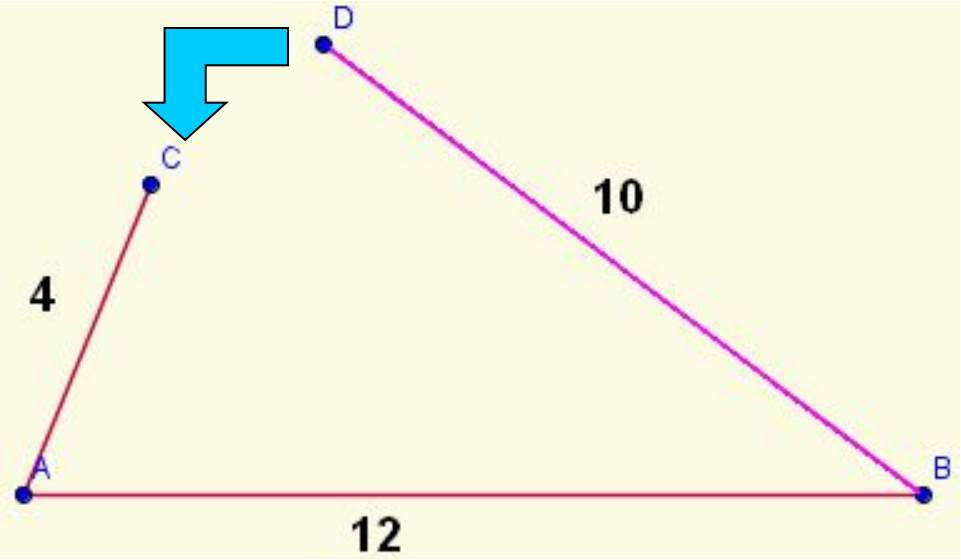
G.5

Triangle Inequalities

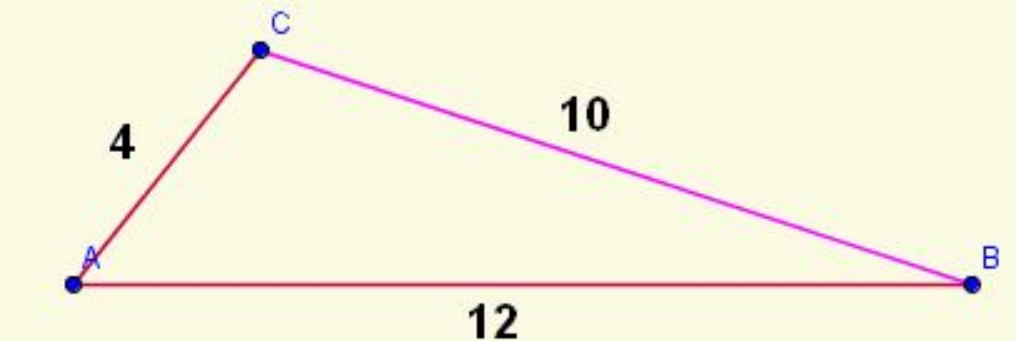
Modified by Lisa Palen

Triangle Inequality Theorem:

Can you make a triangle?

