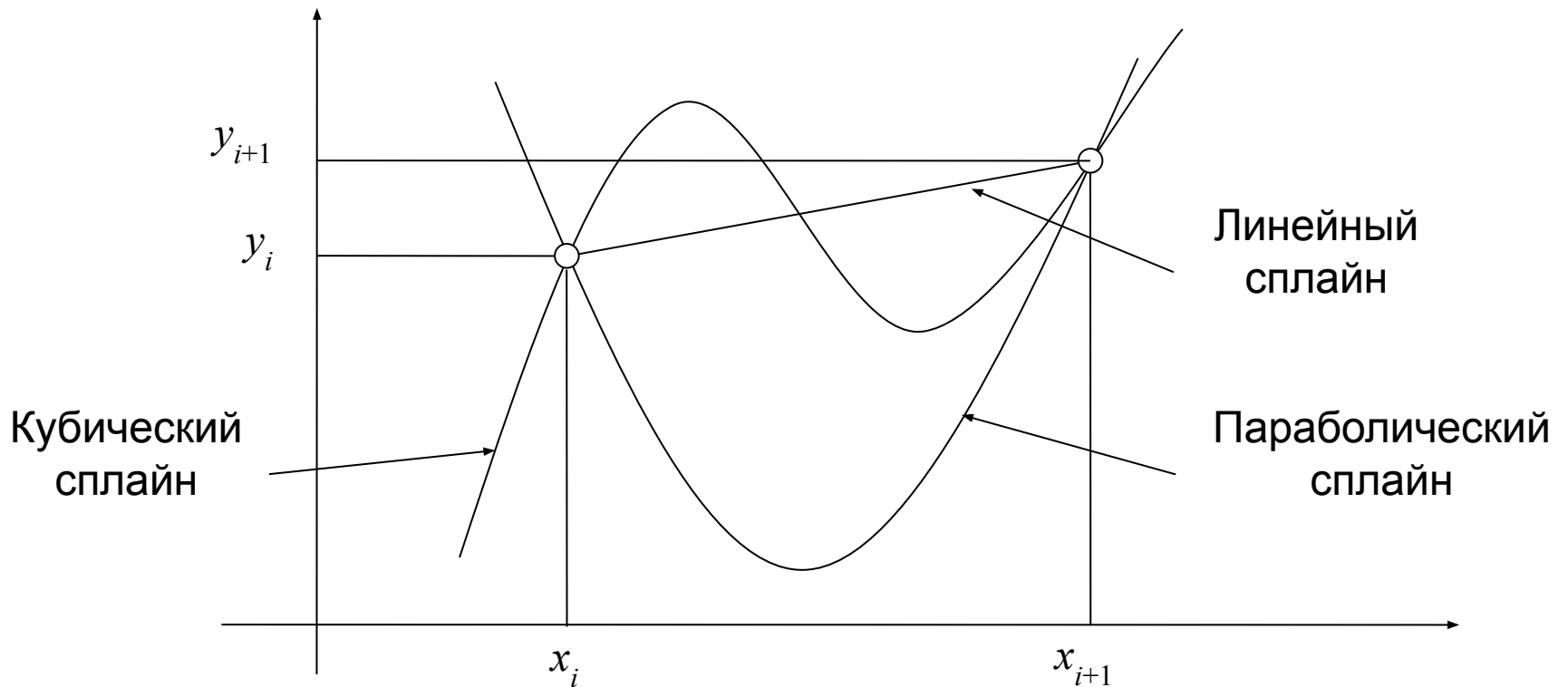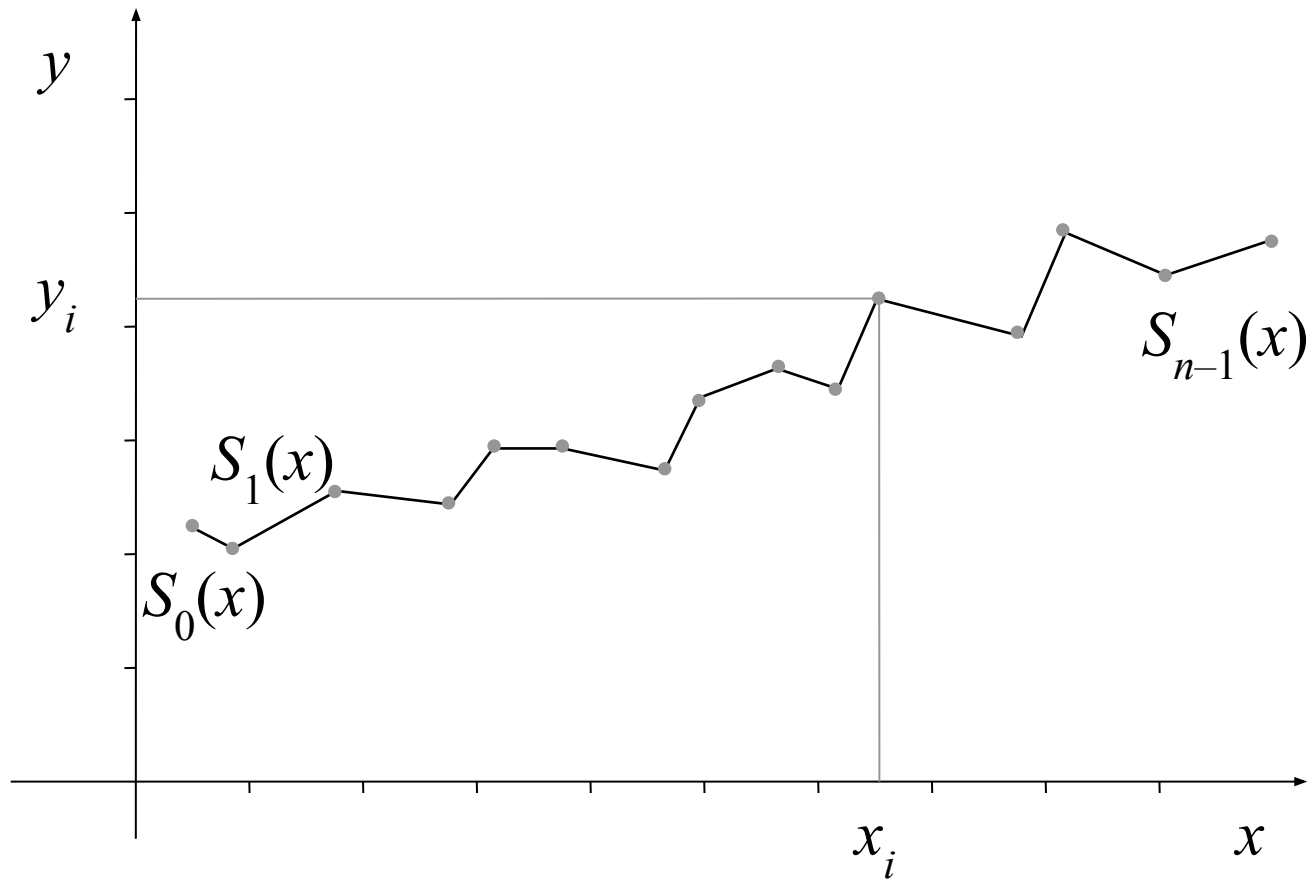# Тема. Приближение сплайнами

