МБОУ СОШ № 6

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Электронный справочник по тригонометрическим формулам

Тригонометрические формулы

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\cos a = \pm\sqrt{1-\sin^2 a}$$

$$\sin a = \pm\sqrt{1-\cos^2 a}$$

$$tga \cdot ctga = 1$$

$$tga = \frac{1}{ctga}$$

$$1 + tg^2 a = \frac{1}{\cos^2 a}$$

$$ctga = \frac{1}{tga}$$

$$1 + ctg^2 a = \frac{1}{\sin^2 a}$$

Назад

Пример

1) *Доказать, что при* $a \neq \pi k, k \in Z$ *справедливо равенство* $1 + ctg^2 a = \frac{1}{\sin^2 a}$

$$1 + ctg^2 a = 1 + \frac{\cos^2 a}{\sin^2 a} = \frac{\sin^2 a + \cos^2 a}{\sin^2 a} = \frac{1}{\sin^2 a}$$

2) *Вычислить* $\sin a$, *если* $\cos a = -\frac{3}{5}$ *и* $\pi < a < \frac{3\pi}{2}$

Воспользуемся формулой $\sin a = \pm\sqrt{1 - \cos^2 a}$

Т.к. $\pi < a < \frac{3\pi}{2}$ *– III квадрант, то* $\sin a < 0$

$$\sin a = -\sqrt{1 - \cos^2 a} = -\sqrt{1 - \frac{9}{25}} = -\frac{4}{5}$$

3) *Вычислить* $ctga$, *если* $tga = 13$

По формуле $ctga = \frac{1}{tga}$ *находим:*

$$ctga = \frac{1}{tga} = \frac{1}{13}$$

Назад

$$\sin(a+\beta)=\sin a\cdot\cos\beta+\cos a\cdot\sin\beta$$

$$\sin(a-\beta)=\sin a\cdot\cos\beta-\cos a\cdot\sin\beta$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$tg(\alpha+\beta)=\frac{tg\alpha+tg\beta}{1-tg\alpha\cdot tg\beta}\qquad tg(\alpha-\beta)=\frac{tg\alpha-tg\beta}{1+tg\alpha\cdot tg\beta}$$

$$ctg(\alpha+\beta)=\frac{ctg\alpha\cdot ctg\beta-1}{ctg\alpha+ctg\beta}\qquad ctg(\alpha-\beta)=\frac{ctg\alpha\cdot ctg\beta+1}{ctg\beta-ctg\alpha}$$

Назад

Пример

1) $\sin 585^0 = \sin(360^0 + 225^0) = \sin 225° = \sin(180^0 + 45°) =$

$= -\sin 45° = -\sqrt{2}/2$

2) $\cos 75^0 = \cos\left(45^0 - 30^0\right) = \cos 45^0 \cdot \cos 30^0 - \sin 45^0 \cdot \sin 30^0 =$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

3) $$tg225^0 = tg\left(180^0 + 45^0\right) = \frac{tg180^0 + tg45^0}{1 - tg180^0 \cdot tg45^0} = \frac{0+1}{1-0\cdot 1} = 1$$

4) $ctg780^0 = ctg\left(2\cdot 360^0 + 60^0\right) = ctg60° = \sqrt{3}/3$

Назад

$$\sin 2\alpha = 2\sin\alpha \cdot \cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$tg\,2\alpha = \frac{2tg\alpha}{1 - tg^2\alpha}$$

$$ctg\,2\alpha = \frac{ctg^2\alpha - 1}{2ctg\alpha}$$

Назад

Пример

$$1)\ \sin 120^0 = \sin\left(2\cdot 60^0\right) = 2\cdot \sin 60^0 \cdot \cos 60^0 = 2\cdot \frac{\sqrt{3}}{2}\cdot\frac{1}{2} = \frac{\sqrt{3}}{2}$$

$$2)\ \cos 240^0 = \cos^2 120^0 - \sin^2 120^0 = \left(\cos^2 60^0 - \sin^2 60^0\right)^2 - \left(2\sin 60^0 \cos 60^0\right)^2 =$$

$$= \left(\frac{3}{4}-\frac{1}{4}\right)^2 - \left(2\cdot\frac{\sqrt{3}}{4}\right)^2 = \frac{1}{4}-\frac{3}{4} = -\frac{1}{2}$$

$$3)\ tg720^0 = tg(2\cdot 360^0 + 0^0) = tg0^0 = 0$$

$$4)\ \textit{если}\ ctg\alpha = \frac{\sqrt{3}}{2},\ \textit{то}\ ctg2\alpha = \frac{ctg^2\alpha - 1}{2ctg\alpha} = \frac{\left(\frac{\sqrt{3}}{2}\right)^2 - 1}{2\cdot\frac{\sqrt{3}}{2}} = \frac{-\frac{1}{4}}{\sqrt{3}} = -\frac{\sqrt{3}}{12}$$

