TIME SERIES MODELS: STATIC MODELS AND MODELS WITH LAGS

$$HOUS = \beta_1 + \beta_2 DPI + \beta_3 PRELHOUS + u$$

In this sequence we will make an initial exploration of the determinants of aggregate consumer expenditure on housing services using the Demand Functions data set.

TIME SERIES MODELS: STATIC MODELS AND MODELS WITH LAGS

$$HOUS = \beta_1 + \beta_2 DPI + \beta_3 PRELHOUS + u$$

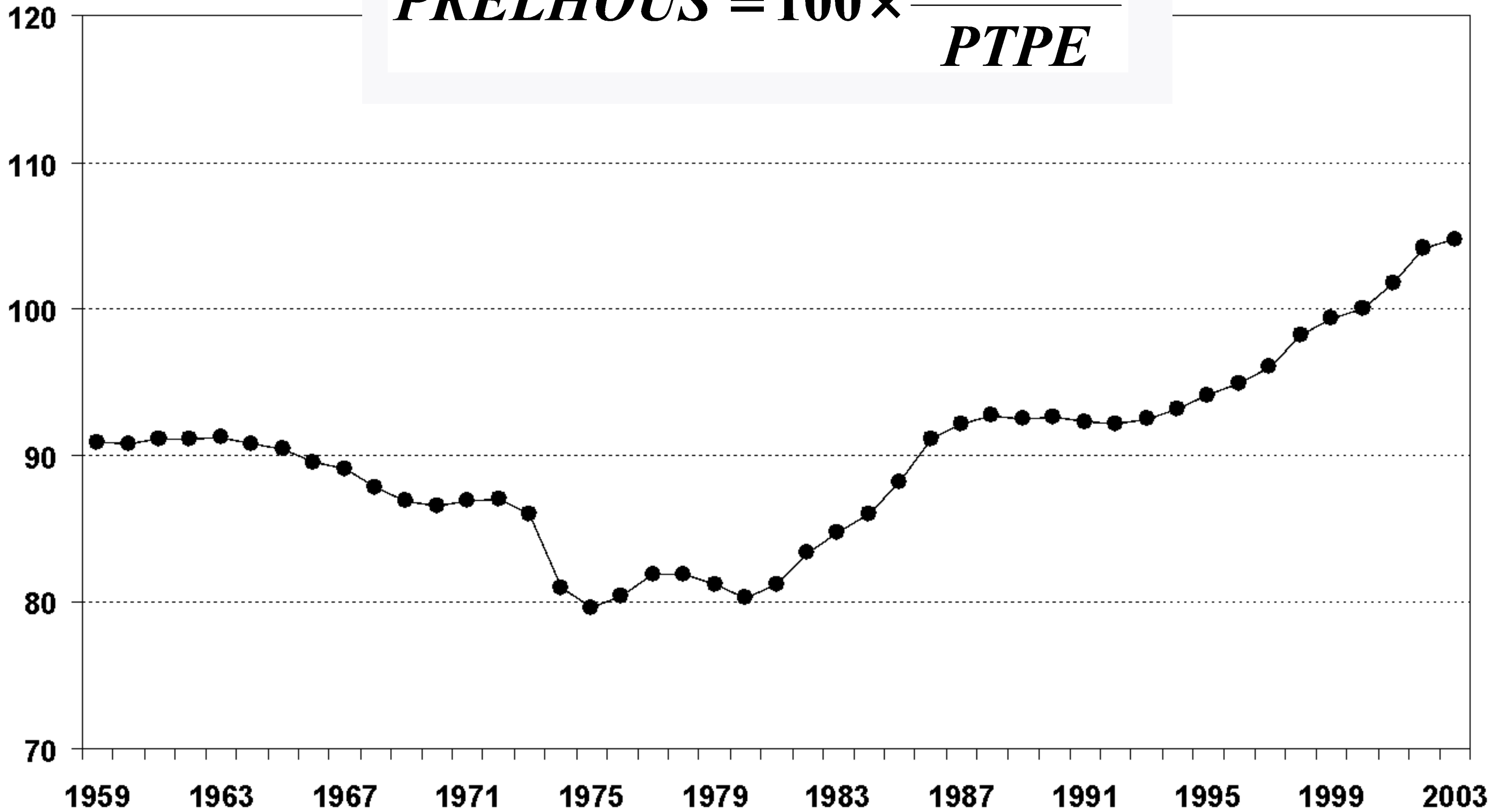
HOUS is aggregate consumer expenditure on housing services and *DPI* is aggregate disposable personal income. Both are measured in $ billion at 2000 constant prices.