Приближение сплайнами

Приближение сплайнами

Постановка задачи.

Сетка (табличные значения функции):

$$\{x_i\}: i = 0,\ 1,\ \ldots,\ n$$

$$\{y_i\}: y_i = f(x_i)$$

Количество узлов – $n + 1$. Количество сплайнов – n: $S_0(x)$, $S_1(x)$, …, $S_{n-1}(x)$. Для $S_i(x)$ область действия $x \in [x_i,\ x_{i+1}]$.

Приближение сплайнами

Постановка задачи.

$$S_i(x) = \sum_{j=0}^{m} c_{ji} \varphi_{ji}(x), \quad \varphi_{ji}(x) = (x - x_i)^j \Rightarrow$$

$$S_i(x) = \sum_{j=0}^{m} c_{ji}(x - x_i)^j, \quad i = 0,1,...,n-1, \quad m = 1,2,3$$

$$a_i = c_{0i}, \ b_i = c_{1i}, \ c_i = c_{2i}, \ d_i = c_{3i}$$

При $m > 1$ требуются *граничные условия*.

Линейные сплайны

$m = 1$:

$$S_i(x) = a_i + b_i(x - x_i), \ \ i = 0,1,\dots,n-1$$

$$S_i(x_{i+1}) = S_{i+1}(x_{i+1}), \ \ i = 0,1,\dots,n-2$$

$$a_i = y_i, \ \ b_i = \frac{\Delta y_i}{\Delta x_i}$$

Здесь $\Delta x_i = x_{i+1} - x_i, \ \Delta y_i = y_{i+1} - y_i.$

Параболические сплайны

$m = 2:$

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2, \quad i = 0, 1, \ldots, n-1$$

$$S_i(x_{i+1}) = S_{i+1}(x_{i+1}),$$

$$S_i'(x_{i+1}) = S_{i+1}'(x_{i+1}), \quad i = 0, 1, \ldots, n-2$$

Граничные условия:

1. $A_0 = f(x_0);$
2. $A_n = f(x_n).$

Параболические сплайны

$$a_i = y_i, \ \ c_i = \frac{b_{i+1} - b_i}{2\Delta x_i}.$$

Если $A_0 = f(x_0)$, то

$$b_0 = A_0, \ \ b_{i+1} = \frac{2\Delta y_i}{\Delta x_i} - b_i, \ \ i = 0,1,...,n-2.$$

Если $A_n = f(x_n)$, то

$$b_n = A_n, \ \ b_i = \frac{2\Delta y_i}{\Delta x_i} - b_{i+1}, \ \ i = n-1, n-2,...,0.$$

Кубические сплайны

$m = 3$:

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3, \ i = 0, 1, ..., n-1$$

$$S_i(x_{i+1}) = S_{i+1}(x_{i+1}), \ S_i'(x_{i+1}) = S_{i+1}'(x_{i+1}),$$

$$S_i''(x_{i+1}) = S_{i+1}''(x_{i+1}), \ i = 0, 1, ..., n-2$$

Граничные условия:

1. $A_0 = f'(x_0), \ A_n = f'(x_n)$;
2. $B_0 = f''(x_0), \ B_n = f''(x_n)$.

Кубические сплайны

$$a_i = y_i,$$

$$b_i = \frac{\Delta y_i}{\Delta x_i} - \frac{\Delta x_i}{6}\left(2M_i + M_{i+1}\right),$$

$$c_i = \frac{1}{2}M_i,$$

$$d_i = \frac{M_{i+1} - M_i}{6\Delta x_i}, \ \ i = 0,1,...,n-1.$$

Кубические сплайны

Если $A_0 = f'(x_0), \; A_n = f'(x_n)$, то

$$b_0 = A_0, \; b_n = A_n$$

$$\tilde{A}M = \tilde{g}$$

$$\tilde{A} = \begin{cases} \tilde{a}_{00} = \dfrac{\Delta x_0}{3}; \; \tilde{a}_{nn} = \dfrac{\Delta x_{n-1}}{3}; \\[3mm] \tilde{a}_{jj} = \dfrac{1}{3}\left(\Delta x_{j-1} + \Delta x_j\right), \;\; j = 1, 2, ..., n-1; \\[3mm] \tilde{a}_{j,j+1} = \tilde{a}_{j+1,j} = \dfrac{1}{6}\Delta x_j, \;\; j = 0, 1, ..., n-1 \end{cases}$$

