# $Arctg, arcctg$. Решение уравнений $tg\ t = a, ctg\ t = a$

# Упражнение:

- Вычислите:

$$ctg\ (arccos\, 0) = 0$$

$$arcsin \frac{\sqrt{3}}{2} + arccos \frac{\sqrt{3}}{2} = \frac{\pi}{2}$$

если $|a| \le 1$, то

$$arccos\, a = t \Leftrightarrow \begin{cases} cos\, t = a, \\ 0 \le t \le \pi \end{cases}$$

$$cos\, t = a$$

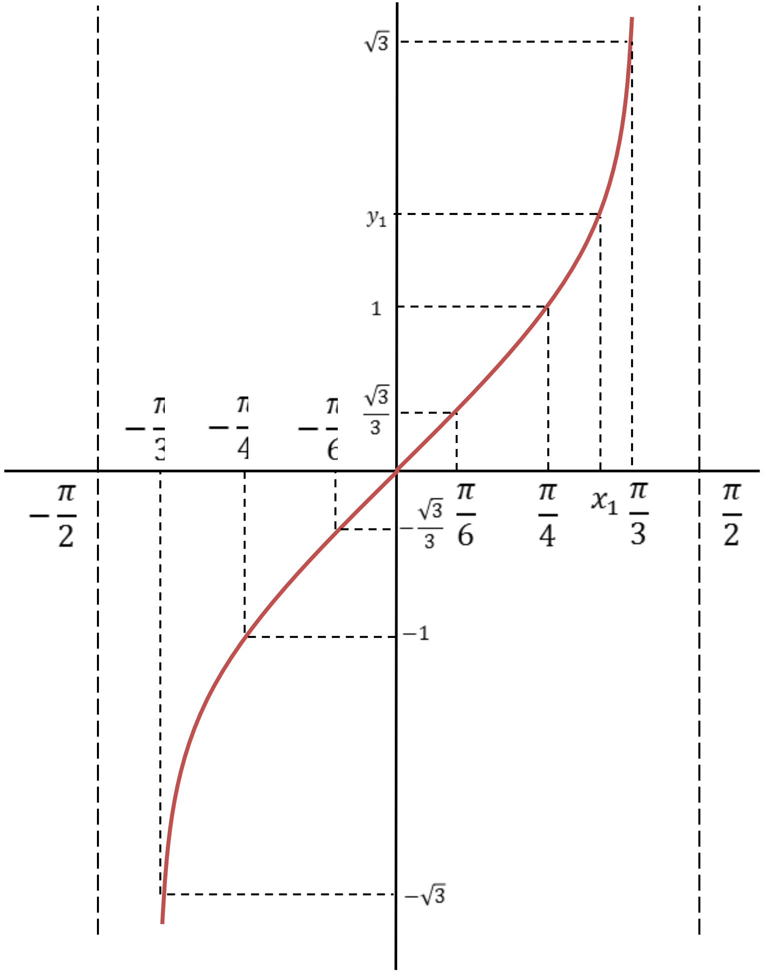
$$t = \pm \arccos a + 2\pi k, k \in Z$$

если $|a| \le 1$, то

$$arcsin\, a = t \Leftrightarrow \begin{cases} sin\, t = a, \\ -\frac{\pi}{2} \le t \le \frac{\pi}{2}. \end{cases}$$

$$sin\, t = a$$

$$t = (-1)^k arcsin\, a + \pi k, k \in Z$$




Функция непрерывна, монотонн

$f(x) \in E(f)$

$x_1 = arctg\, y_1$

$$arctg\, a = t \Leftrightarrow \begin{cases} tg\, t = a, \\ -\frac{\pi}{2} < t < \frac{\pi}{2}. \end{cases}$$

$$arctg\, 1 = \frac{\pi}{4} \Leftrightarrow \begin{cases} \frac{\pi}{4} \in \left(-\frac{\pi}{2};\, \frac{\pi}{2}\right); \\ tg\, \frac{\pi}{4} = 1. \end{cases}$$

$$arcctg\, a = t \Leftrightarrow \begin{cases} ctg\, t = a, \\ 0 < t < \pi. \end{cases}$$

$$arctg\, \sqrt{3} = \frac{\pi}{3} \Leftrightarrow \begin{cases} \frac{\pi}{3} \in \left(-\frac{\pi}{2};\, \frac{\pi}{2}\right); \\ tg\, \frac{\pi}{3} = \sqrt{3}. \end{cases}$$

# Свойства:

$$arctg\,(-a) = -arctg\,a$$

$$arcctg\,(-a) = \pi - arcctg\,a$$

# Пример:

Вычислить

- $$arctg\left(-\frac{\sqrt{3}}{3}\right)+arcctg\,\sqrt{3}+arcctg\,0+arctg\,(-2)+arctg\,2.$$

