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Forecasting Free Cash Flow of an Industrial Enterprise Using Fuzzy Set Tools

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Motivation

- The problem of forecasting companies' cash flows is important in the growing uncertainty of the business environment
- It is also discussed in the literature (Kaplan, Ruback, 1995; Fridson, Alvarez, 2009; Cheng, Czernkowski, 2010; Pae, Yoon, 2012; Ruppert, 2017)
- One of the main objectives of forecasting is to enhance the enterprise's ability to react to changes of the external environment that might affect the performance

Methods of Cash Flow Forecasting

$$FCFF = EBIT(1 - Tax\ Rate) + DA - \Delta NWC - Capex$$

Method	**Applicability**
Linear trend Analytic formulas: polynomial; logarithmic; exponential; geometrical	Used in the context of a company's progressive development Requires regularity, or rhythmicity, unrelated to seasonality: when using cyclical indicators; when forecasting indicators that, after a period of growth/drop, reach a certain level of stability; when a linear trend having clearly defined curvature is present; or at an appropriate stage of an enterprise's life cycle.
The Chained Percentage Ratio Method	Seasonal fluctuation is required
Correlation with macroeconomic indicators	Proven relation with the macroeconomic situation is required
The Brown Method	Forecasting is available for a short period
The Holt-Winters Method	Partitioning of seasonal fluctuation sand main trend to track, analyze and evaluate their mutual influence

Revenues as a Key Element to be Forecasted

- Revenues generated by various centers of financial responsibilities and revenues from varying focus areas generated under the influence of different factors may be forecasted using various methods
- The resulting forecast is based on data obtained on all levels of research starting with the macroeconomic level and down to the level of enterprise. At this stage, variants of macroeconomic and microeconomic dynamics are compared, and the enterprise's response scenarios to changes of the internal and external environments are developed.
- Due to the modern volatility of global and national economies, the significance of forecasting, in general, and enterprise revenue forecasting, in particular (being part of the budgeting process) is greater than ever.
- The models mentioned above are limited, to some extent, and often, their use does not make it possible to obtain the desired result due to certain inherent risks and errors.

Fuzzy Time Series (1)

- When discussing fuzzy time series {$\tilde{Y}(t)$}, it should be noted that such a series includes a number of fuzzy sets X^t, where t = 1, 2, …
- In this case, we used the assumption that the components of the series X^t have linguistic values, and $\tilde{Y}(t)$ is a fuzzy function with argument t which values have fuzzy verbal variables "high", "average", "low", etc.
- The use of logical-linguistic variables makes it possible to take into account qualitative factors that enable to recognize the uncertainty
- Empirical data of the time series $\{Y(t)\}$ must be designated. These are selected with account to the objective of study (revenue of the enterprise, per-capita income, GDP, etc.)

Fuzzy Time Series (2)

- By studying a time series in a fuzzy dynamic environment one is able to formulate a fuzzy function $\tilde{Y}(t)$ with argument t, within the universal set of U, having the set of values designated as fuzzy ranges X^t and membership function $\mu_{Xt}(u_i)$. Thus, (2) is obtained:

$$X^t = \{\mu_X t (u_i)/u_i\},$$

where X^t is fuzzy range; $\mu_{Xt}(u_i)$ is membership function; and t is argument of the function; and the following conditions are met:

$$u_i \in U; \mu_X t (u_i) \in [0, 1].$$

- The conditions considered signify that every point within the interval of u_i will be a member of the set X^t and have the degree of membership of:

$$\mu_{Xt} (u_i) = \mu_i (t),$$

- where $\mu_i (t)$ are given numbers, $i=$; 1, t = [1, 2,…].
- Thus, the complete set or range of number axis will appear as:

$$U = (u_1, u_2, \ldots, u_m),$$

where ,u_i, i = 1,m.

- The fuzzy set of X of the universal set of U may be defined as:

$$X = \{(\mu_x(u_1)/u_2), (\mu_x(u_1)/u_2), \ldots, (\mu_x(u_m)/u_m)\},$$

where $\mu_x(u_i)$ is the membership function which puts its elements u_i as a set of real numbers within the segment of [0, 1] indicating the degree of membership of elements u_i in the set of X, $\mu_x(u_i) \in$ [0, 1];"/" designates the membership of the value μ_x in the element u_m.

Stages for forecasting with Fussy Time Series (1)

Stage 1

Stage 2

Stage 3

Stage 4

Stage 5

Stage 6

Stages for forecasting with Fuzzy Time Series (2)

- At the **first stage**, the boundaries of the time series are defined, and the indicators necessary to solve the forecasting problem are selected. To define the universal *U set,* increment of the considered indicator of the time series throughout the time interval must be determined. It should be noted that boundaries of the universal set *U* coincide with the maximum and minimum values of the indicator increment, however, at the following stages of the forecasting process these boundaries may be expanded for the ease of calculation.
- At the **second stage**, the defined universal *U set* is divided into intervals having equal length.