Кубические сплайны

Если $A_0 = f'(x_0)$, $A_n = f'(x_n)$, то

$$b_0 = A_0, \ b_n = A_n$$

$$\tilde{A}M = \tilde{g}$$

$$\tilde{g} = \begin{cases} \tilde{g}_0 = \dfrac{\Delta y_0}{\Delta x_0} - A_0; \ \tilde{g}_n = A_n - \dfrac{\Delta y_{n-1}}{\Delta x_{n-1}}; \\[4mm] \tilde{g}_i = \dfrac{\Delta y_i}{\Delta x_i} - \dfrac{\Delta y_{i-1}}{\Delta x_{i-1}}, \ \ i = 1, 2, ..., n-1 \end{cases}$$

Кубические сплайны

Если $B_0 = f''(x_0)$, $B_n = f''(x_n)$, то

$$M_0 = B_0, \; M_n = B_n$$

$$AM = g$$

$$M = (M_1, M_2, \ldots, M_{n-1})$$

$$A = \begin{cases} a_{jj} = \tilde{a}_{jj} = \dfrac{1}{3}\left(\Delta x_{j-1} + \Delta x_j\right), \; j = 1, 2, \ldots, n-1; \\[4mm] a_{j,j+1} = a_{j+1,j} = \tilde{a}_{j,j+1} = \tilde{a}_{j+1,j} = \dfrac{1}{6}\Delta x_j, \; j = 1, 2, \ldots, n-2 \end{cases}$$

Кубические сплайны

Если $B_0 = f''(x_0)$, $B_n = f''(x_n)$, то

$$M_0 = B_0, \ M_n = B_n$$

$$AM = g$$

$$g_i = \tilde{g}_i - \beta_1 - \beta_{n-1} = \frac{\Delta y_i}{\Delta x_i} - \frac{\Delta y_{i-1}}{\Delta x_{i-1}} - \beta_1 - \beta_{n-1}, \ \ i = 1, 2, ..., n-1,$$

$$\beta_1 = \begin{cases} \dfrac{\Delta x_0}{6} B_0, \ \ i = 1 \\ 0, \ i \neq 1 \end{cases}; \ \ \beta_{n-1} = \begin{cases} \dfrac{\Delta x_{n-1}}{6} B_n, \ \ i = n-1 \\ 0, \ i \neq n-1 \end{cases}$$

Примеры

Параболический сплайн.

$$f(x) = \sqrt{x}$$

i	0	1	2	3
x	1/4	1	4	9
y	1/2	1	2	3

Результирующая сетка: $\{4/9, 9/4, 25/4\}$

Граничные условия: $f'(x) = \dfrac{1}{2\sqrt{x}} \Rightarrow A_0 = 1, A_3 = \dfrac{1}{6}$

Далее строим сплайны $S_0(x), S_1(x), S_2(x)$.

Примеры

i	0	1	2	3
x	1/4	1	4	9
y	1/2	1	2	3

$$\Delta x_0 = 1 - \frac{1}{4} = \frac{3}{4}; \ \ \Delta x_1 = 4 - 1 = 3; \ \ \Delta x_2 = 9 - 4 = 5;$$

$$\Delta y_0 = 1 - \frac{1}{2} = \frac{1}{2}; \ \ \Delta y_1 = 2 - 1 = 1; \ \ \Delta y_2 = 3 - 2 = 1$$

$$b_0 = 1, \ \ b_1 = \frac{2\Delta y_0}{\Delta x_0} - b_0 = \frac{2 \cdot 1/2}{3/4} - 1 = \frac{1}{3}, \ \ b_2 = \frac{2\Delta y_1}{\Delta x_1} - b_1 =$$

$$= \frac{2 \cdot 1}{3} - \frac{1}{3} = \frac{1}{3}, \ \ b_3 = \frac{2\Delta y_2}{\Delta x_2} - b_2 = \frac{2 \cdot 1}{5} - \frac{1}{3} = \frac{1}{15}$$

Примеры

i	0	1	2	3
x	1/4	1	4	9
y	1/2	1	2	3

$$\Delta x_0 = 1 - \frac{1}{4} = \frac{3}{4}; \ \ \Delta x_1 = 4 - 1 = 3; \ \ \Delta x_2 = 9 - 4 = 5;$$

$$\Delta y_0 = 1 - \frac{1}{2} = \frac{1}{2}; \ \ \Delta y_1 = 2 - 1 = 1; \ \ \Delta y_2 = 3 - 2 = 1$$

$$c_0 = \frac{b_1 - b_0}{2\Delta x_0} = \frac{1/3 - 1}{2 \cdot 3/4} = -\frac{4}{9}, \ \ c_1 = \frac{b_2 - b_1}{2\Delta x_1} = 0,$$

$$c_2 = \frac{b_3 - b_2}{2\Delta x_2} = \frac{1/15 - 1/3}{2 \cdot 5} = -\frac{2}{75}$$

Примеры

i	0	1	2	3
x	1/4	1	4	9
y	1/2	1	2	3
a	1/2	1	2	3
b	1	1/3	1/3	1/15
c	–4/9	0	–2/75	

$$S_0(x) = \frac{1}{2} + 1 \cdot \left(x - \frac{1}{4} \right) - \frac{4}{9} \left(x - \frac{1}{4} \right)^2, \quad x \in \left[\frac{1}{4}, 1 \right]$$

$$S_1(x) = 1 + \frac{1}{3} \cdot (x - 1) + 0(x - 1)^2, \quad x \in [1, 4]$$

$$S_2(x) = 2 + \frac{1}{3} \cdot (x - 4) - \frac{2}{75}(x - 4)^2, \quad x \in [4, 9]$$

Примеры

$$S_0\left(\frac{4}{9}\right) = \frac{1}{2} + \left(\frac{4}{9} - \frac{1}{4}\right) - \frac{4}{9}\left(\frac{4}{9} - \frac{1}{4}\right)^2 = \frac{494}{729} \approx 0.678$$

$$S_1\left(\frac{9}{4}\right) = 1 + \frac{1}{3} \cdot \left(\frac{9}{4} - 1\right) = \frac{17}{12} \approx 1.417$$

$$S_2\left(\frac{25}{4}\right) = 2 + \frac{1}{3} \cdot \left(\frac{25}{4} - 4\right) - \frac{2}{75}\left(\frac{25}{4} - 4\right)^2 = \frac{523}{200} = 2.615$$

