$

1. Solve the problem with $\theta = 0$ by the simplex method.
2. Use the sensitivity analysis procedure to introduce the $\Delta b_i = \alpha_i \theta$ changes to the *right side* column.
3. Increase $\theta$ until one of the **basic** variables has its value in the *right side* column go negative (or until $\theta$ has been increased as far as desired).
4. Use this variable as the leaving basic variable for an iteration of the **dual simplex** method to find the new optimal solution. Return to Step 3.