$$HOUS = \beta_1 + \beta_2 DPI + \beta_3 PRELHOUS + u$$

$$PRELHOUS = 100 \times \frac{PHOUS}{PTPE}$$

PRELHOUS **is a relative price index for housing services constructed by dividing the nominal price index for housing services by the price index for total personal expenditure.**

$$PRELHOUS = 100 \times \frac{PHOUS}{PTPE}$$



Here is a plot of *PRELHOUS* for the sample period, 1959–2003.

```
============================================================
Dependent Variable: HOUS
Method: Least Squares
Sample: 1959 2003
Included observations: 45
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           334.6657   37.26625   8.980396   0.0000
        DPI          0.150925   0.001665   90.65785   0.0000
     PRELHOUS       -3.834387   0.460490  -8.326764   0.0000
============================================================
R-squared            0.996722    Mean dependent var 630.2830
Adjusted R-squared   0.996566    S.D. dependent var 249.2620
S.E. of regression   14.60740    Akaike info criteri8.265274
Sum squared resid    8961.801    Schwarz criterion  8.385719
Log likelihood      -182.9687    F-statistic        6385.025
Durbin-Watson stat   0.337638    Prob(F-statistic)  0.000000
============================================================
```

Here is the regression output using EViews. It was obtained by loading the workfile, clicking on Quick, then on Estimate, and then typing HOUS C DPI PRELHOUS in the box. Note that in EViews you must include C in the command if your model has an intercept.

```
============================================================
Dependent Variable: HOUS
Method: Least Squares
Sample: 1959 2003
Included observations: 45
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
          C          334.6657   37.26625   8.980396   0.0000
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Log likelihood      -182.9687    F-statistic        6385.025
Durbin-Watson stat   0.337638    Prob(F-statistic)  0.000000
============================================================
```

We will start by interpreting the coefficients. The coefficient of *DPI* indicates that if aggregate income rises by $1 billion, aggregate expenditure on housing services rises by $151 million. Is this a plausible figure?

```
============================================================
Dependent Variable: HOUS
Method: Least Squares
Sample: 1959 2003
Included observations: 45
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
        C            334.6657   37.26625   8.980396   0.0000
       DPI           0.150925   0.001665   90.65785   0.0000
     PRELHOUS       -3.834387   0.460490  -8.326764   0.0000
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S.E. of regression   14.60740    Akaike info criteri8.265274
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Log likelihood      -182.9687    F-statistic        6385.025
Durbin-Watson stat   0.337638    Prob(F-statistic)  0.000000
============================================================
```

Possibly. It implies that 15 cents out of the marginal dollar are spent on housing. Housing is the largest category of consumer expenditure, so we would expect a substantial coefficient. Perhaps it is a little low.

```
============================================================
Dependent Variable: HOUS
Method: Least Squares
Sample: 1959 2003
Included observations: 45
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
          C         334.6657   37.26625   8.980396   0.0000
         DPI        0.150925   0.001665   90.65785   0.0000
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Log likelihood     -182.9687    F-statistic        6385.025
Durbin-Watson stat  0.337638    Prob(F-statistic)  0.000000
============================================================
```