Примеры

i	0	1	2	3
x	1/4	1	4	9
y	1/2	1	2	3

$$\Delta x_0 = 1 - \frac{1}{4} = \frac{3}{4}; \ \Delta x_1 = 4 - 1 = 3; \ \Delta x_2 = 9 - 4 = 5;$$

$$\Delta y_0 = 1 - \frac{1}{2} = \frac{1}{2}; \ \Delta y_1 = 2 - 1 = 1; \ \Delta y_2 = 3 - 2 = 1$$

$$b_3 = \frac{1}{6}, \ b_2 = \frac{2\Delta y_2}{\Delta x_2} - b_3 = \frac{2 \cdot 1}{5} - \frac{1}{6} = \frac{7}{30}, \ b_1 = \frac{2\Delta y_1}{\Delta x_1} - b_2 =$$

$$= \frac{2 \cdot 1}{3} - \frac{7}{30} = \frac{13}{30}, \ b_0 = \frac{2\Delta y_0}{\Delta x_0} - b_1 = \frac{2 \cdot 1/2}{3/4} - \frac{13}{30} = \frac{9}{10}$$

Примеры

i	0	1	2	3
x	1/4	1	4	9
y	1/2	1	2	3

$$\Delta x_0 = 1 - \frac{1}{4} = \frac{3}{4}; \ \ \Delta x_1 = 4 - 1 = 3; \ \ \Delta x_2 = 9 - 4 = 5;$$

$$\Delta y_0 = 1 - \frac{1}{2} = \frac{1}{2}; \ \ \Delta y_1 = 2 - 1 = 1; \ \ \Delta y_2 = 3 - 2 = 1$$

$$c_0 = \frac{b_1 - b_0}{2\Delta x_0} = \frac{13/30 - 9/10}{2 \cdot 3/4} = -\frac{14}{45}, \ \ c_1 = \frac{b_2 - b_1}{2\Delta x_1} = \frac{7/39 - 13/30}{2 \cdot 3} =$$

$$= -\frac{1}{30}, \ \ c_2 = \frac{b_3 - b_2}{2\Delta x_2} = \frac{1/6 - 7/30}{2 \cdot 5} = -\frac{1}{150}$$

Примеры

i	0	1	2	3
x	1/4	1	4	9
y	1/2	1	2	3
a	1/2	1	2	3
b	9/10	13/30	7/30	1/6
c	−14/45	−1/30	−1/150	

$$S_0(x) = \frac{1}{2} + \frac{9}{10} \cdot \left(x - \frac{1}{4}\right) - \frac{14}{45}\left(x - \frac{1}{4}\right)^2, \; x \in \left[\frac{1}{4}, 1\right]$$

$$S_1(x) = 1 + \frac{13}{30} \cdot (x - 1) - \frac{1}{30}(x - 1)^2, \; x \in [1, 4]$$

$$S_2(x) = 2 + \frac{7}{30} \cdot (x - 4) - \frac{1}{150}(x - 4)^2, \; x \in [4, 9]$$

Примеры

$$S_0\left(\frac{4}{9}\right) = \frac{1}{2} + \frac{9}{10} \cdot \left(\frac{4}{9} - \frac{1}{4}\right) - \frac{14}{45}\left(\frac{4}{9} - \frac{1}{4}\right)^2 = \frac{967}{1458} \approx 0.663$$

$$S_1\left(\frac{9}{4}\right) = 1 + \frac{13}{30} \cdot \left(\frac{9}{4} - 1\right) - \frac{1}{30}\left(\frac{9}{4} - 1\right)^2 = \frac{143}{96} \approx 1.49$$

$$S_2\left(\frac{25}{4}\right) = 2 + \frac{7}{30} \cdot \left(\frac{25}{4} - 4\right) - \frac{1}{150}\left(\frac{25}{4} - 4\right)^2 = \frac{1993}{800} \approx 2.491$$
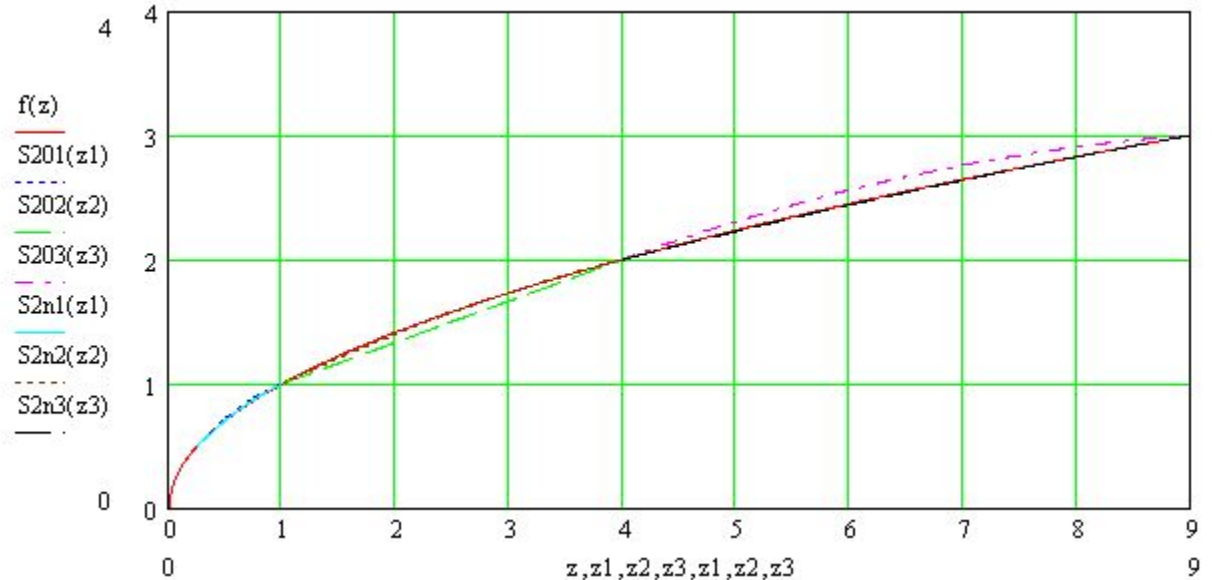
Примеры

