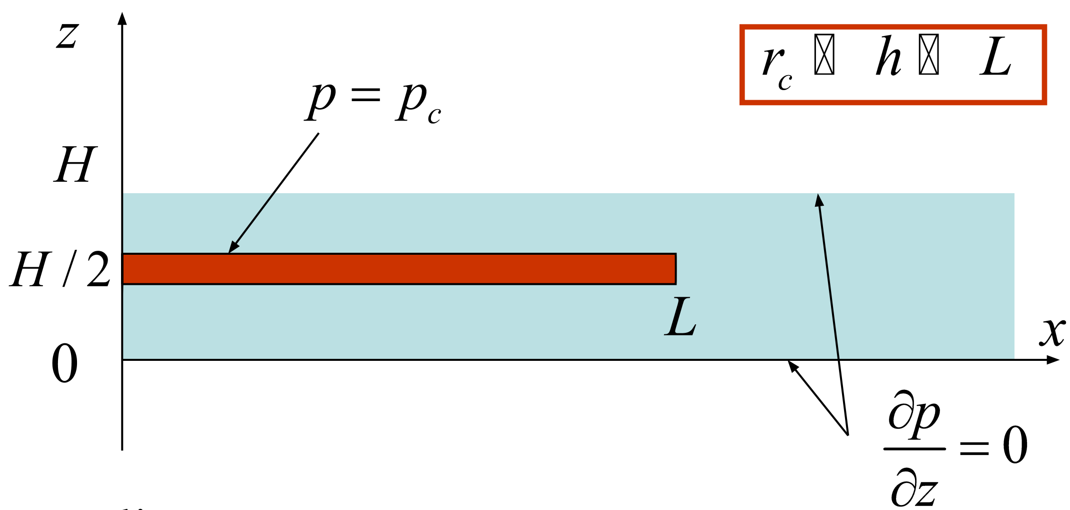
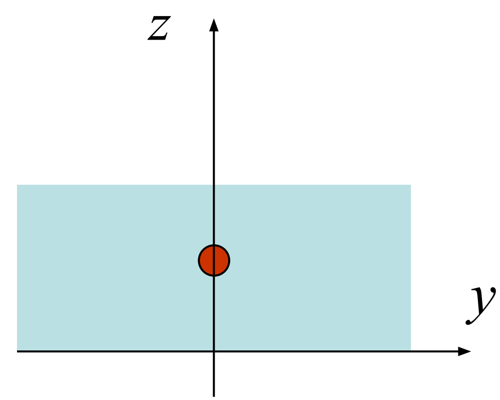
Введение в асимптотические методы.
Лекция 3: продолжение

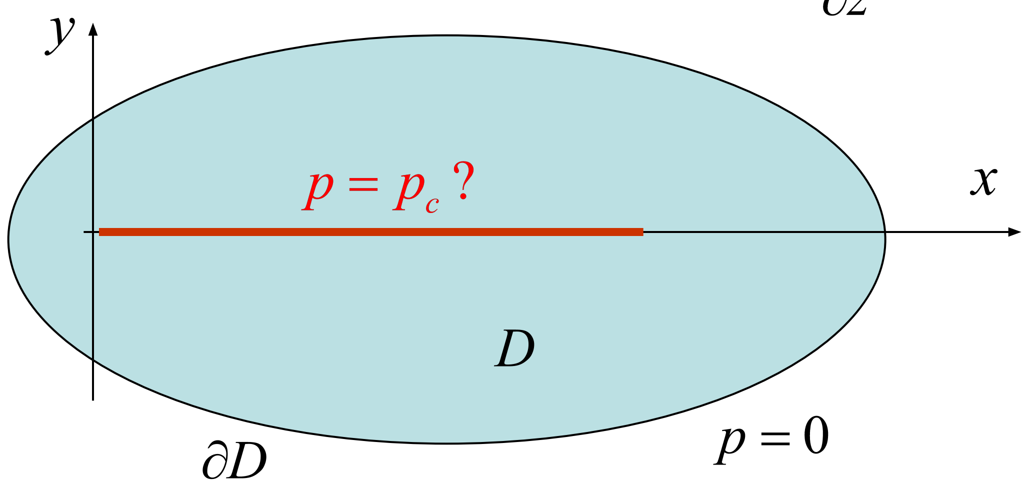
Граничные условия на горизонтальной скважине

