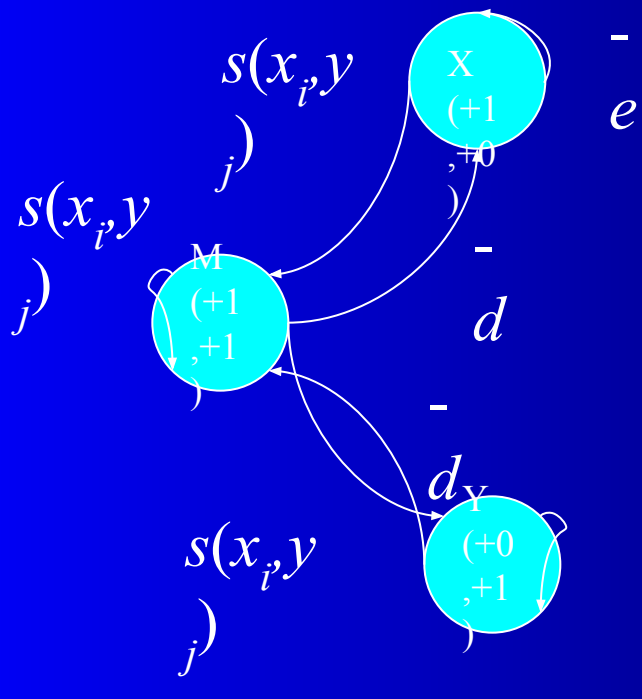
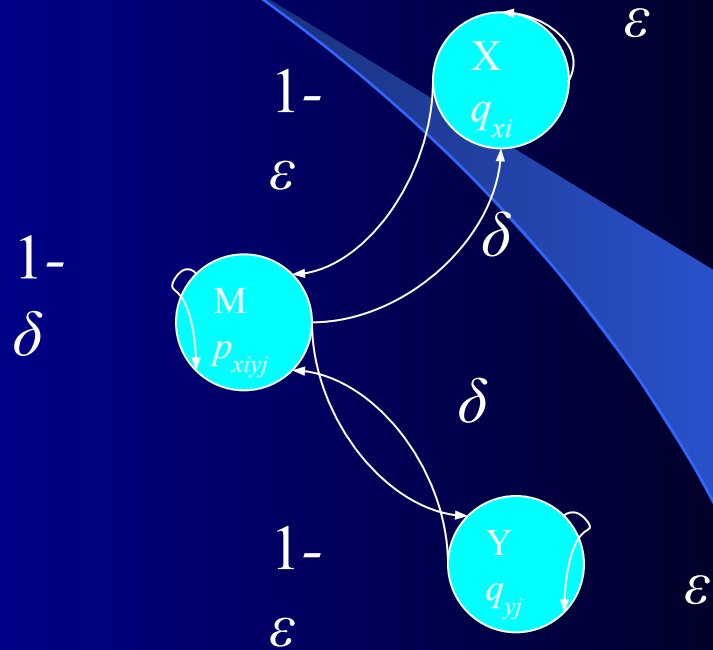