Точность интерполяции:

x	$P(x)$	$L(x)$	$f(x)$	δ
1/4	0.293	0.293	0.5	41.4%
9/4	1.699	1.699	1.5	13.3%
25/4	2.012	2.012	2.5	19.5%

x	$S(x)$	$S(x)$	$f(x)$	δ
4/9	0.678	0.663	2/3	0.6-1.7%
9/4	1.417	1.490	1.5	0.6-5.5%
25/4	2.615	2.491	2.5	0.4-4.6%

Примеры

Точность интерполяции:

Примеры

Кубический сплайн.

$$f(x) = \sqrt{x}$$

i	0	1	2	3
x	1/4	1	4	9
y	1/2	1	2	3

Результирующая сетка: {4/9, 9/4, 25/4}

Гранич. условия: $f''(x) = \dfrac{1}{4\sqrt{x^3}} \Rightarrow B_0 = -2, B_3 = -\dfrac{1}{108}$

Далее строим сплайны $S_0(x)$, $S_1(x)$, $S_2(x)$.

Примеры

i	0	1	2	3
x	1/4	1	4	9
y	1/2	1	2	3

$$\Delta x_0 = 1 - \frac{1}{4} = \frac{3}{4}; \quad \Delta x_1 = 4 - 1 = 3; \quad \Delta x_2 = 9 - 4 = 5;$$

$$\Delta y_0 = 1 - \frac{1}{2} = \frac{1}{2}; \quad \Delta y_1 = 2 - 1 = 1; \quad \Delta y_2 = 3 - 2 = 1$$

$$A = \begin{pmatrix} \frac{1}{3}\left(\Delta x_0 + \Delta x_1\right) & \frac{1}{6}\Delta x_1 \\ \frac{1}{6}\Delta x_1 & \frac{1}{3}\left(\Delta x_1 + \Delta x_2\right) \end{pmatrix} = \begin{pmatrix} \frac{1}{3}\left(\frac{3}{4} + 3\right) & \frac{1}{6}\cdot 3 \\ \frac{1}{6}\cdot 3 & \frac{1}{3}\left(3 + 5\right) \end{pmatrix} = \begin{pmatrix} \frac{5}{4} & \frac{1}{2} \\ \frac{1}{2} & \frac{8}{3} \end{pmatrix}$$

Примеры

i	0	1	2	3
x	1/4	1	4	9
y	1/2	1	2	3

$$\Delta x_0 = 1 - \frac{1}{4} = \frac{3}{4}; \quad \Delta x_1 = 4 - 1 = 3; \quad \Delta x_2 = 9 - 4 = 5;$$

$$\Delta y_0 = 1 - \frac{1}{2} = \frac{1}{2}; \quad \Delta y_1 = 2 - 1 = 1; \quad \Delta y_2 = 3 - 2 = 1$$

$$g = \begin{pmatrix} \dfrac{\Delta y_1}{\Delta x_1} - \dfrac{\Delta y_0}{\Delta x_0} - \dfrac{\Delta x_0}{6} M_0 \\[2mm] \dfrac{\Delta y_2}{\Delta x_2} - \dfrac{\Delta y_1}{\Delta x_1} - \dfrac{\Delta x_2}{6} M_3 \end{pmatrix} = \begin{pmatrix} \dfrac{1}{3} - \dfrac{1/2}{3/4} - \dfrac{3/4}{6} \cdot (-2) \\[2mm] \dfrac{1}{5} - \dfrac{1}{3} - \dfrac{5}{6} \cdot \left(-\dfrac{1}{108} \right) \end{pmatrix} = \begin{pmatrix} -\dfrac{1}{12} \\[2mm] -\dfrac{407}{3240} \end{pmatrix} \approx \begin{pmatrix} -0.0833 \\ -0.1256 \end{pmatrix}$$

Примеры

$$\begin{pmatrix} \dfrac{5}{4} & \dfrac{1}{2} \\ \dfrac{1}{2} & \dfrac{8}{3} \end{pmatrix} \begin{pmatrix} M_1 \\ M_2 \end{pmatrix} = \begin{pmatrix} -0.0833 \\ -0.1256 \end{pmatrix}$$

$$\Rightarrow M_1 = -\frac{1033}{19980} \approx -0.0517; \quad M_2 = -\frac{299}{7992} \approx -0.0374$$

Примеры

$$M_0 = -2; M_1 = -\frac{1033}{19980}; M_2 = -\frac{299}{7992}; M_3 = -\frac{1}{108}$$

$$b_0 = \frac{\Delta y_0}{\Delta x_0} - \frac{\Delta x_0}{6}\left(2M_0 + M_1\right) = \frac{1/2}{3/4} - \frac{3/4}{6}\left(2\cdot(-2) - \frac{1033}{19980}\right) =$$

$$= \frac{187513}{159840} \approx 1.1731; \ b_1 = \frac{\Delta y_1}{\Delta x_1} - \frac{\Delta x_1}{6}\left(2M_1 + M_2\right) =$$

$$= \frac{1}{3} - \frac{3}{6}\left(2\cdot\left(-\frac{1033}{19980}\right) - \frac{299}{7992}\right) = \frac{32267}{79920} \approx 0.4037; \ b_2 =$$

$$= \frac{\Delta y_2}{\Delta x_2} - \frac{\Delta x_2}{6}\left(2M_2 + M_3\right) = \frac{1}{5} - \frac{5}{6}\left(2\cdot\left(-\frac{299}{7992}\right) - \frac{1}{108}\right) = \frac{1349}{4995} \approx 0.2701$$