Stages for forecasting with Fuzzy Time Series (3)

- At the third stage of the analysis, a set of fuzzy sets shall be identified within the previously determined universal *U* set. For this purpose, logical-linguistic variables shall be introduced and appropriate values of these variables determined. In general terms, these variables may be as follows:
- very low level of increment of the forecasted indicator (VLLIFI);
- low level of increment of the forecasted indicator (LLIFI);
- average level of increment of the forecasted indicator (ALIFI);
- stationary level of increment of the forecasted indicator (SLIFI);
- normal level of increment of the forecasted indicator (NLIFI);
- high level of increment of the forecasted indicator (HLIFI); and
- very high level of increment of the forecasted indicator (VHLIFI).

Stages for forecasting with Fuzzy Time Series (4)

- In order to define the fuzzy set A_i within the universal *U set*, membership function shall be used.
- Thus, by consistently adopting the average value u_i of middle point of the intervals as the value of variable *V* we can develop the following representation of fuzzy sets:
- $A_1 = \{(/u_1), (/u_2), (/u_3), (/u_4), (/u_5), (/u_6), (/u_7)\}$
- $A_2 = \{(/u_1), (/u_2), (/u_3), (/u_4), (/u_5), (/u_6), (/u_7)\}$
- $A_3 = \{(/u_1), (/u_2), (/u_3), (/u_4), (/u_5), (/u_6), (/u_7)\}$
- $A_4 = \{(/u_1), (/u_2), (/u_3), (/u_4), (/u_5), (/u_6), (/u_7)\}$
- $A_5 = \{(/u_1), (/u_2), (/u_3), (/u_4), (/u_5), (/u_6), (/u_7)\}$
- $A_6 = \{(/u_1), (/u_2), (/u_3), (/u_4), (/ u_5), (/u_6), (/u_7)\}$
- $A_7 = \{(/u_1), (/u_2), (/u_3), (/u_4), (/ u_5), (/u_6), (/u_7)\}$.

Stages for forecasting with Fuzzy Time Series (5)

- At the **fourth stage**, conversion of numerical values of increment of the forecasted indicator into fuzzy values is performed. At this stage, qualitative representations of the dynamics of increment of the forecasted indicator is accounted for in the form of fussy sets
- At the **fifth stage**, forecasting of the studied indicator is done using fuzzy logic symbols.
- To forecast the value of increment of the studied indicator for the period of *t* resulting in the fuzzy set of *F*(*t*)), fuzzy relation matrix *R*(*t*) must be calculated. This matrix is determined by intersecting the matrix of fuzzy increment of the studied indicator N(t) for the period of (t-1) and the matrix of fuzzy increments of the studied indicator *S*(*t*) for the periods of (*t*-2), (*t*-3), (*t*-4), (*t*-5), (*t*-+).Therefore:

$$R(t)[i,j] = S(t)[i,j] \cap N(t)[i,j] = \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1j} \\ r_{21} & r_{22} & \dots & r_{2j} \\ \dots & \dots & \dots & \dots \\ r_{i1} & r_{i2} & \dots & r_{ij} \end{bmatrix}$$

- In this case $F(t) = [max(r_{11}, r_{21}, \dots, r_{i1})\ max(r_{12}, r_{22}, \dots, r_{i2}) \dots max(r_{1j}, r_{2j}, \dots, r_{ij})]$.

Stages for forecasting with Fuzzy Time Series (6)

- At the sixth stage, conversion of fuzzy values of increment of the studied indicator into precise values is performed. For implementing this, we use a formula:

$$V(t) = \frac{\sum_{i=1}^{7} \mu_t(u_i) u_{cp}^{i}}{\sum_{i=1}^{7} \mu_t(u_i)}$$

- where $\mu_t(u_i)$ are the values of membership function for the considered year and u_{cp}^{i} are coordinates of points dividing intervals of equal length into equal parts u_i that belong to the universal U set.

- In order to obtain the value of the forecasted indicator, the value of the forecasted increment must be added to its value for the last period.

Using the Method for Cash Flows Predicting

Year	Revenue in thousands rubles	Revenue gain in thousands rubles
2009	358987	
2010	396781	37794
2011	429187	32406
2012	498745	69558
2013	517896	19151
2014	629800	11904
2015	690494	60694
2016	693203	2709

According to the proposed method, boundaries of the universal set *U* shall correspond to the maximum and minimum value of increment for the considered year, what makes *U* = [2700; 69700]. Let us divide the defined boundaries into seven equal intervals having the length of 9570 that will be represented as: u_1= [2700, 12270]; u_2= [12270, 218840]; u_3= [21840,31410]; u_4=[31410, 40980]; u_5 = [40980, 50550]; u_6 = [50550, 60120]; u_7 = [60120, 69700], then = 7485; = 17055;

Variables and Their Values

- very low level of revenue gain (VLLRG);
- low level of revenue gain (LLRG);
- average level of revenue gain (ALRG);
- stationary level of revenue gain (SLRG);
- normal level of revenue gain (NLRG);
- high level of revenue gain (HLRG); and
- very high level of revenue gain (VHLRG).