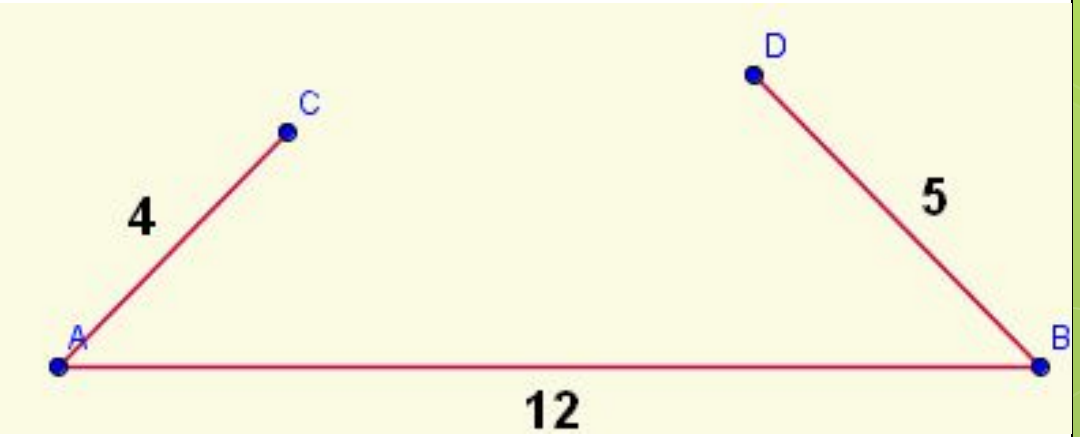


Yes!



Triangle Inequality Theorem:

Can you make a triangle?



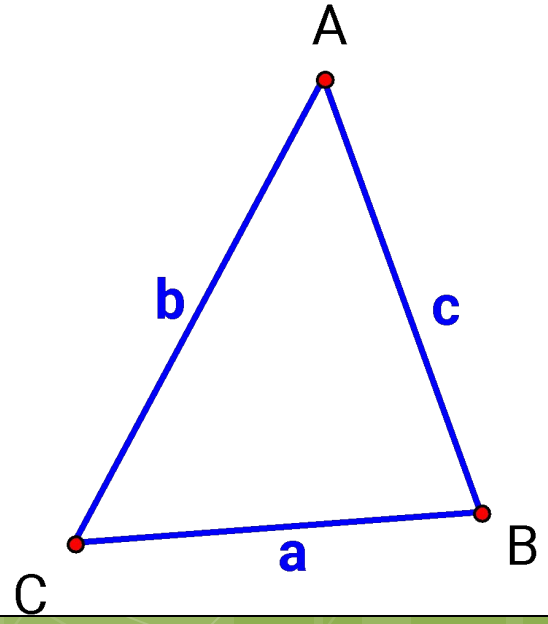
NO

because

$4 + 5 < 12$

Triangle Inequality Theorem:

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



$a + b > c$

$a + c > b$

$b + c > a$

Finding the range of the third side:

Example Given a triangle with sides of length 3 and 7, find the range of possible values for the third side.

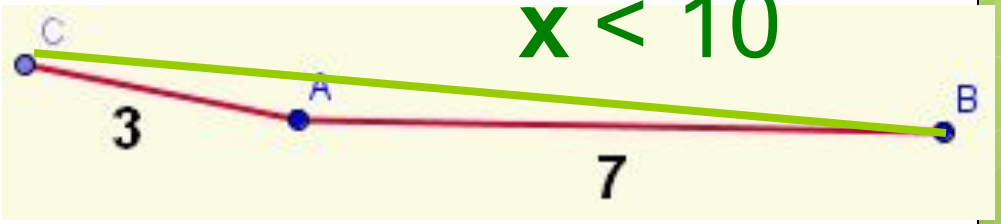
Solution Let x be the length of the third side of the triangle.

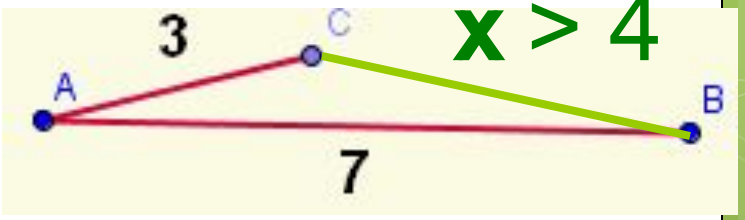
The maximum value:

$x < 3 + 7 = 10$

The minimum value:

$x > 7 - 3 = 4$





So $4 < x < 10$ (x is between 4 and 10.)

