AUTOMATICS and AUTOMATIC CONTROL

LECTURE 4

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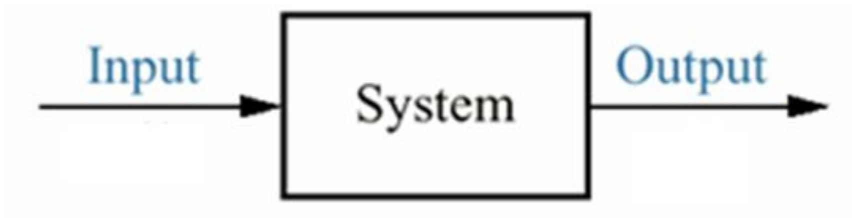
BASIC DYNAMIC ELEMENTS

Any, more or less complex systems (objects) can be presented as a connection of some appropriate basic dynamic elements



1. Proportional (noninertial):

a) Differential equation:

$y(t) = k * u(t)$

b) Transfer function:

$G(s) = k$

c) Step response:

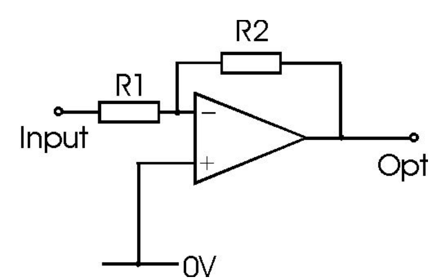
$H(s) = k/s$ $h(t) = k\mathbf{1}(t)$

d) Frequency response:

$P(\omega) = k$ $Q(\omega) = 0$

$L_m(\omega)=20\log k$ $\varphi(\omega)=0$

e) Example:



$k=-R_2/R_1$

BASIC DYNAMIC ELEMENTS

2. **Inertial (1st order):**

a) Differential equation:

$$T \cdot \dot{y}(t) + y(t) = k \cdot u(t)$$

b) Transfer function:

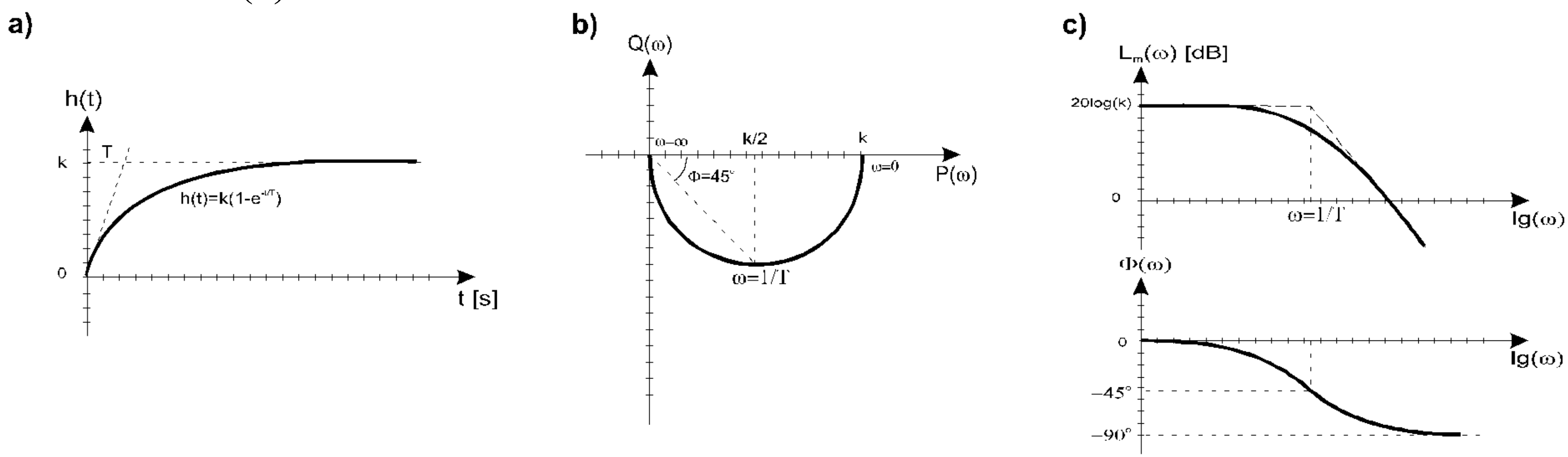
$$G(s) = \frac{Y(s)}{U(s)} = \frac{k}{Ts+1}$$

c) Step response:

$$H(s) = \frac{1}{s}\frac{k}{Ts+1} \qquad h(t) = k\left(1 - e^{-\frac{1}{T}t}\right)$$

d) Frequency response:

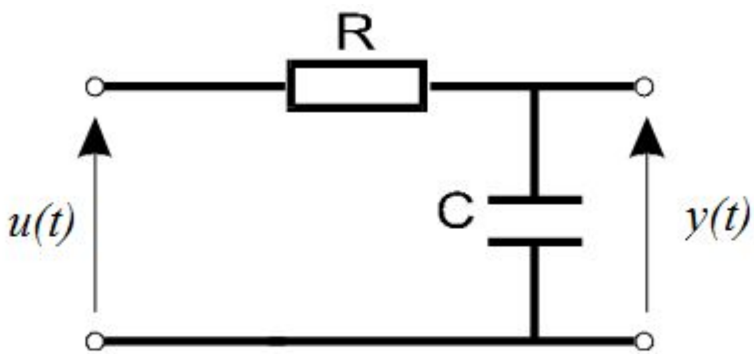
$$P(\omega) = \frac{k}{1+\omega^2 T^2} \qquad Q(\omega) = -\frac{k\omega T}{1+\omega^2 T^2}$$



BASIC DYNAMIC ELEMENTS

2. **Inertial:**

e) Example:



$$RC\frac{dy(t)}{dt} + y(t) = u(t)$$

$$G(s) = \frac{U_2(s)}{U_1(s)} = \frac{Y(s)}{U(s)} = \frac{1}{RCs+1}$$

BASIC DYNAMIC ELEMENTS

3. **Oscilator (2nd order):**

a) Differential equation:

$$T^2 \cdot \ddot{y} + 2d \cdot T \cdot \dot{y} + y = k \cdot u$$

b) Transfer function:

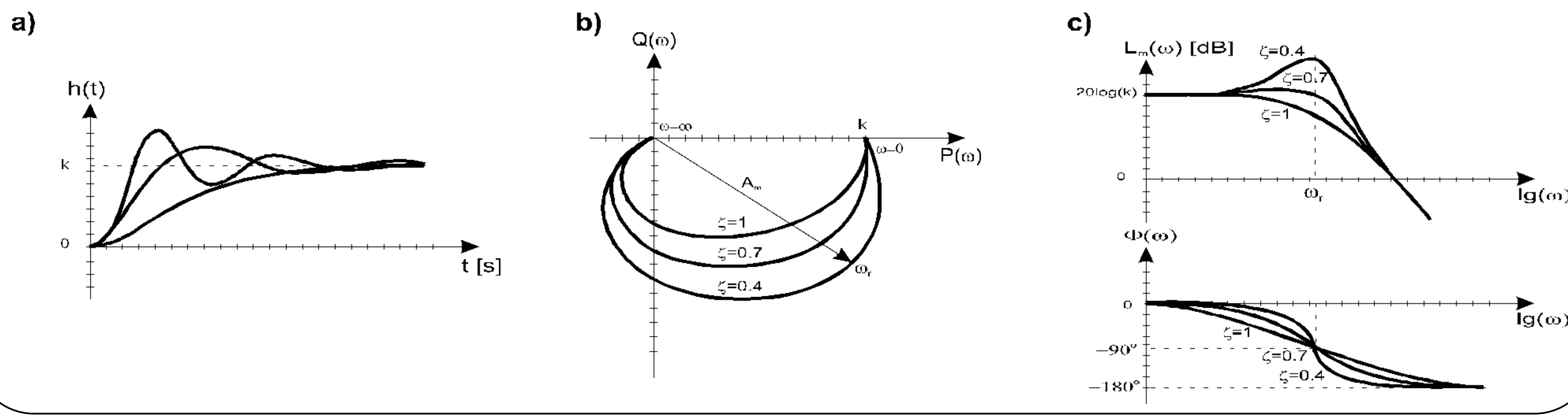
$$G(s) = \frac{Y(s)}{U(s)} = \frac{k}{T^2 s^2 + 2d \cdot T\, s + 1}$$

c) Step response:

$$h(t) = k\left(1 - \frac{1}{\sqrt{1-d}} e^{-\frac{d}{T}t} \sin\left(\frac{\sqrt{1-d^2}}{T_n} t + \varphi\right)\right)$$

$$\varphi = arctg \frac{\sqrt{1-d^2}}{d}, \qquad T_n = T * 2\pi - undumped \quad period$$