1. ПОСТАНОВКА ПРОБЛЕМЫ

$$r_c \ll h \ll L$$







$$\Delta P_3 = 0, \qquad (x, y, z) \in D \times [0, H] \setminus S$$

$$P_3 = 0, \qquad (x, y, z) \in \partial D \times [0, H]$$

$$\partial P_3 / \partial z = 0, \qquad z = 0, \quad z = H$$

$$P_3 = P_c, \qquad (x, y, z) \in \partial S$$

$$P_3 \to P_2 \; ??$$

2. ТЕХНИКА

$$\begin{cases} P_2(x,y) = \int_0^L G_2(x,y;\tilde{x},0)\, q(\tilde{x})\, d\tilde{x} \\ \Delta\ G_2 = \dfrac{1}{H}\delta(x-\tilde{x})\ \delta(y-\tilde{y}) \\ G_2 = 0, \qquad (x,y)\in\partial D \end{cases}$$

$$\begin{cases} P_3(r) = \int_0^L G_3(x,y,z;\tilde{x},0,H/2)\, q(\tilde{x})\, d\tilde{x} \\ \Delta\ G_3 = \delta(x-\tilde{x})\ \delta(y-\tilde{y})\ \delta(z-\tilde{z}) \\ G_3 = 0, \qquad (x,y,z)\in\partial D\times[0,H] \\ \partial G_3/\partial z = 0, \qquad z=0, \quad z=H \end{cases}$$

$$P_3 - P_2 = \int_0^L R(x,y,z;\tilde{x},0,H/2)\, q(\tilde{x})\, d\tilde{x} \qquad\qquad R = G_3 - G_2$$

$$\begin{cases} \Delta R = \delta(x-\tilde{x})\,\delta(y)\left[\delta\left(z-H/2\right)-1/H\right] \\ \partial R/\partial z = 0, \qquad z=0, \quad z=H \\ R\to 0, \quad \sqrt{y^2+(x-\tilde{x})^2}\to\infty \end{cases}$$

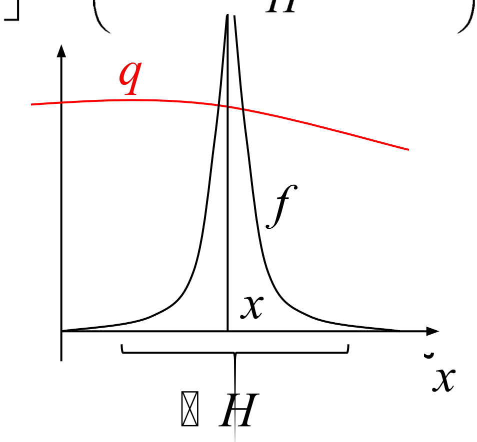
Преобразование Фурье

$$R = \frac{1}{\pi H}\sum_{n=1}^{\infty}\cos\left[\frac{2\pi n}{H}(z-H/2)\right] K_0\left(\frac{2\pi n\sqrt{y^2+(x-\tilde{x})^2}}{H}\right)$$

$$P_c - P_2 = \int_0^L \underbrace{R(x,0,r_c+H/2;\tilde{x},0,H/2)}\, q(\tilde{x})\, d\tilde{x} \ \approx q(x)\underbrace{\int_{-\infty}^{\infty} f(|\tilde{x}|)\ d\tilde{x}}$$

$$f(|x-\tilde{x}|) = \frac{1}{\pi H}\sum_{n=1}^{\infty}\cos\left(2\pi n\left(r_c/H\right)\right) K_0(2\pi n\,|x-\tilde{x}|/H)$$

$$\frac{1}{2\pi}\ln\left(\frac{H}{2\pi r_c}\right)$$



3. РЕЗУЛЬТАТ

$$\Delta_2 P_2 = 0, \qquad (x, y) \in D$$

$$P_2 = 0, \qquad (x, y) \in \partial D$$

$$\left[\partial P_2 / \partial n\right] = H^{-1} q(x), \; y = 0, \; 0 < x < L \quad \Rightarrow \quad P_c - P_2 = \alpha \left[\frac{\partial P_2}{\partial n}\right] \qquad \alpha = \frac{H}{2\pi} \ln\left(\frac{H}{2\pi r_c}\right)$$

Пример:

$L_1 = L_2 = 50 \qquad H = 10 \qquad r_c = 0.1$

Δ	q_0	q_1	q_2	δ_1 %	δ_2 %
100	17.54	17.37	21.03	1.01	19.8
50	20.54	20.30	25.56	1.19	24.4
10	28.41	27.91	40.29	1.78	41.81
2	34.13	33.27	58.69	2.49	72
0.5	36.44	35.22	74.53	3.35	104