Примеры

$$M_0 = -2; M_1 = -\frac{1033}{19980}; M_2 = -\frac{299}{7992}; M_3 = -\frac{1}{108}$$

$$c_0 = \frac{1}{2}M_0 = \frac{1}{2}\cdot(-2) = -1;$$

$$c_1 = \frac{1}{2}M_1 = \frac{1}{2}\cdot\left(-\frac{1033}{19980}\right) = -\frac{1033}{39960} \approx -0.0259;$$

$$c_2 = \frac{1}{2}M_2 = \frac{1}{2}\cdot\left(-\frac{299}{7992}\right) = -\frac{299}{15984} \approx -0.0187$$

Примеры

$$M_0 = -2; M_1 = -\frac{1033}{19980}; M_2 = -\frac{299}{7992}; M_3 = -\frac{1}{108}$$

$$d_0 = \frac{M_1 - M_0}{6\Delta x_0} = \frac{-\dfrac{1033}{19980} + 2}{6 \cdot 3/4} = \frac{38927}{89910} \approx 0.4330;$$

$$d_1 = \frac{M_2 - M_1}{6\Delta x_1} = \frac{-\dfrac{299}{7992} + \dfrac{1033}{19980}}{6 \cdot 3} = \frac{571}{719280} \approx 0.0008;$$

$$d_2 = \frac{M_3 - M_2}{6\Delta x_2} = \frac{-\dfrac{1}{108} + \dfrac{299}{7992}}{6 \cdot 5} = \frac{5}{5328} \approx 0.0009$$

Примеры

i	0	1	2	3
x	1/4	1	4	9
y	1/2	1	2	3
a	1/2	1	2	3
b	1.1731	0.4037	0.2701	
c	–1	–0.0259	–0.0187	
d	0.4330	–0.0008	0.0009	

$$S_0(x) = \frac{1}{2} + 1.1731 \cdot \left(x - \frac{1}{4} \right) - 1 \cdot \left(x - \frac{1}{4} \right)^2 + 0.4330 \cdot \left(x - \frac{1}{4} \right)^2, \ \ x \in \left[\frac{1}{4}, 1 \right]$$

$$S_1(x) = 1 + 0.4037 \cdot (x - 1) - 0.0259 \cdot (x - 1)^2 - 0.0008 \cdot (x - 1)^3, \ \ x \in [1, 4]$$

$$S_2(x) = 2 + 0.2701 \cdot (x - 4) - 0.0187 \cdot (x - 4)^2 + 0.0009 \cdot (x - 4)^3, \ \ x \in [4, 9]$$

Примеры

$$S_0\left(\frac{4}{9}\right) = \frac{1}{2} + 1.1731\cdot\left(\frac{4}{9}-\frac{1}{4}\right) - 1\cdot\left(\frac{4}{9}-\frac{1}{4}\right)^2 + 0.4330\cdot\left(\frac{4}{9}-\frac{1}{4}\right)^2 \approx$$

$$\approx 0.6935$$

$$S_1\left(\frac{9}{4}\right) = 1 + 0.2701\cdot\left(\frac{9}{4}-1\right) - 0.0259\cdot\left(\frac{9}{4}-1\right)^2 - 0.0008\cdot\left(\frac{9}{4}-1\right)^3 \approx$$

$$\approx 1.4658$$

$$S_2\left(\frac{25}{4}\right) = 2 + 0.3176\cdot\left(\frac{25}{4}-4\right) - 0.0187\cdot\left(\frac{25}{4}-4\right)^2 + 0.0009\cdot\left(\frac{25}{4}-4\right)^3 \approx$$

