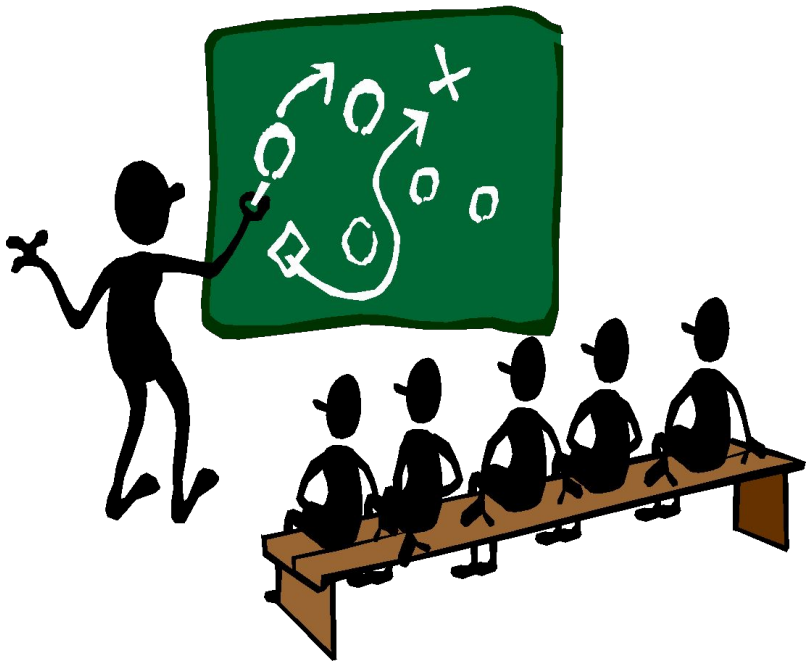
Objectives for Section 11.3 Derivatives of Products and Quotients

The student will be able to calculate:

- the derivative of a product of two functions, and
- the derivative of a quotient of two functions.



Derivatives of Products

Theorem 1 (Product Rule)

If $f(x) = F(x) \cdot S(x)$, and if $F'(x)$ and $S'(x)$ exist, then

$$f'(x) = F(x) \cdot S'(x) + F'(x) \cdot S(x),$$

or

$$f'(x) = F\frac{dS}{dx} + \frac{dF}{dx}S$$

In words: The derivative of the product of two functions is the first function times the derivative of the second function plus the second function times the derivative of the first function.

Example

Find the derivative of $y = 5x^2(x^3 + 2)$.

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Solution:

Let $F(x) = 5x^2$, so $F'(x) = 10x$
Let $S(x) = x^3 + 2$, so $S'(x) = 3x^2$.

Then

$$f'(x) = F(x) \cdot S'(x) + F'(x) \cdot S(x)$$

$$= 5x^2 \cdot 3x^2 + 10x \cdot (x^3 + 2)$$

$$= 15x^4 + 10x^4 + 20x = 25x^4 + 20x.$$

Derivatives of Quotients

Theorem 2 (Quotient Rule)

If $f(x) = T(x) / B(x)$, and if $T'(x)$ and $B'(x)$ exist, then

$$f'(x) = \frac{B(x) \cdot T'(x) - T(x) \cdot B'(x)}{[B(x)]^2} \quad \text{or} \quad \frac{dy}{dx} = \frac{B\dfrac{dT}{dx} - T\dfrac{dB}{dx}}{B^2}$$

In words: The derivative of the quotient of two functions is the bottom function times the derivative of the top function minus the top function times the derivative of the bottom function, all over the bottom function squared.

Example

Find the derivative of $y = 3x / (2x + 5)$.

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Solution:

Let $T(x) = 3x$, so $T'(x) = 3$
Let $B(x) = 2x + 5$, so $B'(x) = 2$.

Then

$$f'(x) = \frac{B(x) \cdot T'(x) - T(x) \cdot B'(x)}{[B(x)]^2}$$

$$= \frac{(2x+5) \cdot 3 - 3x \cdot 2}{(2x+5)^2} = \frac{15}{(2x+5)^2}$$

Tangent Lines

Let $f(x) = (2x - 9)(x^2 + 6)$. Find the equation of the line tangent to the graph of $f(x)$ at $x = 3$.

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Solution: First, find $f'(x)$:

$$f'(x) = (2x - 9)(2x) + (2)(x^2 + 6)$$

Then find $f(3)$ and $f'(3)$:

$$f(3) = -45 \quad f'(3) = 12$$

The tangent has slope 12 and goes through the point (3, -45).

Using the point-slope form $y - y_1 = m(x - x_1)$, we get

$$y - (-45) = 12(x - 3) \text{ or } y = 12x - 81$$

Summary

Product Rule:

$$\frac{d}{dx}\left(F(x)\cdot S(x)\right) = F'(x)\cdot S(x) + F(x)\cdot S'(x)$$

Quotient Rule:

$$\frac{d}{dx}\left(\frac{T(x)}{B(x)}\right) = \frac{B(x)\cdot T'(x) - T(x)\cdot B'(x)}{[B(x)]^2}$$