AUTOMATICS and AUTOMATIC CONTROL

LECTURE 3

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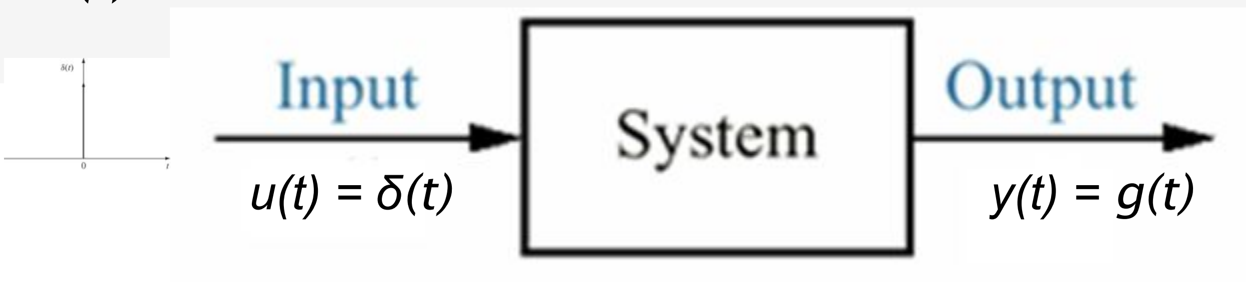
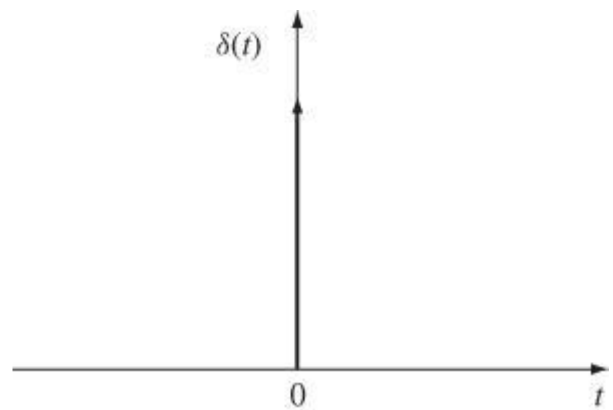
IMPULSE RESPONSE

The impulse response of the system, denoted by *g(t)* is the transient output response *y* of the system when its input is fed with an ideal Dirac impulse *u(t) = δ(t)*



For linear system with transfer function:



$$G(s) = \frac{Y(s)}{U(s)} \qquad Y(s) = U(s)G(s)$$

When : *u(t) = δ(t)* $\Rightarrow U(s) = 1$

$$Y(s) = G(s)$$

then :

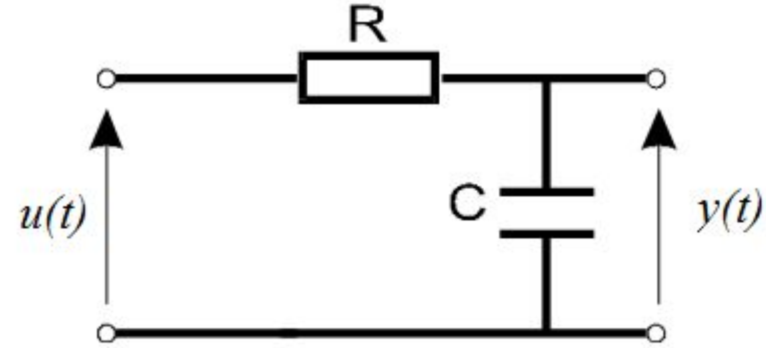
$$g(t) = L^{-1}[G(s)]$$

IMPULSE RESPONSE

Example: RC circuit:

$$RC\frac{dy(t)}{dt}+y(t)=u(t)$$

$$G(s)=\frac{1}{RCs+1}$$

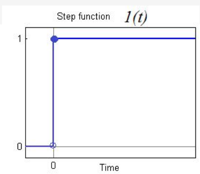


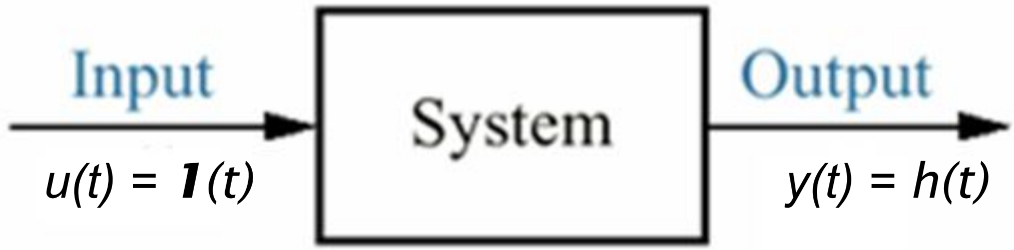
$$g(t)=L^{-1}\left[\frac{1}{RCs+1}\right]=L^{-1}\left[\frac{\frac{1}{RC}}{s+\frac{1}{RC}}\right]=\frac{1}{RC}L^{-1}\left[\frac{1}{s+\frac{1}{RC}}\right]$$

$$g(t)=\frac{1}{RC}e^{-\frac{1}{RC}t}$$

STEP RESPONSE

The step response of the system, denoted by *h(t)* is the transient output response *y* of the system when its input is fed with a unit step signal *u(t)* = ***1***(*t*)





For linear system with transfer function:

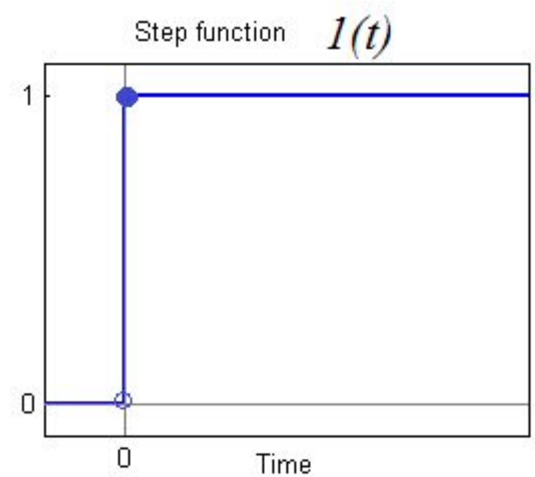
$$G(s) = \frac{Y(s)}{U(s)} \qquad Y(s) = U(s)G(s)$$

When : $u(t) = \mathbf{1}(t) \Rightarrow U(s) = \frac{1}{s}$

$$Y(s) = \frac{1}{s}G(s) = H(s)$$

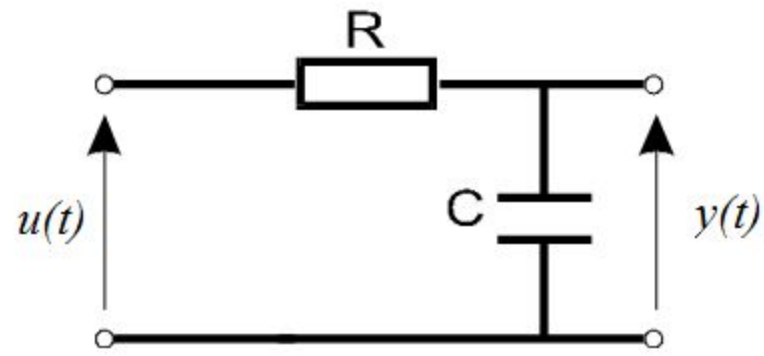
then :

$$h(t) = L^{-1}\left[\frac{1}{s}G(s)\right]$$



STEP RESPONSE

Example: RC circuit:



$$RC\frac{dy(t)}{dt} + y(t) = u(t)$$

$$G(s) = \frac{1}{RCs+1}$$

$$h(t) = L^{-1}\left[\frac{1}{s}\frac{1}{RCs+1}\right]$$

After partial fractions decomposition : $$\frac{1}{s}\frac{1}{RCs+1} = \frac{A}{s} + \frac{B}{RCs+1} = \frac{1}{s} - \frac{RC}{RCs+1}$$

$$h(t) = L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{1}{s+\frac{1}{RC}}\right]$$

$$h(t) = 1 - e^{-\frac{1}{RC}t}$$

Initial and final value of step response?