$$\approx 2.5236$$

Примеры

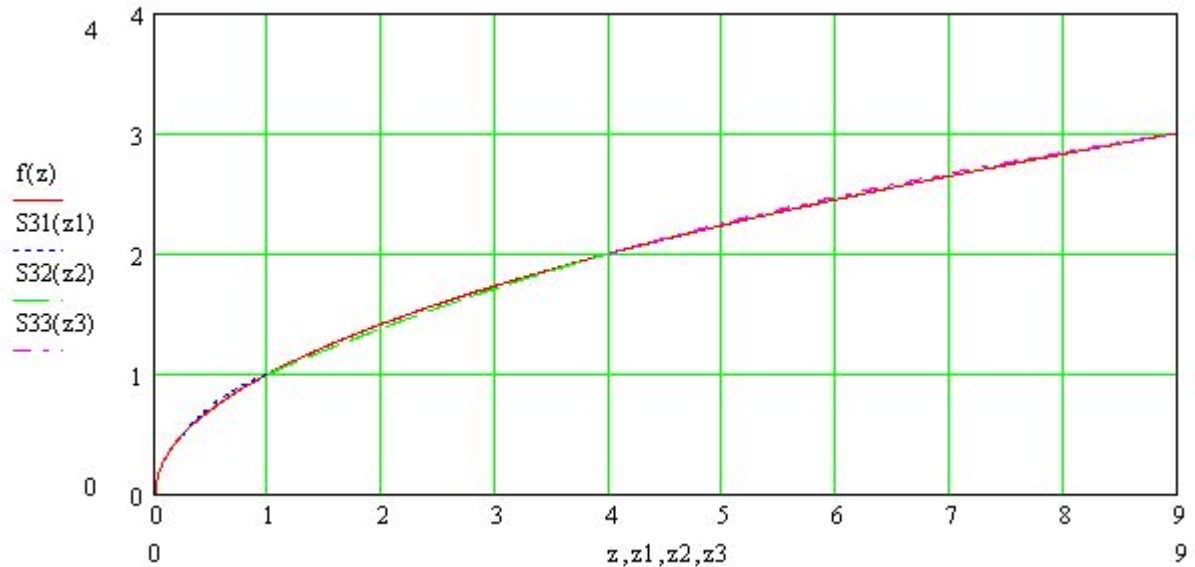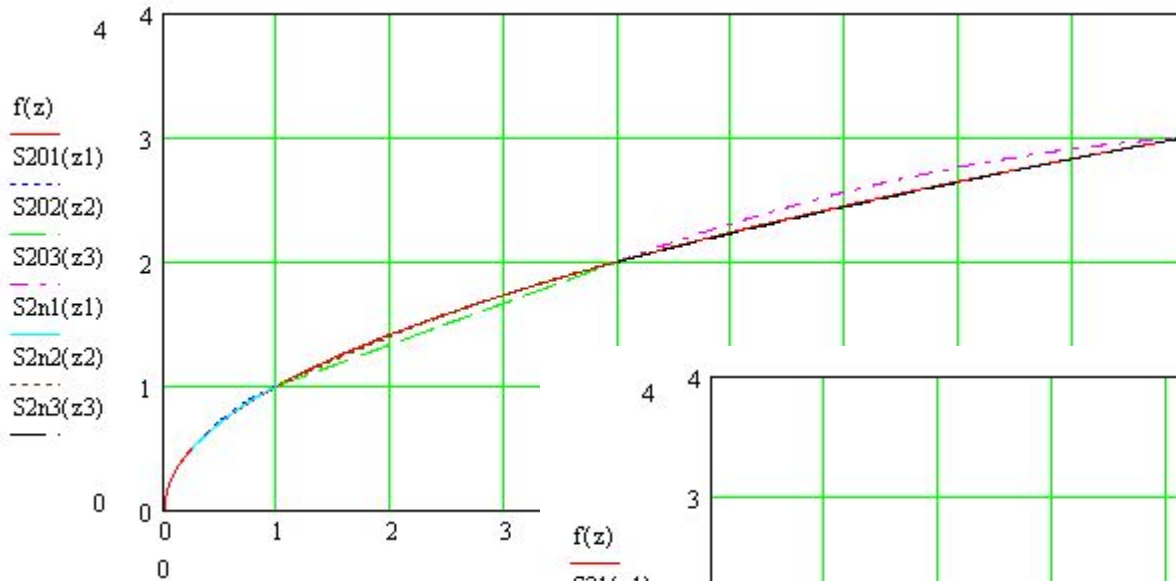Точность интерполяции:

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25/4	2.012	2.012	2.5	19.5%

x	$S(x)$	$S(x)$	$S(x)$	$f(x)$	δ_2	δ_3
4/9	0.678	0.663	0.694	2/3	0.6-1.7%	4.10%
9/4	1.417	1.490	1.466	1.5	0.6-5.5%	2.27%
25/4	2.615	2.491	2.524	2.5	0.4-4.6%	0.96%

Примеры

Точность интерполяции:

Примеры

Кубический сплайн.

$$f(x) = \sqrt{x}$$

i	0	1	2	3
x	1/4	1	4	9
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Результирующая сетка: {4/9, 9/4, 25/4}

Гранич. условия:

$$f'(x) = \frac{1}{2\sqrt{x}} \Rightarrow A_0 = 1, A_3 = \frac{1}{6}$$

Далее строим сплайны $S_0(x), S_1(x), S_2(x)$.

$$A = \begin{pmatrix} \dfrac{1}{3}\Delta x_0 & \dfrac{1}{6}\Delta x_0 & 0 & 0 \\[2mm] \dfrac{1}{6}\Delta x_0 & \dfrac{1}{3}\left(\Delta x_0 + \Delta x_1\right) & \dfrac{1}{6}\Delta x_1 & 0 \\[2mm] 0 & \dfrac{1}{6}\Delta x_1 & \dfrac{1}{3}\left(\Delta x_1 + \Delta x_2\right) & \dfrac{1}{6}\Delta x_2 \\[2mm] 0 & 0 & \dfrac{1}{6}\Delta x_2 & \dfrac{1}{3}\Delta x_2 \end{pmatrix} =$$

$$= \begin{pmatrix} \dfrac{1}{3}\cdot\dfrac{3}{4} & \dfrac{1}{6}\cdot\dfrac{3}{4} & 0 & 0 \\[2mm] \dfrac{1}{6}\cdot\dfrac{3}{4} & \dfrac{1}{3}\left(\dfrac{3}{4}+3\right) & \dfrac{1}{6}\cdot 3 & 0 \\[2mm] 0 & \dfrac{1}{6}\cdot 3 & \dfrac{1}{3}\left(3+5\right) & \dfrac{1}{6}\cdot 5 \\[2mm] 0 & 0 & \dfrac{1}{6}\cdot 5 & \dfrac{1}{3}\cdot 5 \end{pmatrix} = \begin{pmatrix} \dfrac{1}{4} & \dfrac{1}{8} & 0 & 0 \\[2mm] \dfrac{1}{8} & \dfrac{5}{4} & \dfrac{1}{2} & 0 \\[2mm] 0 & \dfrac{1}{2} & \dfrac{8}{3} & \dfrac{5}{6} \\[2mm] 0 & 0 & \dfrac{5}{6} & \dfrac{5}{3} \end{pmatrix}$$

Примеры

$$g = \begin{pmatrix} \dfrac{\Delta y_0}{\Delta x_0} - A_0 \\[2mm] \dfrac{\Delta y_1}{\Delta x_1} - \dfrac{\Delta y_0}{\Delta x_0} \\[2mm] \dfrac{\Delta y_2}{\Delta x_2} - \dfrac{\Delta y_1}{\Delta x_1} \\[2mm] A_3 - \dfrac{\Delta y_2}{\Delta x_2} \end{pmatrix} = \begin{pmatrix} \dfrac{1/2}{3/4} - 1 \\[2mm] \dfrac{1}{3} - \dfrac{1/2}{3/4} \\[2mm] \dfrac{1}{5} - \dfrac{1}{3} \\[2mm] \dfrac{1}{6} - \dfrac{1}{5} \end{pmatrix} = \begin{pmatrix} -\dfrac{1}{3} \\[2mm] -\dfrac{1}{3} \\[2mm] -\dfrac{2}{15} \\[2mm] -\dfrac{1}{30} \end{pmatrix}$$

$$M_0 = -\frac{2948}{2325} \approx -1.2680; \; M_1 = -\frac{304}{2325} \approx -0.1308;$$

$$M_2 = -\frac{53}{2325} \approx -0.0228; \; M_3 = -\frac{4}{465} \approx -0.0086$$