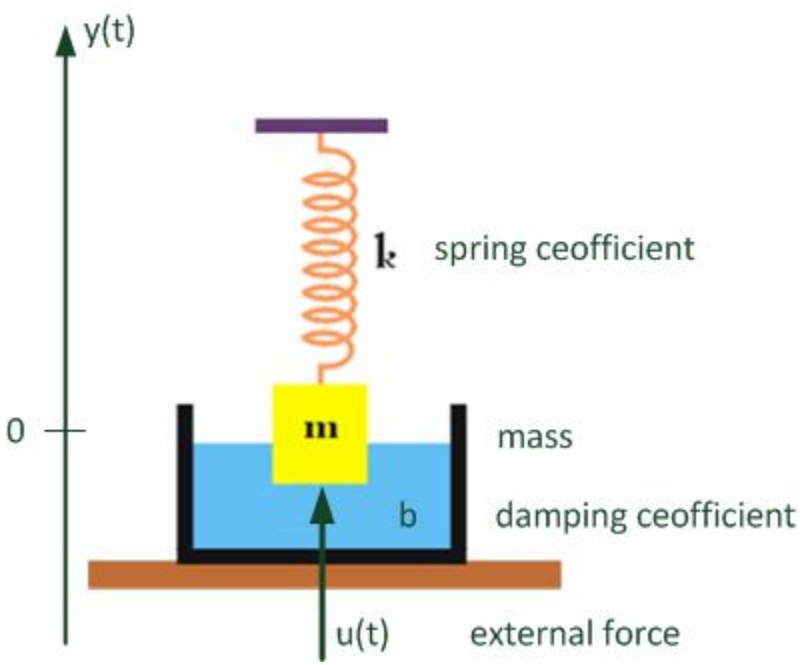
d) Frequency response:



BASIC DYNAMIC ELEMENTS

3. **Oscilator (2nd order):**

e) Example1: **damped harmonic oscillator**



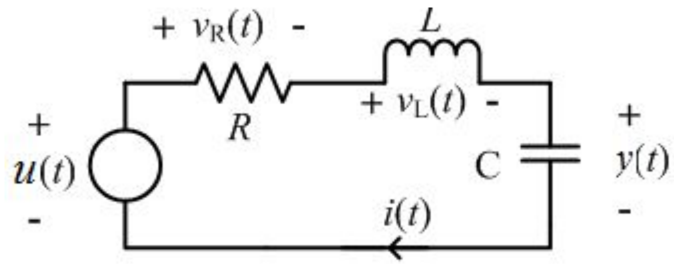
$$m\frac{d^2y}{dt^2}+b\frac{dy}{dt}+ky=u$$

$$G(s)=\frac{1}{ms^2+bs+k}=\frac{\frac{1}{k}}{\frac{m}{k}s^2+\frac{b}{k}s+1}$$

BASIC DYNAMIC ELEMENTS

3. **Oscilator (2nd order):**

e) Example 2: **RLC circuit**



$$\begin{cases} u(t) = i(t)R + L\dfrac{di(t)}{dt} + y(t) \\ i(t) = C\dfrac{dy(t)}{dt} \end{cases}$$

$$G(s) = \frac{1}{LCs^2 + RCs + 1}$$

$$LC\frac{dy^2(t)}{dt^2} + RC\frac{dy(t)}{dt} + y(t) = u(t)$$

BASIC DYNAMIC ELEMENTS

4. Integrator (ideal integrator):

a) Differential equation:

$$y(t) = \frac{1}{T_i}\int_0^t u(t)dt$$

b) Transfer function:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{T_i s}$$

c) Step response:

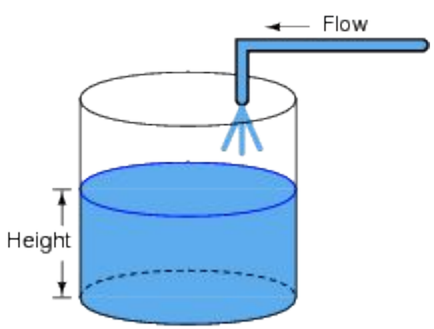
$$H(s) = \frac{1}{s}\frac{1}{T_i s} \qquad h(t) = \frac{1}{T_i}t$$

d) Frequency response:

$$P(\omega) = 0 \qquad Q(\omega) = -\frac{1}{\omega T}$$

$$L_m(\omega) = -20\log\omega T_i \qquad \varphi(\omega) = -\pi/2$$

e) Example: (water flow q to the tank with water level area C_h):



$$h(t) = \frac{1}{C_h}\int_0^t q(t)dt$$

BASIC DYNAMIC ELEMENTS

5. **Real integrator (with inertia):**

a) Differential equation:

$$T\frac{dy(t)}{dt}+y(t)=\frac{1}{T_i}\int_0^t u(t)dt$$

b) Transfer function:

$$G(s)=\frac{Y(s)}{U(s)}=\frac{1}{T_i s(Ts+1)}$$

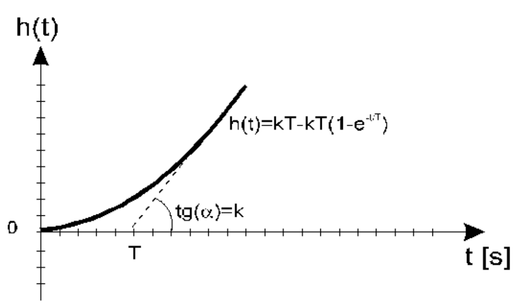
c) Step response:

$$H(s)=\frac{1}{s}\frac{1}{T_i s(Ts+1)} \qquad h(t)=\frac{1}{T_i}t-\frac{1}{T_i}T\left(1-e^{-\frac{1}{T}t}\right)$$

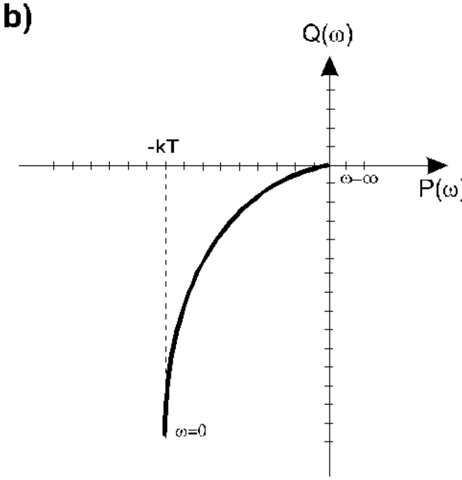
d) Frequency response:

$$P(\omega)=-\frac{T}{T_i(1+\omega^2T^2)} \qquad Q(\omega)=-\frac{1}{T_i\omega(1+\omega^2T^2)}$$

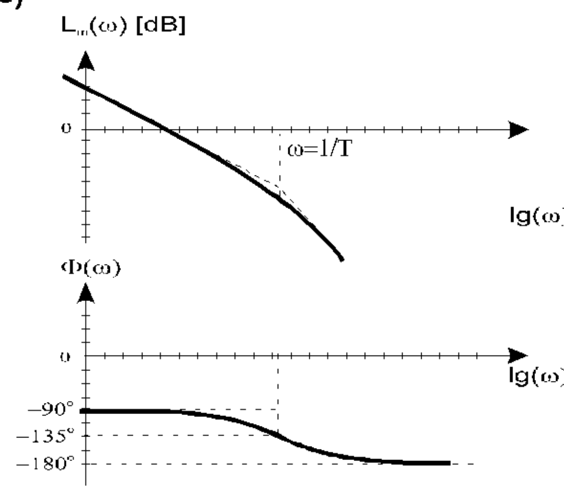
a)



b)