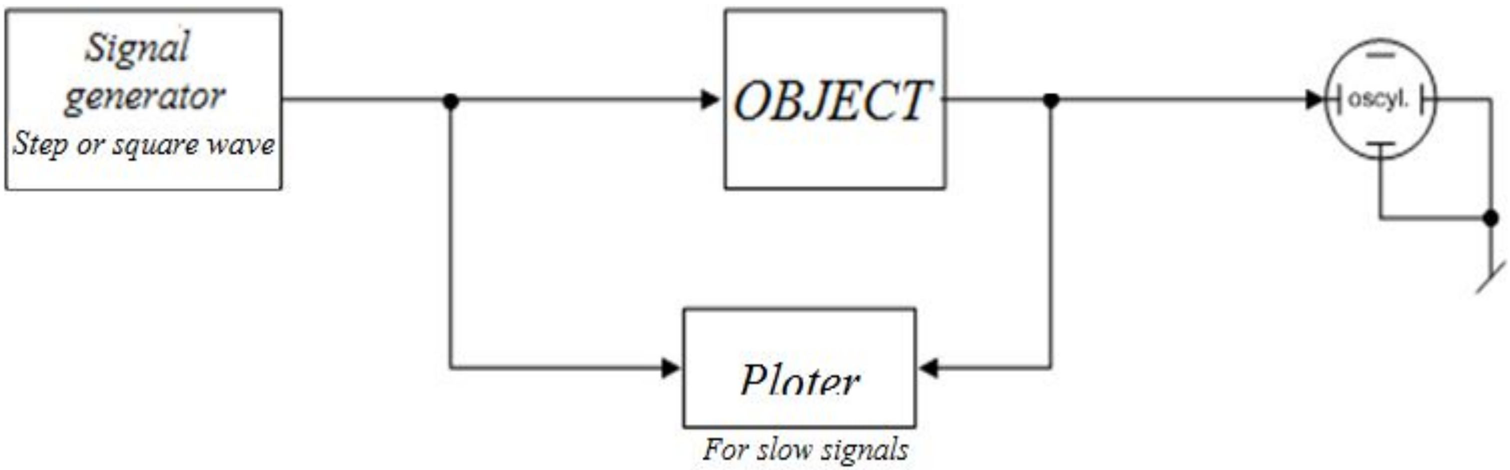
STEP RESPONSE VERSUS IMPULSE RESPONSE

$$H(s) = \frac{1}{s} G(s) \quad \Rightarrow \quad h(t) = \int_0^t g(t)\,dt$$

$$G(s) = sH(s) \quad \Rightarrow \quad g(t) = \frac{dh(t)}{dt}$$

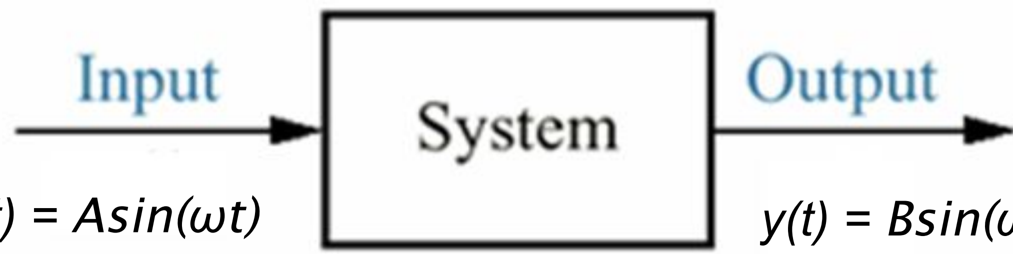
STEP RESPONSE

Device for an experimental obtaining the step response:



FREQUENCY RESPONSES

Frequency response is the quantitative measure of the output spectrum of a system or device in response to a stimulus, and is used to characterize the dynamics of the system. It is a measure of amplitude and phase of the output as a function of frequency, in comparison to the input.



$u(t) = A\sin(\omega t)$ $y(t) = B\sin(\omega t + \varphi)$

If a sine wave is injected into a system at a given frequency, a linear system will respond at that same frequency with a certain amplitude and a certain phase angle relative to the input

where:

$\omega = 2\pi f \quad [\frac{rad}{s}]$ - angular frequency

FREQUENCY RESPONSES

$$G(s)|_{s=j\omega} \quad \Rightarrow \quad G(j\omega) = \frac{\hat{Y}}{\hat{U}} = \frac{Y(j\omega)}{U(j\omega)}$$

$$G(j\omega) = \frac{B \cdot e^{j(\omega t+\Phi)}}{A \cdot e^{j\omega t}} = \frac{B}{A} e^{j\Phi} = |G(j\omega)| \cdot e^{j\Phi(\omega)} = \mathrm{Re}G(j\omega) + j\,\mathrm{Im}G(j\omega) = |G(j\omega)| \cdot e^{j\arg G(j\omega)}$$

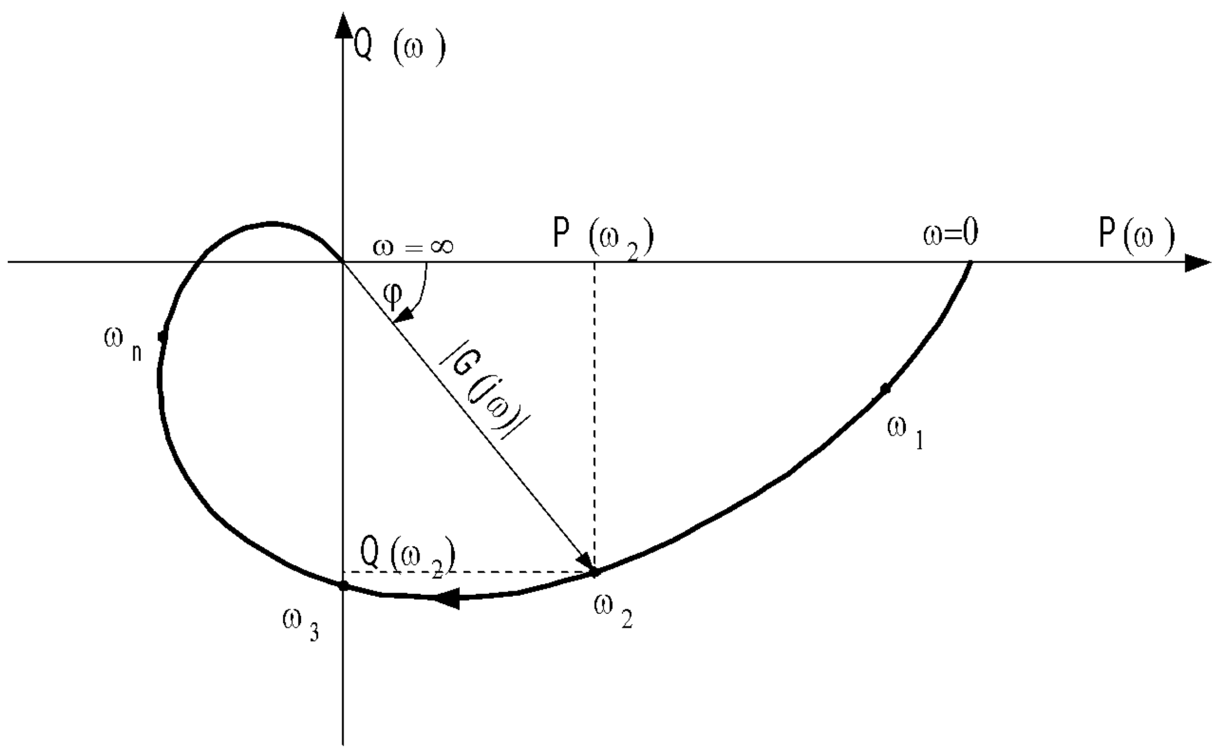
where:

$$|G(j\omega)| = \sqrt{[\mathrm{Re}G(j\omega)]^2 + [\mathrm{Im}G(j\omega)]^2} \overset{mark.}{=} \sqrt{P^2(\omega) + Q^2(\omega)}$$

$$\arg G(j\omega) = \Phi(\omega) = arc\,\mathrm{tg}\frac{\mathrm{Im}[G(j\omega)]}{\mathrm{Re}[G(j\omega)]} = arc\,\mathrm{tg}\,\frac{Q(\omega)}{P(\omega)}$$

FREQUENCY RESPONSES

- **Nyquist plot -** the graph of the frequency respons with coordinates *P(ω)* = Re *[G(j ω)]* and *Q(ω)* = Im *[G(j ω)]*



FREQUENCY RESPONSES

• **Nyquist plot** - EXAMPLE

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2}{10s + 1} \quad \Rightarrow \quad G(j\omega) = \frac{2}{1 + j10\omega}$$

$$P(\omega) = \frac{2}{1 + 100\omega^2} \qquad Q(\omega) = -\frac{20\omega}{1 + 100\omega^2}$$

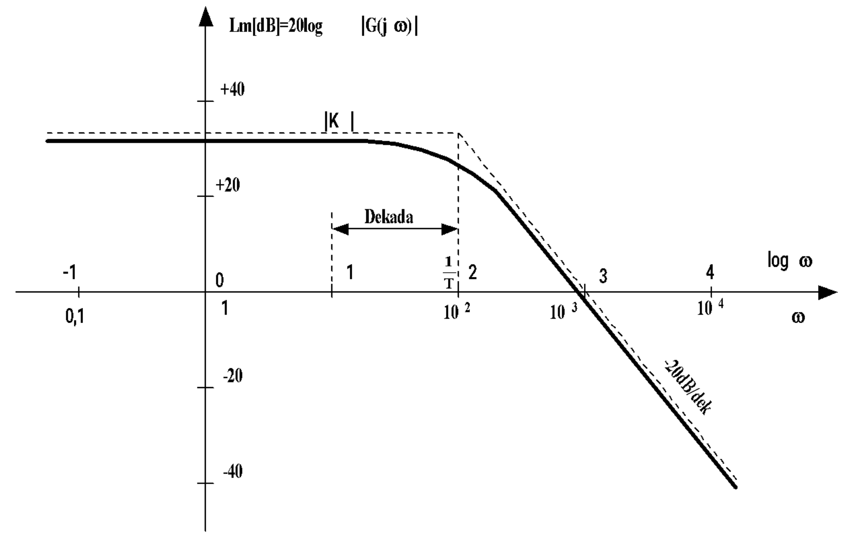
ω	**0**	**1/20**	**1/10**	**1/2**	**1**	**2**	**∞**
P(ω)	2	8/5	1	1/13	2/101	2/401	0
Q(ω)	0	-4/5	-1	-5/13	-20/101	-40/401	0

FREQUENCY RESPONSES

- **Bode plots -** present the frequency characteristics separetely in the form of:

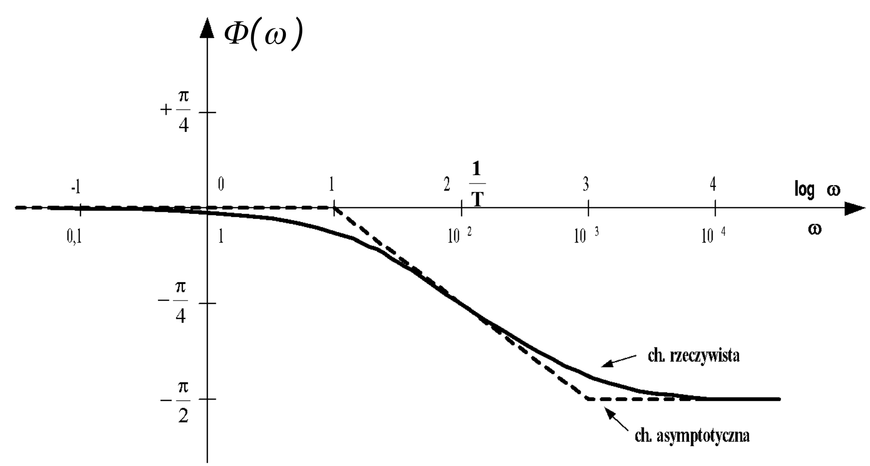
a) magnitude:

$$L_m(\omega) = 20\,log\left|G(j\omega)\right| \text{ [dB]}$$



b) phase:

$$\Phi(\omega)$$



FREQUENCY RESPONSES

• **Bode plots** - EXAMPLE

$$G(s)=\frac{Y(s)}{U(s)}=\frac{k}{Ts+1} \quad \Rightarrow \quad G(j\omega)=\frac{k}{1+j\omega T}$$

$$L_m(\omega)=20\log\left|G(j\omega)\right|=20\log\left|\frac{k}{1+j\omega T}\right|=20\log\frac{|k|}{|1+j\omega T|}=20\log\frac{k}{\sqrt{1+\omega^2T^2}}=$$

$$=20\log k-20\log\sqrt{1+\omega^2T^2}$$

$$-20\log\sqrt{\omega^2T^2+1}=\begin{cases}0 & for \quad \omega T<<1\\ -20\log(\omega T) & for \quad \omega T>>1\end{cases}$$

$$-20\log(\omega T)=-20\log\omega-20\log T$$

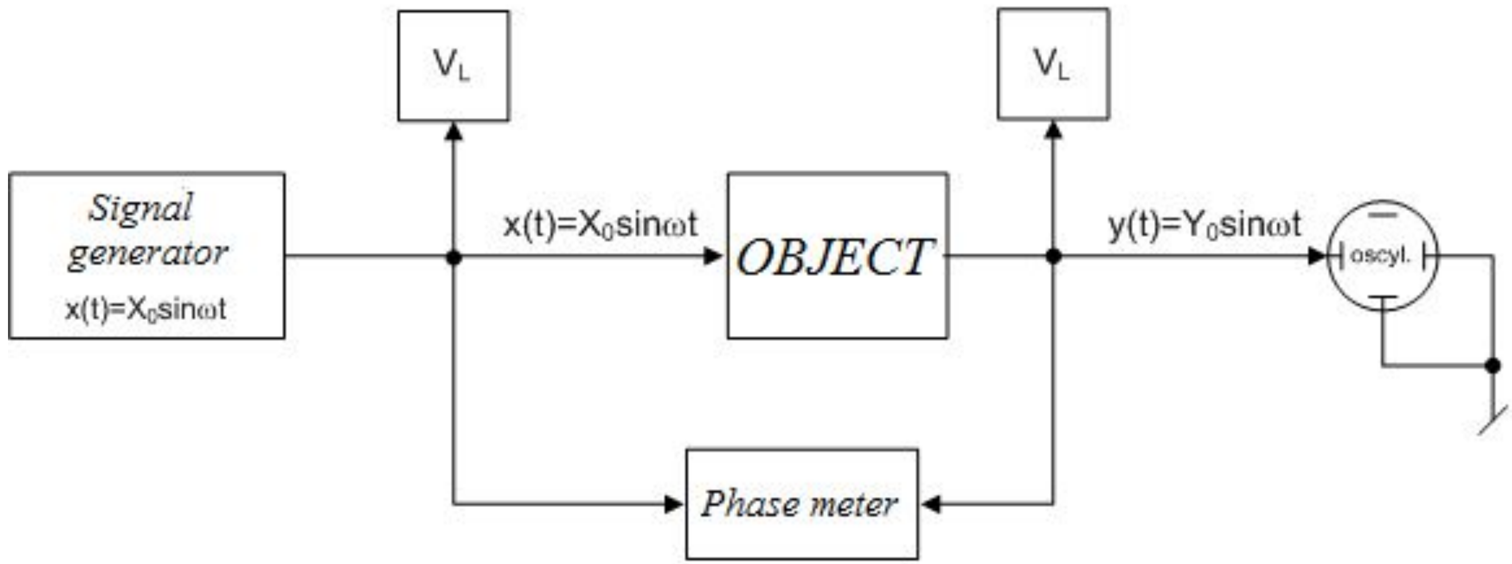
$$when: \quad \omega=1/T \quad then: \quad -20\log(\omega T)=0$$