The coefficient of *PRELHOUS* indicates that a one-point increase in this price index causes expenditure on housing to fall by $3.84 billion. It is not easy to determine whether this is plausible. At least the effect is negative.

```
============================================================
Dependent Variable: HOUS
Method: Least Squares
Sample: 1959 2003
Included observations: 45
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
           C        334.6657   37.26625   8.980396   0.0000
         DPI        0.150925   0.001665   90.65785   0.0000
     PRELHOUS      -3.834387   0.460490  -8.326764   0.0000
============================================================
R-squared           0.996722    Mean dependent var 630.2830
Adjusted R-squared  0.996566    S.D. dependent var 249.2620
S.E. of regression  14.60740    Akaike info criteri8.265274
Sum squared resid   8961.801    Schwarz criterion  8.385719
Log likelihood     -182.9687    F-statistic        6385.025
Durbin-Watson stat  0.337638    Prob(F-statistic)  0.000000
============================================================
```

The constant has no meaningful interpretation. (Literally, it indicates that $335 billion would be spent on housing services if aggregate income and the price series were both 0.)

```
============================================================
Dependent Variable: HOUS
Method: Least Squares
Sample: 1959 2003
Included observations: 45
============================================================
     Variable     Coefficient Std. Error t-Statistic  Prob.
============================================================
        C          334.6657   37.26625   8.980396   0.0000
       DPI         0.150925   0.001665   90.65785   0.0000
     PRELHOUS     -3.834387   0.460490  -8.326764   0.0000
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S.E. of regression 14.60740   Akaike info criteri8.265274
Sum squared resid  8961.801   Schwarz criterion  8.385719
Log likelihood    -182.9687   F-statistic        6385.025
Durbin-Watson stat 0.337638   Prob(F-statistic)  0.000000
============================================================
```

The explanatory power of the model appears to be excellent. The coefficient of *DPI* has a very high *t* statistic, that of price is also high, and R^2 is close to a perfect fit.

$$HOUS = \beta_1 DPI^{\beta_2} PRELHOUS^{\beta_3} v$$

Constant elasticity functions are usually considered preferable to linear functions in models of consumer expenditure. Here β_2 is the income elasticity and β_3 is the price elasticity for expenditure on housing services.

$$HOUS = \beta_1 DPI^{\beta_2} PRELHOUS^{\beta_3} v$$

$$LGHOUS = \log \beta_1 + \beta_2 LGDPI + \beta_3 LGPRHOUS + \log v$$

We linearize the model by taking logarithms. We will regress *LGHOUS*, the logarithm of expenditure on housing services, on *LGDPI*, the logarithm of disposable personal income, and *LGPRHOUS*, the logarithm of the relative price index for housing services.

```
============================================================
Dependent Variable: LGHOUS
Method: Least Squares
Sample: 1959 2003
Included observations: 45
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C            0.005625   0.167903   0.033501   0.9734
       LGDPI          1.031918   0.006649   155.1976   0.0000
     LGPRHOUS        -0.483421   0.041780  -11.57056   0.0000
============================================================
R-squared             0.998583    Mean dependent var 6.359334
Adjusted R-squared    0.998515    S.D. dependent var 0.437527
S.E. of regression    0.016859    Akaike info criter-5.263574
Sum squared resid     0.011937    Schwarz criterion -5.143130
Log likelihood        121.4304    F-statistic         14797.05
Durbin-Watson stat    0.633113    Prob(F-statistic)   0.000000
============================================================
```

Here is the regression output. The estimate of the income elasticity is 1.03. Is this plausible?

```
============================================================
Dependent Variable: LGHOUS
Method: Least Squares
Sample: 1959 2003
Included observations: 45
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           0.005625   0.167903   0.033501   0.9734
       LGDPI         1.031918   0.006649   155.1976   0.0000
     LGPRHOUS       -0.483421   0.041780  -11.57056   0.0000
============================================================
R-squared            0.998583    Mean dependent var 6.359334
Adjusted R-squared   0.998515    S.D. dependent var 0.437527
S.E. of regression   0.016859    Akaike info criter-5.263574
Sum squared resid    0.011937    Schwarz criterion -5.143130
Log likelihood       121.4304    F-statistic         14797.05
Durbin-Watson stat   0.633113    Prob(F-statistic)   0.000000
============================================================
```

