GAUSS METHOD

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \cdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{cases}$$

$$\widetilde{A} = \left(\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right)$$

$$\left(\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ 0 & a'_{22} & \cdots & a'_{2n} & b'_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & a'_{mn} & b'_m \end{array}\right)$$

$$\begin{cases} 5x+3y+2z=-1 \\ x-10y+5z=2 \\ -9x-y-8z=0 \end{cases}$$

$b_1 \neq 0$ & $b_2 \neq 0$ => non**homogeneous system**
Δ=-716≠0=> nondegenerate matrix & **consistent** system (**has solution**) & **definite** system (**has the only solution**)

$$\tilde{A} = \left(\begin{array}{ccc|c} 5 & 3 & 2 & -1 \\ 1 & -10 & 5 & 2 \\ -9 & -1 & -8 & 0 \end{array} \right)$$

$$\overset{I \leftrightarrow II}{\approx} \left(\begin{array}{ccc|c} 1 & -10 & 5 & 2 \\ 5 & 3 & 2 & -1 \\ -9 & -1 & -8 & 0 \end{array} \right)$$

$$\overset{I\leftrightarrow II}{\approx}\left(\begin{array}{ccc|c} 1 & -10 & 5 & 2 \\ 5 & 3 & 2 & -1 \\ -9 & -1 & -8 & 0 \end{array}\right) \underset{I\cdot(-9)-III\rightarrow III}{\overset{I\cdot 5-II\rightarrow II}{\approx}} \left(\begin{array}{ccc|c} 1 & -10 & 5 & 2 \\ 0 & -53 & 23 & 11 \\ 0 & 91 & -37 & -18 \end{array}\right)$$

$$\overset{II\cdot 91-III\cdot(-53)\rightarrow III}{\approx}\left(\begin{array}{ccc|c} 1 & -10 & 5 & 2 \\ 0 & -53 & 23 & 11 \\ 0 & 0 & 132 & 47 \end{array}\right)$$

$$132z = 47$$

$$z = \frac{47}{132}$$

$$-53y + 23z = 11$$

$$y = \frac{11 - 23\cdot\frac{47}{132}}{-53}$$

$$y = \frac{11 - 23\cdot\frac{47}{132}}{-53}$$

$$y = -\frac{7}{132}$$

$$x - 10y + 5z = 2$$

$$x = 2 - 5\cdot\frac{47}{132} + 10\cdot\left(-\frac{7}{132}\right)$$

$$x = -\frac{41}{132}$$

$$Ответ: \left(-\frac{41}{132}; -\frac{7}{132}; \frac{47}{132}\right)$$

$b_1 \neq 0$ & $b_2 \neq 0$ => non**homogeneous system**

Δ=-716≠0=> nondegenerate matrix & **consistent** system (**has solution**) & **definite** system (**has the only solution**)

$$\begin{cases} 9a-2b+7c+d=8 \\ -7a-9b+7c+4d=3 \\ -9a+4b+9c+5d=1 \\ -15a-2c+2d=-6 \end{cases}$$

$b_1 \neq 0$ & $b_2 \neq 0$ => non**homogeneous system**

Δ=0=> degenerate matrix

$$\left(\begin{array}{cccc|c} 9 & -2 & 7 & 1 & 8 \\ -7 & -9 & 7 & 4 & 3 \\ -9 & 4 & 9 & 5 & 1 \\ -15 & 0 & -2 & 2 & -6 \end{array}\right)$$

$$\underset{\substack{I+III \\ I\cdot 15+IV\cdot 9}}{\overset{I\cdot(-7)-II\cdot 9}{\approx}} \left(\begin{array}{cccc|c} 9 & -2 & 7 & 1 & 8 \\ 0 & 95 & -112 & -43 & -83 \\ 0 & 2 & 16 & 6 & 9 \\ 0 & -30 & 87 & 33 & 66 \end{array}\right) \underset{II\cdot 30+IV\cdot 95}{\overset{II\cdot 2-III\cdot 95}{\approx}} \left(\begin{array}{cccc|c} 9 & -2 & 7 & 1 & 8 \\ 0 & 95 & -112 & -43 & -83 \\ 0 & 0 & -1744 & -656 & -1021 \\ 0 & 0 & 4905 & 1845 & 3780 \end{array}\right)$$

$$\overset{III\cdot 4905+IV\cdot 1744}{\approx} \left(\begin{array}{cccc|c} 9 & -2 & 7 & 1 & 8 \\ 0 & 95 & -112 & -43 & -83 \\ 0 & 0 & -1744 & -656 & -1021 \\ 0 & 0 & 0 & 0 & 1584315 \end{array}\right)$$

$$0 \cdot d = 1584315$$

inconsistent system (**has no solution**)

$$\begin{cases} -4x_1+9x_2+9x_3+5x_4=-6 \\ -9x_1+9x_2-2x_3-9x_4=0 \\ -3x_1-9x_2-6x_3=3 \\ -27x_1-3x_2-4x_3+x_4=-2 \end{cases}$$