Finding the range of the third side:

Given *The lengths of two sides of a triangle*

- Since the third side cannot be larger than the other two added together, we find the maximum value by adding the two sides.
- Since the third side and the smallest side given cannot be larger than the other side, we find the minimum value by subtracting the two sides.

Difference < Third Side < Sum

Finding the range of the third side:

Example Given a triangle with sides of length a and b, find the range of possible values for the third side.

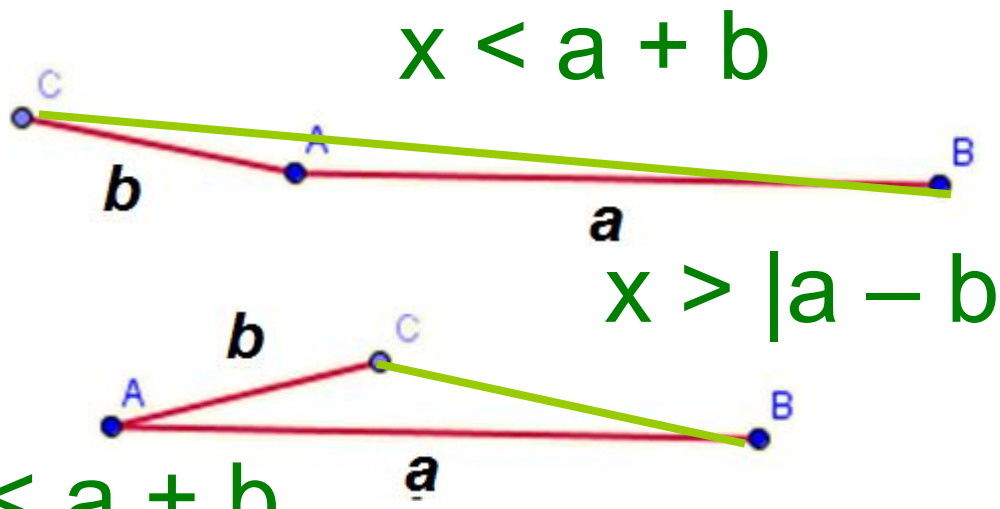
Solution Let x be the length of the third side of the triangle.

The maximum value:

$x < a + b$

The minimum value:

$x > |a - b|$

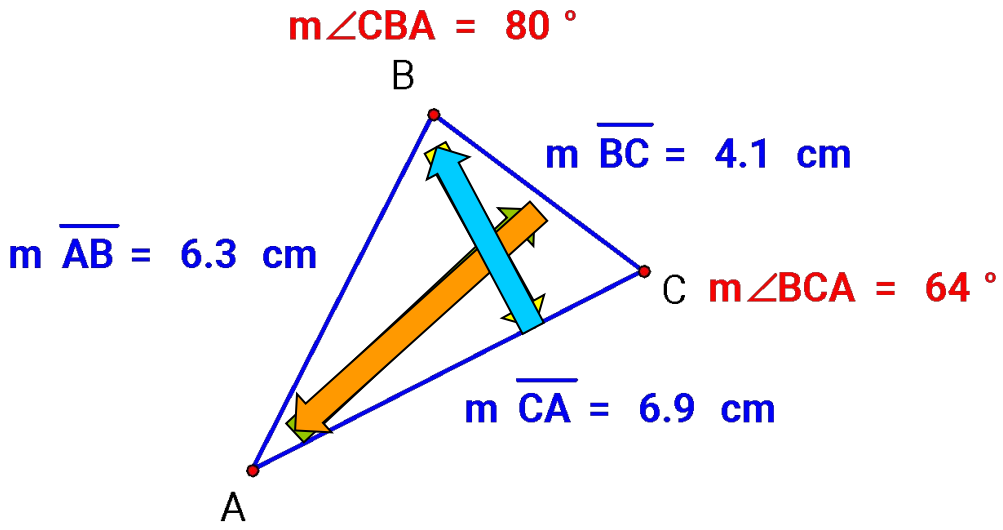


So $|a - b| < x < a + b$

(x is between |a – b| and a + b.)