Probably. Housing is an essential category of consumer expenditure, and necessities generally have elasticities lower than 1. But it also has a luxury component, in that people tend to move to more desirable housing as income increases.

```
============================================================
Dependent Variable: LGHOUS
Method: Least Squares
Sample: 1959 2003
Included observations: 45
============================================================
     Variable     Coefficient Std. Error t-Statistic  Prob.
============================================================
        C           0.005625   0.167903   0.033501   0.9734
      LGDPI         1.031918   0.006649   155.1976   0.0000
     LGPRHOUS      -0.483421   0.041780  -11.57056   0.0000
============================================================
R-squared           0.998583    Mean dependent var 6.359334
Adjusted R-squared  0.998515    S.D. dependent var 0.437527
S.E. of regression  0.016859    Akaike info criter-5.263574
Sum squared resid   0.011937    Schwarz criterion -5.143130
Log likelihood      121.4304    F-statistic        14797.05
Durbin-Watson stat  0.633113    Prob(F-statistic)  0.000000
============================================================
```

Thus an elasticity near 1 seems about right. The price elasticity is 0.48, suggesting that expenditure on this category is not very price elastic.

```
============================================================
Dependent Variable: LGHOUS
Method: Least Squares
Sample: 1959 2003
Included observations: 45
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
            C         0.005625   0.167903   0.033501   0.9734
        LGDPI         1.031918   0.006649   155.1976   0.0000
     LGPRHOUS        -0.483421   0.041780  -11.57056   0.0000
============================================================
R-squared               0.998583    Mean dependent var 6.359334
Adjusted R-squared      0.998515    S.D. dependent var 0.437527
S.E. of regression      0.016859    Akaike info criter-5.263574
Sum squared resid       0.011937    Schwarz criterion -5.143130
Log likelihood          121.4304    F-statistic       14797.05
Durbin-Watson stat      0.633113    Prob(F-statistic)  0.000000
============================================================
```

Again, the constant has no meaningful interpretation.

```
============================================================
Dependent Variable: LGHOUS
Method: Least Squares
Sample: 1959 2003
Included observations: 45
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
            C         0.005625   0.167903   0.033501   0.9734
        LGDPI         1.031918   0.006649   155.1976   0.0000
     LGPRHOUS        -0.483421   0.041780  -11.57056   0.0000
============================================================
R-squared            0.998583    Mean dependent var 6.359334
Adjusted R-squared   0.998515    S.D. dependent var 0.437527
S.E. of regression   0.016859    Akaike info criter-5.263574
Sum squared resid    0.011937    Schwarz criterion -5.143130
Log likelihood       121.4304    F-statistic         14797.05
Durbin-Watson stat   0.633113    Prob(F-statistic)   0.000000
============================================================
```

The explanatory power of the model appears to be excellent.

Current and lagged values of the logarithm of disposable personal income

Year	*LGDPI*	*LGDPI*(–1)
1959	5.4914	—
1960	5.5426	5.4914
1961	5.5898	5.5426
1962	5.6449	5.5898
1963	5.6902	5.6449
1964	5.7371	5.6902
......	......	......
......	......	......
1999	6.8861	6.8553
2000	6.9142	6.8861
2001	6.9410	6.9142
2002	6.9679	6.9410
2003	6.9811	6.9679

Next, we will introduce some simple dynamics. Expenditure on housing is subject to inertia and responds slowly to changes in income and price. Accordingly we will consider specifications of the model where it depends on lagged values of income and price.

Current and lagged values of the logarithm of disposable personal income

Year	*LGDPI*	*LGDPI*(–1)
1959	5.4914	—
1960	5.5426	5.4914
1961	5.5898	5.5426
1962	5.6449	5.5898
1963	5.6902	5.6449
1964	5.7371	5.6902
......	......	......
......	......	......
1999	6.8861	6.8553
2000	6.9142	6.8861
2001	6.9410	6.9142
2002	6.9679	6.9410
2003	6.9811	6.9679

A variable *X* lagged one time period has values that are simply the previous values of *X*, and it is conventionally denoted *X*(–1). Here *LGDPI*(–1) has been derived from *LGDPI*. You can see, for example, that the value of *LGDPI*(–1) in 2003 is just the value of *LGDPI* in 2002.