Назад

$$\sin^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{2}$$

$$\cos^2\frac{\alpha}{2}=\frac{1+\cos\alpha}{2}$$

$$tg^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{1+\cos\alpha}$$

$$ctg^2\frac{\alpha}{2}=\frac{1+\cos\alpha}{1-\cos\alpha}$$

Назад

Пример

$$1)\sin^2 15^0=\frac{1-\cos 30^0}{2}=\frac{1-\frac{\sqrt{3}}{2}}{2}=\frac{\frac{2}{2}-\frac{\sqrt{3}}{2}}{2}=\frac{2-\sqrt{3}}{4}$$

$$2)2\cos^2\frac{\pi}{8}-1=2\cdot\frac{1+\cos\frac{\pi}{4}}{2}-1=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$$

$$3)\text{если}\ \cos\alpha=0{,}6\ \text{и}\ 0<\alpha<\frac{\pi}{2},\ \text{то}\ tg\frac{\alpha}{2}=\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}=\sqrt{\frac{0{,}4}{1{,}6}}=$$

$$=\sqrt{\frac{1}{4}}=\frac{1}{2}$$

$$4)\text{если}\ \cos\alpha=-0{,}02\ \text{и}\ 0<\alpha<\pi,\ \text{то}\ \cos\frac{\alpha}{2}=\sqrt{\frac{1+\cos\alpha}{2}}=\sqrt{\frac{1-0{,}02}{2}}=0{,}7$$

Назад

$$\sin(a + 2\pi k) = \sin a \qquad \cos(a + 2\pi k) = \cos a \qquad k \in Z$$

$$\cos\left(\frac{\pi}{2} - a\right) = \sin a \qquad \cos\left(\frac{\pi}{2} + a\right) = -\sin a$$

$$\cos(\pi - a) = -\cos a \qquad \cos(\pi + a) = -\cos a$$

$$\cos\left(\frac{3\pi}{2} - a\right) = -\sin a \qquad \cos\left(\frac{3\pi}{2} + a\right) = \sin a$$

$$\sin\left(\frac{\pi}{2} - a\right) = \cos a \qquad \sin\left(\frac{\pi}{2} + a\right) = \cos a$$

$$\sin(\pi - a) = \sin a \qquad \sin(\pi + a) = -\sin a$$

$$\sin\left(\frac{3\pi}{2} - a\right) = -\cos a \qquad \sin\left(\frac{3\pi}{2} + a\right) = -\cos a$$

$$tg(a + \pi k) = tga \qquad ctg(a + \pi k) = ctga$$

$$tg\left(\frac{\pi}{2} - a\right) = ctga \qquad tg\left(\frac{\pi}{2} + a\right) = -ctga$$

$$ctg\left(\frac{\pi}{2} - a\right) = tga \qquad ctg\left(\frac{\pi}{2} + a\right) = -tga$$

Назад

Пример

1) *Вычислить* $\sin 930^0$

$$\sin 930^0 = \sin(3\cdot 360^0 - 150^0) = -\sin 150^0 = -\sin(180^0 - 30^0) = -\sin 30^0 = -\frac{1}{2}$$

2) *Вычислить* $tg\frac{11\pi}{3}$

$$tg\frac{11\pi}{3} = tg\left(4\pi - \frac{\pi}{3}\right) = tg\left(-\frac{\pi}{3}\right) = -tg\frac{\pi}{3} = -\sqrt{3}$$

3) *Вычислить* $tg\frac{13\pi}{4}$

$$tg\frac{13\pi}{4} = tg\left(3\pi + \frac{\pi}{4}\right) = tg\frac{\pi}{4} = 1$$

4) *Вычислить* $\cos\frac{15\pi}{4}$

$$\cos\frac{15\pi}{4} = \cos\left(4\pi - \frac{\pi}{4}\right) = \cos\left(-\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

Назад

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$tg\alpha + tg\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$$

$$tg\alpha - tg\beta = \frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}$$

Назад

Пример

$$1)\cos 105^0+\cos 75^0=2\cos\frac{105^0+75^0}{2}\cos\frac{105^0-75^0}{2}=2\cos 90^0\cos 15^0=0;$$

$$2)\cos\frac{11\pi}{12}-\cos\frac{5\pi}{12}=-2\sin\frac{\frac{11\pi}{12}+\frac{5\pi}{12}}{2}\sin\frac{\frac{11\pi}{12}-\frac{5\pi}{12}}{2}=-2\sin\frac{2\pi}{3}\sin\frac{\pi}{4}=$$

$$=-2\cdot\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}=-\frac{\sqrt{6}}{2};$$

$$3)\sin 105^0+\sin 165^0=2\sin\frac{105^0+165^0}{2}\cos\frac{105^0-165^0}{2}=2\sin 135^0\cos(-30)^0=$$

$$=2\cdot\sin(180^0-45^0)\cdot\cos 30^0=2\cdot\sin 45^0\cdot\cos 30^0=2\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}=\frac{\sqrt{6}}{2};$$

$4)\sin 10^0+\sin 50^0=2\sin((10°+50°)/2)\ \cos((10°-50°)/2)=2\sin 30^0\cos(-20°)=$
$=2\cdot 1/2\cos 20^0=\cos 20^0$

Назад

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$$

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right]$$

Назад

Пример

1) Преобразовать произведение в сумму

$$\cos x \cos 6x = \frac{1}{2}[\cos(x - 6x) + \cos(x + 6x)]$$

$$= \frac{1}{2}\cos 5x + \frac{1}{2}\cos 7x$$

2) Решить уравнение: $\cos x \cos 2x = \sin x \sin 2x$

$$\frac{1}{2}[\cos(x - 2x) + \cos(x + 2x)]$$

$$= \frac{1}{2}[\cos(x - 2x) - \cos(x + 2x)]$$

$$\cos(-x) + \cos 3x = \cos(-x) - \cos 3x$$

$$2\cos 3x = 0$$

$$\cos 3x = 0$$

$$x = \frac{\pi}{6} + \frac{\pi n}{3}, n \in Z$$

Назад