НММ выравнивание



Конечный автомат FSA



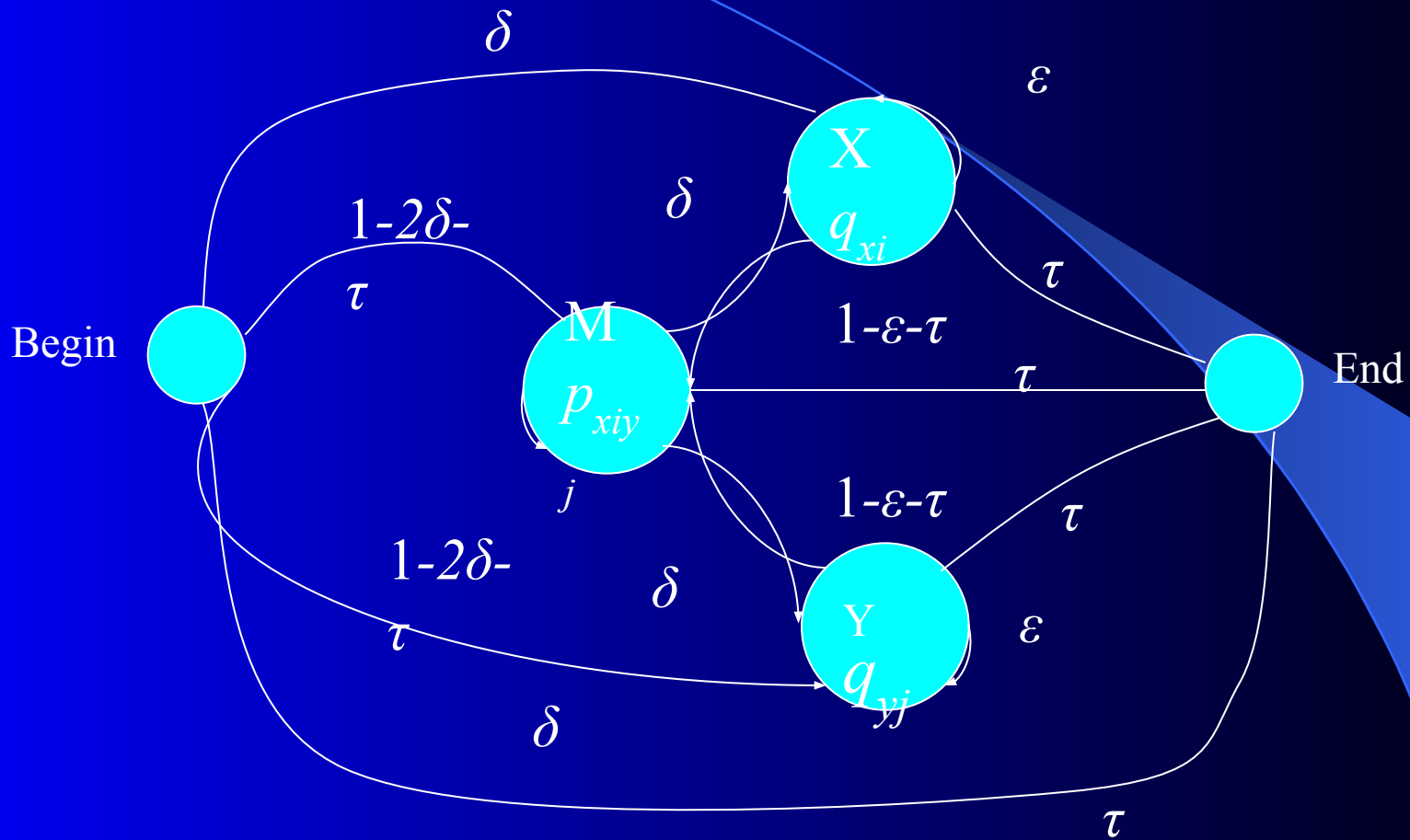
HMM

Рекурсия FSA

$$V^M(i, j) = s(x_i, y_j) + \max \begin{cases} V^M(i-1, j-1) \\ V^X(i-1, j-1) \\ V^Y(i-1, j-1) \end{cases}$$

$$V^X(i, j) = \max \begin{cases} V^M(i-1, j) - d \\ V^X(i-1, j) - e \end{cases}$$

$$V^Y(i, j) = \max \begin{cases} V^M(i, j-1) - d \\ V^Y(i, j-1) - e \end{cases}$$



Алгоритм Витерби

– Начало:

- $v^M(0, 0) = 1$. $v^X(0, 0) = v^Y(0, 0) = 0$ $v^*(-1, j) = v^*(i, -1) = 0$.

– Рекурсия: $i = 0, \dots, n, j = 0, \dots, m$, except for $(0, 0)$;

$$v^M(i, j) = p_{x_i y_j} \max \begin{cases} (1 - 2\delta - \tau)v^M(i-1, j-1) \\ (1 - \varepsilon - \tau)v^X(i-1, j-1) \\ (1 - \varepsilon - \tau)v^Y(i-1, j-1) \end{cases}$$

$$v^X(i, j) = q_{x_i} \max \begin{cases} \delta v^M(i-1, j) \\ \varepsilon v^X(i-1, j) \end{cases}$$

$$v^Y(i, j) = q_{y_j} \max \begin{cases} \delta v^M(i, j-1) \\ \varepsilon v^Y(i, j-1) \end{cases}$$

– Вывод: $v^E = \tau \max(v^M(n, m), v^X(n, m), v^Y(n, m))$

Полная вероятность выравниваний

- Алгоритм: Forward для парных HMMs

- Начало:

- $f^M(0, 0) = 1, f^X(0,0) = f^Y(0,0) = 0.$
- All $f(i,-1), f(-1, j)$ are set to 0.

- Рекурсия: $i = 0, \dots, n, j = 0, \dots, m$ except $(0,0)$;

$$f^M(i, j) = p_{x_i y_j} [(1 - 2\delta - \tau) f^M(i-1, j-1) + (1 - \varepsilon - \tau)(f^X(i-1, j-1) + f^Y(i-1, j-1))];$$

$$f^X(i, j) = q_{x_i} [\delta f^M(i-1, j) + \varepsilon f^X(i-1, j)];$$

$$f^Y(i, j) = q_{y_j} [\delta v^M(i, j-1) + \varepsilon v^X(i, j-1)].$$

- Вывод:

$$f^E(n, m) = \tau [f^M(n, m) + f^X(n, m) + f^Y(n, m)];$$

Вероятность выровненных x_i и y_j

$$P(x_i \diamond y_j | x, y) = \frac{P(x, y, x_i \diamond y_j)}{P(x, y)}$$

Forward algorithm

$$\begin{aligned} P(x, y, x_i \diamond y_j) &= P(x_{1\dots i}, y_{1\dots j}, x_i \diamond y_j) P(x_{i+1\dots n}, y_{j+1\dots m} | x_{1\dots i}, y_{1\dots j}, x_i \diamond y_j) \\ &= P(x_{1\dots i}, y_{1\dots j}, x_i \diamond y_j) P(x_{i+1\dots n}, y_{j+1\dots m} | x_i \diamond y_j) \end{aligned}$$

Forward algorithm

Backward algorithm

Backward Algorithm

- Алгоритм: Backward для парных HMMs

- Начало:

- $b^M(n, m) = b^X(n, m) = b^Y(n, m) = \tau$.
 - All $b^*(i, m+1), b^*(n+1, j)$ are set to 0.

- Рекурсия: $i = 1, \dots, n, j = 1, \dots, m$ except (n, m) ;

$$b^M(i, j) = (1 - 2\delta - \tau) p_{x_{i+1}y_{j+1}} b^M(i+1, j+1) + \delta [q_{x_{i+1}} b^X(i+1, j) + q_{y_{j+1}} b^Y(i, j+1)];$$

$$b^X(i, j) = (1 - \varepsilon - \tau) p_{x_{i+1}y_{j+1}} b^M(i+1, j+1) + \varepsilon q_{x_{i+1}} b^X(i+1, j);$$

$$b^Y(i, j) = (1 - \varepsilon - \tau) p_{x_{i+1}y_{j+1}} b^M(i+1, j+1) + \varepsilon q_{y_{j+1}} b^Y(i, j+1).$$