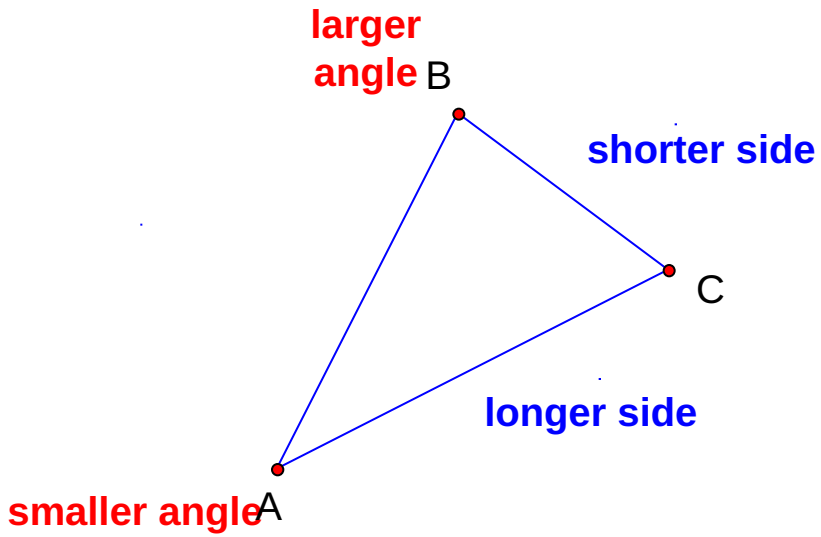
In a Triangle:

- The smallest angle is opposite the smallest side.
- The largest angle is opposite the largest side.
- The smallest side is opposite the smallest angle.
- The largest side is opposite the largest angle.



Theorem

- If one *angle* of a triangle is larger than a second angle, then the *side* opposite the first angle is larger than the side opposite the second angle.



Theorem

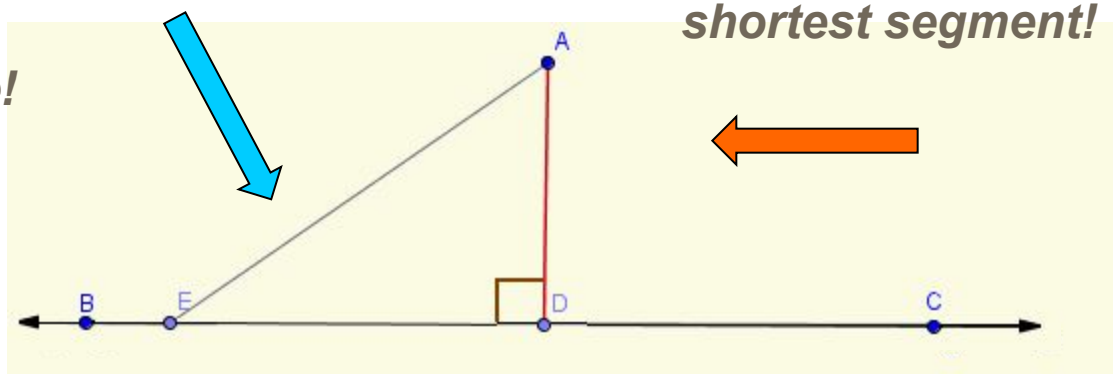
- If one *side* of a triangle is larger than a second side, then the *angle* opposite the first side is larger than the angle opposite the second side.

Corollary #1:

The perpendicular segment from a point to a line is the shortest segment from the point to the line.

This side is longer because it is opposite the largest angle!

This is the shortest segment!



Corollary #2:

The perpendicular segment from a point to a plane is the shortest segment from the point to the plane.

This side is longer because it is opposite the largest angle!

This is the shortest segment!