Conversion of Fuzzy Sets

Let us define a fuzzy set A_i within the universal set U according to the formula discussed above where c=0.0001, and we will obtain a representation of fuzzy sets:

A1 = {(1/u1), (0,97/u2), (0,87/u3), (0,75/u4), (0,63/u5), (0,53/u6), (0,43/u7)}
A2 = {(0,97/u1), (1/u2), (0,97/u3), (0,87/u4), (0,75/u5), (0,63/u6), (0,53/u7)}
A3 = {(0,87/u1), (0,97/u2), (1/u3), (0,97/u4), (0,87/u5), (0,75/u6), (0,63/u7)}
A4 = {(0,75/u1), (0,87/u2), (0,97/u3), (1/u4), (0,97/u5), (0,87/u6), (0,75/u7)}
A5 = {(0,63/u1), (0,75/u2), (0,87/u3), (0,97/u4), (1/u5), (0,97/u6), (0,87/u7)}
A6 = {(0,53/u1), (0,63/u2), (0,75/u3), (0,87/u4), (0,97/ *u5),* (1/u6), (0,97/u7)}
A7 = {(0,43/u1), (0,53/u2), (0,63/u3), (0,75/u4), (0,87/ *u5),* (0,97/u6),(1/u7)}.

Revenue Gain	Conversion into Fuzzy Sets
37794	a1 = {(1/u), (0,95/u), (0,85/u), (0,73/u), (0,62/u), (0,51/u), (0,42/u)}
32406	a2 = {(1/u), (0,98/u), (0,90/u), (0,79/u), (0,66/u), (0,55/u), (0,45/u)}
69558	a3 = {(0,73/u), (0,85/u), (0,95/u), (1/u), (0,97/u), (0,89/u), (0,77/u)}
19151	a4 = {(0,83/u), (0,97/u), (1/u), (0,95/u), (0,85/u), (0,73/u), (0,61/u)}
11904	a5 = {(0,85/u), (0,94/u), (1/u), (0,98/u), (0,90/u), (0,78/u), (0,66/u)}
60694	a6 = {(0,62/u), (0,74/u), (0,86/u), (0,96/u), (1/u), (0,98/u), (0,88/u)}
2709	a7 = {(0,40/u), (0,48/u), (0,58/u), (0,70/u), (0,82/u), (0,93/u), (1/u)}

- When comparing the matrices, the fuzzy relation matrix for 2013 will have the following form:

$$R(2013) = \begin{vmatrix} 0{,}73 & 0{,}85 & 0{,}82 & 0{,}69 & 0{,}58 & 0{,}48 & 0{,}40 \\ 0{,}73 & 0{,}85 & 0{,}91 & 0{,}80 & 0{,}68 & 0{,}56 & 0{,}46 \\ 0{,}73 & 0{,}85 & 0{,}89 & 0{,}77 & 0{,}65 & 0{,}54 & 0{,}45 \\ 0{,}73 & 0{,}85 & 0{,}85 & 0{,}73 & 0{,}62 & 0{,}51 & 0{,}43 \\ 0{,}73 & 0{,}85 & 0{,}90 & 0{,}79 & 0{,}66 & 0{,}55 & 0{,}42 \end{vmatrix} =$$

Thus, we get F (2013)= |0,73 0,85 0,91 0,80 0,68 0,56 0,46|,
what makes

$$V(2013) = \frac{0.73 * 226{,}45 + 0{,}85 * 415 + 0{,}91 * 605 + 0{,}80 * 795 + 0{,}68 * 985 + 0{,}56 * 1175 + 0{,}46 * 1369}{0{,}73 + 0{,}85 + 0{,}91 + 0{,}68 + 0{,}56 + 0{,}46} = 1731{,}9\ thousand\ ruble$$

By using this calculation, we obtain V(2014)=1670.2 thousand rubles; V(2015)=1764.8 thousand rubles; and V(2016)=2568 thousand rubles.

Revenues Forecasting

Forecasting Method	2013	2014	2015	2016	Average Approximation Error in %
Precise Time Series Method	518,743.5	630,676.0	690,013.6	69,314.7	19.72
Fuzzy Time Series Method	519,627.9	631,470.2	692,258.8	695,771	5.98

Conclusion

- The proposed forecasting method based on fuzzy sets is an addition to the existing quantitative methods of forecasting. Application of this method promotes the reduction of approximation error below the level of reduction achieved when forecasting using statistical methods. This indicates that this model has practical significance and enhances the accuracy of forecast.
- Application of forecasting using the fuzzy set method and linear model demonstrated that the fuzzy set method made it possible to bring average approximation error down by 13.7%.
- This allows for the conclusion that forecast accuracy is increased

Thank you for your attention
Your questions?