$$P(\omega)=\frac{k}{1+\omega^2T^2} \qquad Q(\omega)=-\frac{k\omega T}{1+\omega^2T^2}$$

$$\varphi(\omega)=arctg\frac{Q(\omega)}{P(\omega)}=arctg(-\omega T)=-arctg(\omega T)$$

FREQUENCY RESPONSES

Device for an experimental obtaining the frequency response characteristics:



FREQUENCY RESPONSES

- **Bode plots –** objects connected in series

$$G(j\omega) = |G_1(j\omega)|e^{j\varphi_1/\omega/} \cdot |G_2(j\omega)|e^{j\varphi_2/\omega/} \ldots |G_n(j\omega)|e^{j\varphi_n/\omega/} =$$

$$= |G_1(j\omega)| \cdot |G_2(j\omega)| \ldots |G_r(j\omega)| e^{j\varphi_1/\omega/+\varphi_2/\omega/+\ldots+\varphi_r/\omega/}$$

magnitude plot:

$$L_m[G(j\omega)] = 20\log|G(j\omega)| = 20\log|G_1(j\omega)| + 20\log|G_2(j\omega)| + \ldots + 20\log|G_r(j\omega)| =$$

$$= L_m[G_1(j\omega)] + L_m[G_2(j\omega)] + \ldots + L_m[G_r(j\omega)]$$

phase plot:

$$\varphi(\omega) = \varphi_1(\omega) + \varphi_2(\omega) + \ldots + \varphi_r(\omega)$$

FREQUENCY RESPONSES

- **Bode plots –** objects connected in series EXAMPLE

$$G(s) = \frac{k_1}{T_1 s + 1} \frac{k_2}{T_2 s + 1} = \frac{k}{(T_1 s + 1)(T_2 s + 1)} \qquad \Rightarrow \qquad G(j\omega) = \frac{k}{(1 + j\omega T_1)(1 + j\omega T_2)}$$

$$L_m(\omega) = 20\log\left|G(j\omega)\right| = 20\log\frac{|k|}{|1 + j\omega T_1|} + 20\log\frac{1}{|1 + j\omega T_2|}$$

$$\varphi(\omega) = \varphi_1(\omega) + \varphi_2(\omega) = -arctg(\omega T_1) + (-arctg(\omega T_2))$$

EXERCISE:

$$G(s) = \frac{s}{(0.1s + 1)(0.01s + 1)}$$

THANK YOU