Current and lagged values of the logarithm of disposable personal income

Year	*LGDPI*	*LGDPI*(–1)
1959	5.4914	—
1960	5.5426	5.4914
1961	5.5898	5.5426
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1963	5.6902	5.6449
1964	5.7371	5.6902
......	......	......
......	......	......
1999	6.8861	6.8553
2000	6.9142	6.8861
2001	6.9410	6.9142
2002	6.9679	6.9410
2003	6.9811	6.9679

Similarly for the other years. Note that *LGDPI*(–1) is not defined for 1959, given the data set. Of course, in this case, we could obtain it from the 1960 issues of the *Survey of Current Business*.

Current and lagged values of the logarithm of disposable personal income

Year	*LGDPI*	*LGDPI*(–1)	*LGDPI*(–2)
1959	5.4914	—	—
1960	5.5426	5.4914	—
1961	5.5898	5.5426	5.4914
1962	5.6449	5.5898	5.5426
1963	5.6902	5.6449	5.5898
1964	5.7371	5.6902	5.6449
......	......	......	......
......	......	......	......
1999	6.8861	6.8553	6.8271
2000	6.9142	6.8861	6.8553
2001	6.9410	6.9142	6.8861
2002	6.9679	6.9410	6.9142
2003	6.9811	6.9679	6.9410

Similarly, *LGDPI*(–2) is *LGDPI* lagged 2 time periods. *LGDPI*(–2) in 2003 is the value of *LGDPI* in 2001, and so on. Generalizing, *X*(–*s*) is *X* lagged *s* time periods.

```
============================================================
Dependent Variable: LGHOUS
Method: Least Squares
Sample(adjusted): 1960 2003
Included observations: 44 after adjusting endpoints
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           0.019172   0.148906   0.128753   0.8982
     LGDPI(-1)       1.006528   0.005631   178.7411   0.0000
   LGPRHOUS(-1)     -0.432223   0.036461  -11.85433   0.0000
============================================================
R-squared            0.998917    Mean dependent var 6.379059
Adjusted R-squared   0.998864    S.D. dependent var 0.421861
S.E. of regression   0.014218    Akaike info criter-5.602852
Sum squared resid    0.008288    Schwarz criterion -5.481203
Log likelihood       126.2628    F-statistic         18906.98
Durbin-Watson stat   0.919660    Prob(F-statistic)   0.000000
============================================================
```

Here is a logarithmic regression of current expenditure on housing on lagged income and price. Note that EViews, in common with most regression applications, recognizes *X*(–1) as being the lagged value of *X* and there is no need to define it as a distinct variable.

```
============================================================
Dependent Variable: LGHOUS
Method: Least Squares
Sample(adjusted): 1960 2003
Included observations: 44 after adjusting endpoints
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C           0.019172   0.148906   0.128753   0.8982
     LGDPI(-1)       1.006528   0.005631   178.7411   0.0000
   LGPRHOUS(-1)     -0.432223   0.036461  -11.85433   0.0000
============================================================
R-squared            0.998917    Mean dependent var 6.379059
Adjusted R-squared   0.998864    S.D. dependent var 0.421861
S.E. of regression   0.014218    Akaike info criter-5.602852
Sum squared resid    0.008288    Schwarz criterion -5.481203
Log likelihood       126.2628    F-statistic        18906.98
Durbin-Watson stat   0.919660    Prob(F-statistic)  0.000000
============================================================
```

The estimate of the lagged income and price elasticities are 1.01 and 0.43, respectively.

Alternative dynamic specifications, housing services

Variable	(1)	(2)
LGDPI	1.03	—
	(0.01)	
LGDPI(–1)	—	1.01
		(0.01)
LGDPI(–2)	—	—
LGPRHOUS	–0.48	—
	(0.04)	
LGPRHOUS(–1)	—	–0.43
		(0.04)
LGPRHOUS(–2)	—	—
R^2	0.9985	0.9989

The regression results will be summarized in a table for comparison. The results of the lagged-values regression are virtually identical to those of the current-values regression.