$b_1 \neq 0$ & $b_2 \neq 0$ => non**homogeneous** **system**

Δ=0=> degenerate matrix

$$\left(\begin{array}{cccc|c} -4 & 9 & 9 & 5 & -6 \\ -9 & 9 & -2 & -9 & 0 \\ -3 & -9 & -6 & 0 & 3 \\ -27 & -3 & -4 & 1 & -2 \end{array}\right)$$

$$\underset{\substack{I\cdot 3-II\cdot 9-III\cdot 4 \\ I\cdot 27-IV\cdot 4}}{\overset{I\cdot 9-II\cdot 4}{\approx}} \left(\begin{array}{cccc|c} -4 & 9 & 9 & 5 & -6 \\ 0 & 45 & 89 & 81 & -54 \\ 0 & 63 & 51 & 15 & -30 \\ 0 & 255 & 259 & 131 & -154 \end{array}\right)$$

$$\underset{II\cdot 255-IV\cdot 45}{\overset{II\cdot 63-III\cdot 45}{\approx}} \left(\begin{array}{cccc|c} -4 & 9 & 9 & 5 & -6 \\ 0 & 45 & 89 & 81 & -54 \\ 0 & 0 & 3312 & 4428 & -2052 \\ 0 & 0 & 11040 & 14760 & -6840 \end{array}\right)$$

$$\overset{III\cdot 11040-IV\cdot 3312}{\approx} \left(\begin{array}{cccc|c} -4 & 9 & 9 & 5 & -6 \\ 0 & 45 & 89 & 81 & -54 \\ 0 & 0 & 3312 & 4428 & -2052 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

consistent system (**has solution**) & in**definite** system (**has more than one solution**)

$$0 \cdot x_4 = 0$$

$$\left(\begin{array}{cccc|c} -4 & 9 & 9 & 5 & -6 \\ 0 & 45 & 89 & 81 & -54 \\ 0 & 0 & 3312 & 4428 & -2052 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$0 \cdot x_4 = 0$$

$$x_4 = t$$

$$3312x_3 + 4428t = -2052$$

$$x_3 = \frac{-57 - 123t}{92}$$

$$45x_2 + 89 \cdot \frac{-57 - 123t}{92} + 81t = -54$$

$$x_2 = \frac{7 + 233t}{276}$$

$$-4x_1 + 9 \cdot \frac{7 + 233t}{276} + 9 \cdot \frac{-57 - 123t}{92} + 5t = -6$$

$$x_1 = \frac{15 + 13t}{92}$$

$$\left(\frac{15 + 13t}{92}; \frac{7 + 233t}{276}; \frac{-57 - 123t}{92}; t\right)$$ **general solution**

$$\left(\frac{15}{92}; \frac{7}{276}; -\frac{57}{92}; 0\right)$$

$$\left(\frac{7}{23}; \frac{60}{69}; -\frac{45}{23}; 1\right)$$ **particular solutions**

$$\begin{cases} 3a+b+c+d=3 \\ a+8b-3c=6 \\ 7a+b-c-d=8 \end{cases} \qquad \left(\begin{array}{cccc|c} 3 & 1 & 1 & 1 & 3 \\ 1 & 8 & -3 & 0 & 6 \\ 7 & 1 & -1 & -1 & 8 \end{array}\right)$$

$$\overset{I \leftrightarrow II}{\approx} \left(\begin{array}{cccc|c} 1 & 8 & -3 & 0 & 6 \\ 3 & 1 & 1 & 1 & 3 \\ 7 & 1 & -1 & -1 & 8 \end{array}\right) \quad \underset{I\cdot 7-III \leftrightarrow III}{\overset{I\cdot 3-II \leftrightarrow II}{\approx}} \left(\begin{array}{cccc|c} 1 & 8 & -3 & 0 & 6 \\ 0 & 23 & -10 & -1 & 15 \\ 0 & 55 & -20 & 1 & 34 \end{array}\right)$$

$$\overset{II\cdot 55-III\cdot 23 \leftrightarrow III}{\approx} \left(\begin{array}{cccc|c} 1 & 8 & -3 & 0 & 6 \\ 0 & 23 & -10 & -1 & 15 \\ 0 & 0 & -90 & -78 & 43 \end{array}\right)$$

$$d=t$$

$$-90c-78d=43$$

$$c=\frac{-43-78t}{90}$$

$$23b-10c-d=15$$

$$23b-10\cdot\frac{-43-78t}{90}-t=15$$

$$b=\frac{92-69t}{207}$$

$$a+8\cdot\frac{92-69t}{207}-3\cdot\frac{-43-78t}{90}=6$$

$$a=\frac{2093+138t}{2070}$$

$$\left(\frac{2093+138t}{2070};\frac{92-69t}{207};\frac{-43-78t}{90};t\right)$$

$$\left(\frac{2369}{2070};\frac{-46}{207};\frac{-199}{90};2\right)$$