INVERSE OF THE MATRIX A

$A \cdot A^{-1} = A^{-1} \cdot A = E$

$$A_{n\times n} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

$$A^{-1} = \frac{1}{\Delta} \cdot \begin{pmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{pmatrix}$$

$$A_{2\times 2} = \begin{pmatrix} 5 & -2 \\ -10 & 6 \end{pmatrix}$$

$$\Delta = \begin{vmatrix} 5 & -2 \\ -10 & 6 \end{vmatrix} = 10 \neq 0$$

$$A_{11} = 6 \qquad A_{21} = 2$$

$$A_{12} = 10 \qquad A_{22} = 5$$

$$A^{-1} = \frac{1}{10} \cdot \begin{pmatrix} 6 & 2 \\ 10 & 5 \end{pmatrix} = \begin{pmatrix} \frac{1}{10} \cdot 6 & \frac{1}{10} \cdot 2 \\ \frac{1}{10} \cdot 10 & \frac{1}{10} \cdot 5 \end{pmatrix} = \begin{pmatrix} 0{,}6 & 0{,}2 \\ 1 & 0{,}5 \end{pmatrix}$$

$$A_{3\times 3} = \begin{pmatrix} -1 & -2 & 0 \\ 8 & -9 & 0 \\ -9 & 9 & -1 \end{pmatrix}$$

$$\Delta = \begin{vmatrix} -1 & -2 & 0 \\ 8 & -9 & 0 \\ -9 & 9 & -1 \end{vmatrix} = -25 \neq 0$$

$$A_{11} = 9 \quad A_{21} = -2 \quad A_{31} = 0$$

$$A_{12} = 8 \quad A_{22} = 1 \quad A_{32} = 0$$

$$A_{13} = -9 \quad A_{23} = 27 \quad A_{33} = 25$$

$$A^{-1} = \frac{1}{-25} \cdot \begin{pmatrix} 9 & -2 & 0 \\ 8 & 1 & 0 \\ -9 & 27 & 25 \end{pmatrix} = \begin{pmatrix} -\frac{1}{25}\cdot 9 & -\frac{1}{25}\cdot(-2) & -\frac{1}{25}\cdot 0 \\ -\frac{1}{25}\cdot 8 & -\frac{1}{25}\cdot 1 & -\frac{1}{25}\cdot 0 \\ -\frac{1}{25}\cdot(-9) & -\frac{1}{25}\cdot 27 & -\frac{1}{25}\cdot 25 \end{pmatrix} = \begin{pmatrix} -0{,}36 & 0{,}08 & 0 \\ -0{,}32 & -0{,}04 & 0 \\ 0{,}36 & -1{,}08 & -1 \end{pmatrix}$$

SIMULTANEOUS LINEAR ALGEBRAIC EQUATIONS

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{cases}$$

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \qquad X=\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \qquad B=\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

$$AX=B \qquad \tilde{A}=\left(\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right)$$

Cramer

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n \end{cases}$$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$\Delta_{x_1} = \begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \end{vmatrix} \quad \Delta_{x_2} = \begin{vmatrix} a_{11} & b_1 & \cdots & a_{1n} \\ a_{21} & b_2 & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & b_n & \cdots & a_{nn} \end{vmatrix} \quad \cdots \quad \Delta_{x_n} = \begin{vmatrix} a_{11} & a_{12} & \cdots & b_1 \\ a_{21} & a_{22} & \cdots & b_2 \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & b_n \end{vmatrix}$$

$$x_1 = \frac{\Delta_{x_1}}{\Delta},\ x_2 = \frac{\Delta_{x_2}}{\Delta},\ \cdots\ x_n = \frac{\Delta_{x_n}}{\Delta}$$