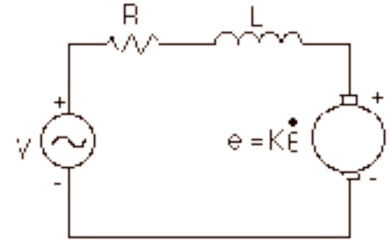
c)

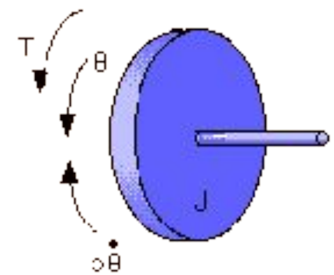


BASIC DYNAMIC ELEMENTS

5**. Real integrator (with inertia):**

e) Example: DC motor





$$\begin{cases} u(t) = i(t)R + c_e\omega(t) & M_e = k_i i(t) \\ M_e(t) - M_0 = J\dfrac{d\omega(t)}{dt} & \omega(t) = \dfrac{d\alpha(t)}{dt} \end{cases}$$

$$\frac{JR}{k_i}\frac{d\alpha^2(t)}{dt^2} + c_e\frac{d\alpha(t)}{dt} = u(t) \qquad T_m = \frac{JR}{k_i c_e}, \qquad k = \frac{1}{c_e}$$

$$T_m\frac{d\alpha^2(t)}{dt^2} + \frac{d\alpha(t)}{dt} = ku(t) \qquad \Rightarrow \qquad T_m\frac{d\alpha(t)}{dt} + \alpha(t) = k\int_0^t u(t)dt$$

$$G(s) = \frac{k}{s(T_m s + 1)}$$

BASIC DYNAMIC ELEMENTS

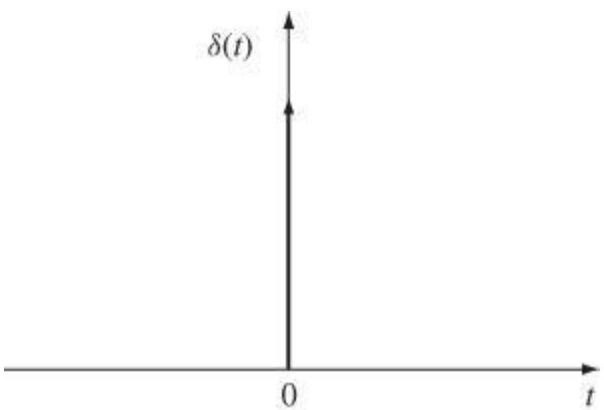
6**. Differentiator (ideal):**

a) Differential equation:

$$y(t) = T_d \frac{du(t)}{dt}$$

b) Transfer function:

$$G(s) = \frac{Y(s)}{U(s)} = T_d s$$



c) Step response:

$$H(s) = T_d \qquad h(t) = T_d \delta(t)$$

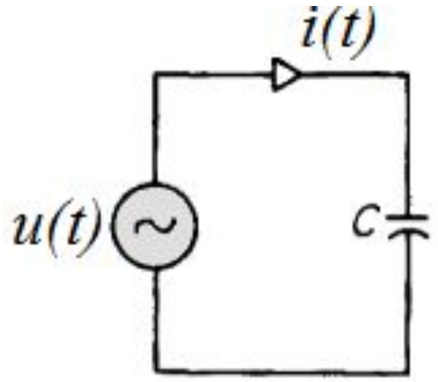
d) Frequency response:

$$P(\omega) = 0 \qquad Q(\omega) = T_d \omega$$

BASIC DYNAMIC ELEMENTS

6**. Differentiator (ideal)**

e) Example: Ideal capasitor



$$i(t) = C\frac{du(t)}{dt}$$

$$G(s) = \frac{I(s)}{U(s)} = sC$$

BASIC DYNAMIC ELEMENTS

7. Real differentiator (with inertia):

a) Differential equation:

$$T\frac{dy(t)}{dt} + y(t) = T_d\frac{du(t)}{dt}$$

b) Transfer function:

$$G(s) = \frac{T_d s}{Ts+1}$$

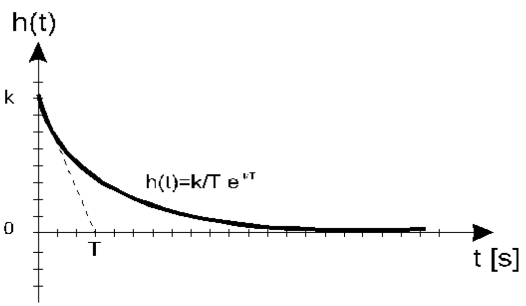
c) Step response:

$$G(s) = \frac{T_d}{Ts+1} \qquad h(t) = \frac{T_d}{T}e^{-\frac{1}{T}t}$$

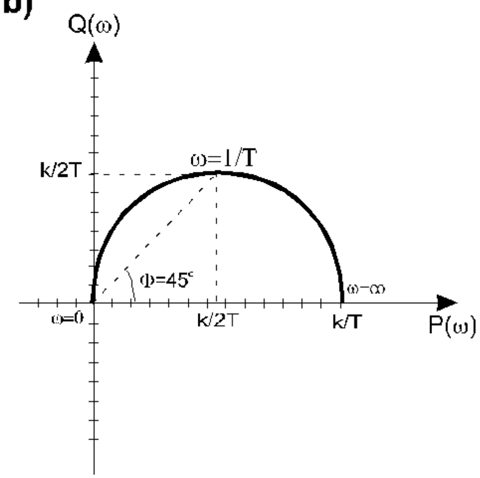
d) Frequency response:

$$P(\omega) = \frac{kT\omega^2}{1+(\omega T)^2} \qquad Q(\omega) = \frac{k\omega}{1+(\omega T)^2}$$

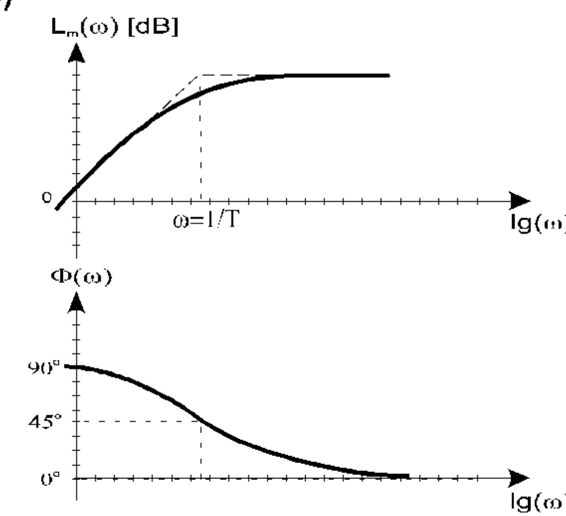
a)



b)



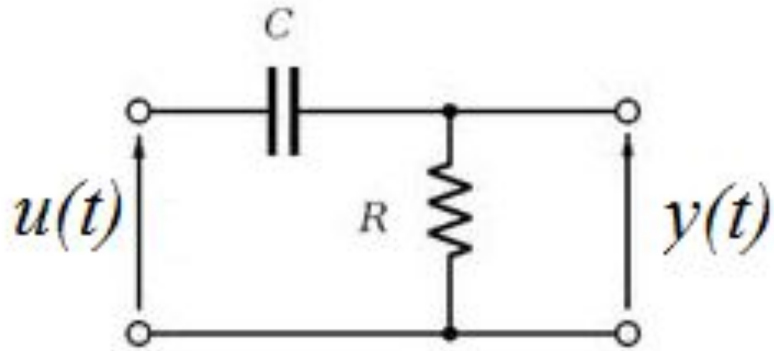
c)



BASIC DYNAMIC ELEMENTS

7. Real differentiator (with inertia):

e) Example: RC circuit



$$\begin{cases} u(t) = \dfrac{1}{C}\displaystyle\int_0^t i(t)dt + Ri(t) \\ y(t) = Ri(t) \end{cases}$$

$$\frac{1}{RC}\int_0^t y(t)dt + y(t) = u(t)$$

$$G(s) = \frac{RCs}{RCs+1}$$

$$RC\frac{dy(t)}{dt} + y(t) = RC\frac{du(t)}{dt}$$

BASIC DYNAMIC ELEMENTS

8. **Delay**

a) Differential equation:

$$y(t) = k \cdot u(t - T_o)$$

b) Transfer function:

$$G(s) = \frac{Y(s)}{U(s)} = ke^{-sT_o}$$

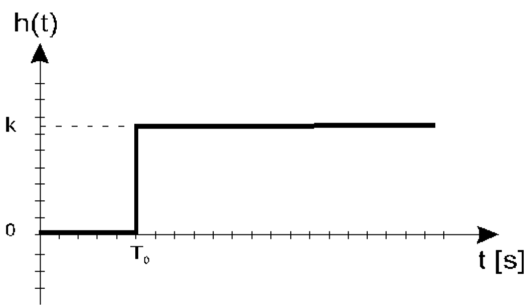
c) Step response:

$$H(s) = \frac{1}{s} ke^{-sT_0} \qquad h(t) = k1(t - T_0)$$

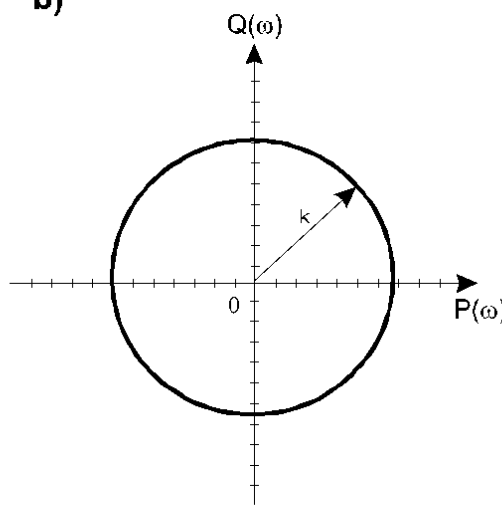
d) Frequency response:

$$P(\omega) = k\cos(\omega T_0) \qquad Q(\omega) = -k\sin(\omega T_0)$$

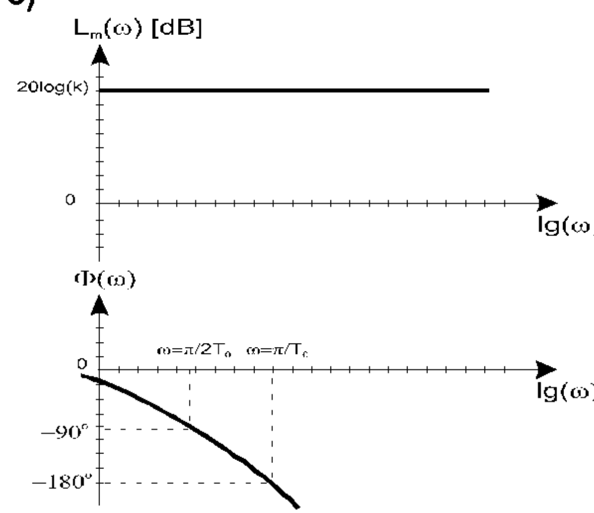
a)



b)



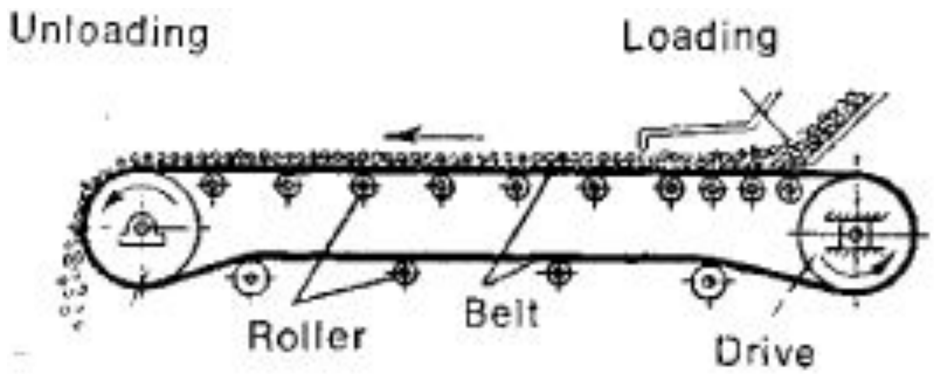
c)



BASIC DYNAMIC ELEMENTS

8. **Delay**

e) Example: conveyor (transporter)



$$T_0 = \frac{V}{l}$$

THANK YOU