Примеры

$$M_0 = -\frac{2948}{2325}; M_1 = -\frac{304}{2325}; M_2 = -\frac{53}{2325}; M_3 = -\frac{4}{465}$$

$$b_0 = A_0 = 1; \ b_1 = \frac{\Delta y_1}{\Delta x_1} - \frac{\Delta x_1}{6}\left(2M_1 + M_2\right) =$$

$$= \frac{1}{3} - \frac{3}{6}\left(2\cdot\left(-\frac{304}{2325}\right) - \frac{53}{2325}\right) = \frac{737}{1550} \approx 0.4755; \ b_2 =$$

$$= \frac{\Delta y_2}{\Delta x_2} - \frac{\Delta x_2}{6}\left(2M_2 + M_3\right) = \frac{1}{5} - \frac{5}{6}\left(2\cdot\left(-\frac{53}{2325}\right) - \frac{4}{465}\right) =$$

$$= \frac{38}{155} \approx 0.2452$$

Примеры

$$M_0 = -\frac{2948}{2325}; M_1 = -\frac{304}{2325}; M_2 = -\frac{53}{2325}; M_3 = -\frac{4}{465}$$

$$c_0 = \frac{1}{2} M_0 = \frac{1}{2} \cdot \left(-\frac{2948}{2325} \right) = -\frac{1474}{2325} \approx -0.6340;$$

$$c_1 = \frac{1}{2} M_1 = \frac{1}{2} \cdot \left(-\frac{304}{2325} \right) = -\frac{152}{2325} \approx -0.0654;$$

$$c_2 = \frac{1}{2} M_2 = \frac{1}{2} \cdot \left(-\frac{53}{2325} \right) = -\frac{53}{4650} \approx -0.0114$$

Примеры

$$M_0 = -\frac{2948}{2325}; M_1 = -\frac{304}{2325}; M_2 = -\frac{53}{2325}; M_3 = -\frac{4}{465}$$

$$d_0 = \frac{M_1 - M_0}{6\Delta x_0} = \frac{-\dfrac{304}{2325} + \dfrac{2948}{2325}}{6 \cdot 3/4} = \frac{5288}{20925} \approx 0.2527;$$

$$d_1 = \frac{M_2 - M_1}{6\Delta x_1} = \frac{-\dfrac{53}{2325} + \dfrac{304}{2325}}{6 \cdot 3} = \frac{251}{41850} \approx 0.0060;$$

$$d_2 = \frac{M_3 - M_2}{6\Delta x_2} = \frac{-\dfrac{4}{465} + \dfrac{53}{2325}}{6 \cdot 5} = \frac{11}{23250} \approx 0.0005$$

Примеры

i	0	1	2	3
x	1/4	1	4	9
y	1/2	1	2	3
a	1/2	1	2	3
b	1	0.4755	0.2452	
c	–0.6340	–0.0654	–0.0114	
d	0.2527	0.0060	0.0005	

$$S_0(x) = \frac{1}{2} + 1 \cdot \left(x - \frac{1}{4}\right) - 0.6340 \cdot \left(x - \frac{1}{4}\right)^2 + 0.2527 \cdot \left(x - \frac{1}{4}\right)^2, \ x \in \left[\frac{1}{4}, 1\right]$$

$$S_1(x) = 1 + 0.4755 \cdot (x - 1) - 0.0654 \cdot (x - 1)^2 + 0.0040 \cdot (x - 1)^3, \ x \in [1, 4]$$

$$S_2(x) = 2 + 0.2452 \cdot (x - 4) - 0.0114 \cdot (x - 4)^2 + 0.0005 \cdot (x - 4)^3, \ x \in [4, 9]$$

Примеры

$$S_0\left(\frac{4}{9}\right) = \frac{1}{2} + 1 \cdot \left(\frac{4}{9} - \frac{1}{4}\right) - 0.6340 \cdot \left(\frac{4}{9} - \frac{1}{4}\right)^2 + 0.2527 \cdot \left(\frac{4}{9} - \frac{1}{4}\right)^2 \approx$$

$$\approx 0.6723$$

$$S_1\left(\frac{9}{4}\right) = 1 + 0.4755 \cdot \left(\frac{9}{4} - 1\right) - 0.0654 \cdot \left(\frac{9}{4} - 1\right)^2 + 0.0060 \cdot \left(\frac{9}{4} - 1\right)^3 \approx$$

$$\approx 1.5039$$

$$S_2\left(\frac{25}{4}\right) = 2 + 0.2452 \cdot \left(\frac{25}{4} - 4\right) - 0.0114 \cdot \left(\frac{25}{4} - 4\right)^2 + 0.0005 \cdot \left(\frac{25}{4} - 4\right)^3 \approx$$

$$\approx 2.4993$$

Примеры

Точность интерполяции:

x	$P(x)$	$L(x)$	$f(x)$	δ
1/4	0.293	0.293	0.5	41.4%
9/4	1.699	1.699	1.5	13.3%
25/4	2.012	2.012	2.5	19.5%

x	$S(x)$	$S(x)$	$S(x)$	$S(x)$	$f(x)$	δ_2	δ_3	δ_3
4/9	0.678	0.663	0.693	0.672	2/3	0.6-1.7%	3.95%	0.80%
9/4	1.417	1.490	1.474	1.504	1.5	0.6-5.5%	1.73%	0.27%
25/4	2.615	2.491	2.569	2.499	2.5	0.4-4.6%	2.76%	0.04%