$$\begin{cases} 4x - 5y = -8 \\ -3x - 2y = 7 \end{cases}$$

$$\Delta = \begin{vmatrix} 4 & -5 \\ -3 & -2 \end{vmatrix} = -23 \neq 0$$

$$\Delta_x = \begin{vmatrix} -8 & -5 \\ 7 & -2 \end{vmatrix} = 51$$

$$\Delta_y = \begin{vmatrix} 4 & -8 \\ -3 & 7 \end{vmatrix} = 4$$

$$x = \frac{\Delta_x}{\Delta} = -\frac{51}{23}$$

$$y = \frac{\Delta_y}{\Delta} = -\frac{4}{23}$$

$$\begin{cases} -\alpha - 9\beta + 9\gamma = -6 \\ -10\alpha - 5\beta + 5\gamma = -9 \\ 8\alpha - 2\beta + \gamma = 3 \end{cases}$$

$$\Delta = \begin{vmatrix} -1 & -9 & 9 \\ -10 & -5 & 5 \\ 8 & -2 & 1 \end{vmatrix} = 85 \neq 0$$

$$\Delta_\alpha = \begin{vmatrix} -6 & -9 & 9 \\ -9 & -5 & 5 \\ 3 & -2 & 1 \end{vmatrix} = 51 \quad \Delta_\beta = \begin{vmatrix} -1 & -6 & 9 \\ -10 & -9 & 5 \\ 8 & 3 & 1 \end{vmatrix} = 102 \quad \Delta_\gamma = \begin{vmatrix} -1 & -9 & -6 \\ -10 & -5 & -9 \\ 8 & -2 & 3 \end{vmatrix} = 51$$

$$\alpha = \frac{\Delta_\alpha}{\Delta} = \frac{51}{85} = 0{,}6; \beta = \frac{\Delta_\beta}{\Delta} = \frac{102}{85} = 1{,}2; \gamma = \frac{\Delta_\gamma}{\Delta} = \frac{51}{85} = 0{,}6$$

$$\begin{cases} -8a-5b-8c+6d=4 \\ 4a+3b+8c+5d=-10 \\ -5a+7b-4d=-2 \\ -4a-8b-8c+5d=7 \end{cases}$$

$$\Delta=\begin{vmatrix} -8 & -5 & -8 & 6 \\ 4 & 3 & 8 & 5 \\ -5 & 7 & 0 & -4 \\ -4 & -8 & -8 & 5 \end{vmatrix}=200\neq 0$$

$$\Delta_a=\begin{vmatrix} 4 & -5 & -8 & 6 \\ -10 & 3 & 8 & 5 \\ -2 & 7 & 0 & -4 \\ 7 & -8 & -8 & 5 \end{vmatrix}=184 \qquad \Delta_b=\begin{vmatrix} -8 & 4 & -8 & 6 \\ 4 & -10 & 8 & 5 \\ -5 & -2 & 0 & -4 \\ -4 & 7 & -8 & 5 \end{vmatrix}=56$$

$$\Delta_c=\begin{vmatrix} -8 & -5 & 4 & 6 \\ 4 & 3 & -10 & 5 \\ -5 & 7 & -2 & -4 \\ -4 & -8 & 7 & 5 \end{vmatrix}=-343 \qquad \Delta_d=\begin{vmatrix} -8 & -5 & -8 & 4 \\ 4 & 3 & 8 & -10 \\ -5 & 7 & 0 & -2 \\ -4 & -8 & -8 & 7 \end{vmatrix}=-32$$

$$a=\frac{\Delta_a}{\Delta}=\frac{184}{200}=0{,}92;\ b=\frac{\Delta_b}{\Delta}=\frac{56}{200}=0{,}28;\ c=\frac{\Delta_c}{\Delta}=-\frac{343}{200}=-1{,}715;\ d=\frac{\Delta_d}{\Delta}=-\frac{32}{200}=-0{,}16$$

$a=0{,}92; b=0{,}28; c=-1{,}715; d=-0{,}16.$ *Answer* : $(0{,}92;0{,}28;-1{,}715;-0{,}16)$