Решение:

$$arctg\left(-\frac{\sqrt{3}}{3}\right)=-\frac{\pi}{6}$$

$$arcctg\,\sqrt{3}=\frac{\pi}{6}$$

$$arcctg\,0=\frac{\pi}{2}$$

$$arctg\,(-2)=-arctg\,2$$

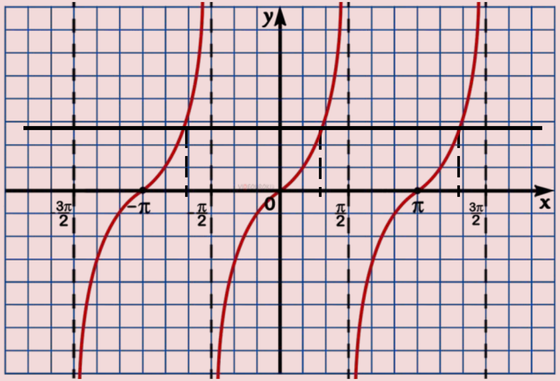
$$-\frac{\pi}{6}+\frac{\pi}{6}+\frac{\pi}{2}-arctg\,2+arctg\,2=\frac{\pi}{2}$$

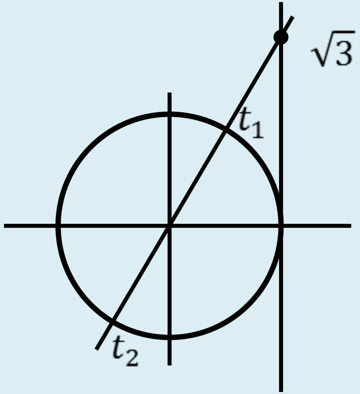
Ответ: $\frac{\pi}{2}$.

# Пример:

- Решить уравнение $tg\ t = \sqrt{3}$.

Решение:

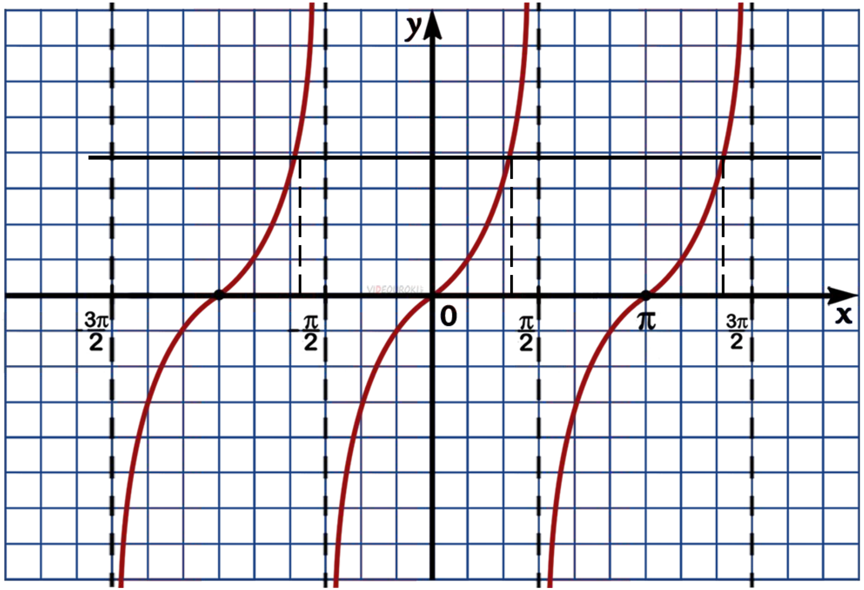



$$t = arctg\sqrt{3} + \pi k, k \in Z$$

$$t = \frac{\pi}{3} + \pi k, k \in Z$$

$$\left.\begin{aligned} t_1&: \frac{\pi}{3} + 2\pi k, k \in Z \\ t_2&: \frac{\pi}{3} + \pi + 2\pi k, k \in Z \end{aligned}\right\} \Leftrightarrow t = \frac{\pi}{3} + \pi k, k \in Z$$



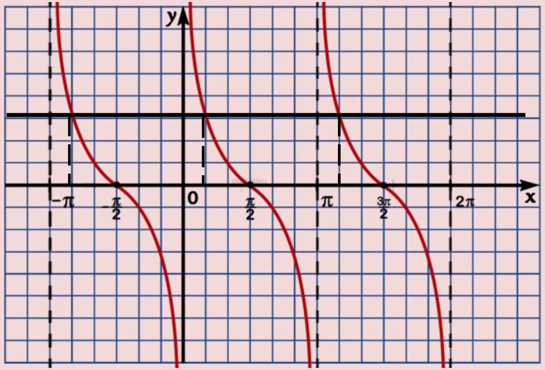

$$tg\, t = a$$

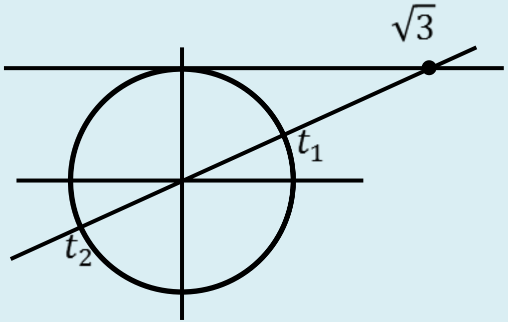
$$t = arctg\, a + \pi k, k \in Z$$

# Пример:

- Решить уравнение $ctg\ t = \sqrt{3}$.

Решение:

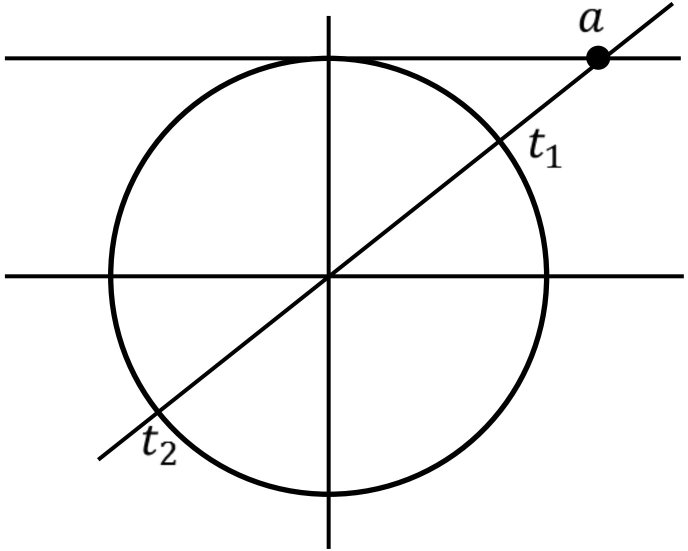



$$t = arcctg\sqrt{3} + \pi k, k \in Z$$

$$t = \frac{\pi}{6} + \pi k, k \in Z$$

$$\left.\begin{aligned} t_1&: \frac{\pi}{6} + 2\pi k, k \in Z \\ t_2&: \frac{\pi}{6} + \pi + 2\pi k, k \in Z \end{aligned}\right\} \Leftrightarrow t = \frac{\pi}{6} + \pi k, k \in Z$$




# $ctg\, t = a$

$t_1$: $arcctg\, a + 2\pi k, k \in Z$

$t_2$: $arcctg\, a + \pi + 2\pi k, k \in Z$

$$t = arcctg\, a + \pi k, k \in Z$$

# Пример:

- Решить уравнение

$tg^2x - 6tg\,x + 5 = 0.$

Решение:

$tg\,x = b$

$b^2 - 6b + 5 = 0$

$b_1 = 1, b_2 = 5$

$tg\,x = 1$ $\qquad$ $tg\,x = 5$

$x = \frac{\pi}{4} + \pi k, k \in Z$ $\qquad$ $x = arctg\,5 + \pi k, k \in Z$

Ответ: $\frac{\pi}{4} + \pi k, k \in Z, arctg\,5 + \pi k, k \in Z.$

# Пример:

- Решить уравнение
$ctg^2 x - 2ctg\, x - 3 = 0.$

Решение:

$ctg\, x = b$

$b^2 - 2b - 3 = 0$

$b_1 = -1, b_2 = 3$

$ctg\, x = -1$ $\qquad$ $ctg\, x = 3$

$x = \frac{3\pi}{4} + \pi k, k \in Z$ $\qquad$ $x = arcctg\, 3 + \pi k, k \in Z$

Ответ: $\frac{3\pi}{4} + \pi k, k \in Z, arcctg\, 3 + \pi k, k \in Z.$

# Пример:

$ctg(2\pi + x) = \boldsymbol{ctg\ x}$

$tg\left(\frac{\pi}{2} + x\right) = \boldsymbol{-ctg\ x}$

- Решить уравнение

$2ctg(2\pi + x) - tg\left(\frac{\pi}{2} + x\right) = \sqrt{3}$.

Решение:

$2\,ctg\ x - (-ctg\ x)$

$2ctg\ x + ctg\ x = \sqrt{3}$

$3ctg\ x = \sqrt{3}$

$ctg\ x = \frac{\sqrt{3}}{3} \Rightarrow x = \frac{\pi}{3} + \pi k, k \in Z$

Ответ: $\frac{\pi}{3} + \pi k, k \in Z$.