Alternative dynamic specifications, housing services

Variable	(1)	(2)	(3)
LGDPI	1.03	—	—
	(0.01)		
LGDPI(–1)	—	1.01	—
		(0.01)	
LGDPI(–2)	—	—	0.98
			(0.01)
LGPRHOUS	–0.48	—	—
	(0.04)		
LGPRHOUS(–1)	—	–0.43	—
		(0.04)	
LGPRHOUS(–2)	—	—	–0.38
			(0.04)
R^2	0.9985	0.9989	0.9988

So also are the results of regressing *LGHOUS* on *LGDPI* and *LGPRHOUS* lagged two years.

Alternative dynamic specifications, housing services

Variable	(1)	(2)	(3)	(4)
LGDPI	1.03	—	—	0.33
	(0.01)			(0.15)
LGDPI(–1)	—	1.01	—	0.68
		(0.01)		(0.15)
LGDPI(–2)	—	—	0.98	—
			(0.01)	
LGPRHOUS	–0.48	—	—	–0.09
	(0.04)			(0.17)
LGPRHOUS(–1)	—	–0.43	—	–0.36
		(0.04)		(0.17)
LGPRHOUS(–2)	—	—	–0.38	—
			(0.04)	
R^2	0.9985	0.9989	0.9988	0.9990

One approach to discriminating between the effects of current and lagged income and price is to include both in the equation. Since both may be important, failure to do so may give rise to omitted variable bias.

Alternative dynamic specifications, housing services

Variable	(1)	(2)	(3)	(4)
LGDPI	1.03	—	—	0.33
	(0.01)			(0.15)
LGDPI(–1)	—	1.01	—	0.68
		(0.01)		(0.15)
LGDPI(–2)	—	—	0.98	—
			(0.01)	
LGPRHOUS	–0.48	—	—	–0.09
	(0.04)			(0.17)
LGPRHOUS(–1)	—	–0.43	—	–0.36
		(0.04)		(0.17)
LGPRHOUS(–2)	—	—	–0.38	—
			(0.04)	
R^2	0.9985	0.9989	0.9988	0.9990

With the current values of income and price, and their values lagged one year, we see that lagged income has a higher coefficient than current income. This is plausible, since we expect inertia in the response.

Alternative dynamic specifications, housing services

Variable	(1)	(2)	(3)	(4)
LGDPI	1.03	—	—	0.33
	(0.01)			(0.15)
LGDPI(–1)	—	1.01	—	0.68
		(0.01)		(0.15)
LGDPI(–2)	—	—	0.98	—
			(0.01)	
LGPRHOUS	–0.48	—	—	–0.09
	(0.04)			(0.17)
LGPRHOUS(–1)	—	–0.43	—	–0.36
		(0.04)		(0.17)
LGPRHOUS(–2)	—	—	–0.38	—
			(0.04)	
R^2	0.9985	0.9989	0.9988	0.9990

The price side of the model exhibits similar behavior.

Alternative dynamic specifications, housing services

Correlation Matrix

	LGDPI	LGDPI(-1)
LGDPI	1.000000	0.999345
LGDPI(-1)	0.999345	1.000000

				(4)
LGPRHOUS	–0.48 (0.04)	—	—	–0.09 (0.17)
LGPRHOUS(–1)	—	–0.43 (0.04)	—	–0.36 (0.17)
LGPRHOUS(–2)	—	—	–0.38 (0.04)	—
R^2	0.9985	0.9989	0.9988	0.9990

However there is a problem of multcollinearity caused by the high correlation between current and lagged values. The correlation is particularly high for current and lagged income.

Alternative dynamic specifications, housing services

```
           Correlation Matrix
====================================
              LGPRHOUS   LGPRHOUS(-1)
====================================
  LGPRHOUS    1.000000     0.977305
LGPRHOUS(-1)  0.977305     1.000000
====================================
```

				(4)
LGPRHOUS	–0.48	—	—	–0.09
	(0.04)			(0.17)
LGPRHOUS(–1)	—	–0.43	—	–0.36
		(0.04)		(0.17)
LGPRHOUS(–2)	—	—	–0.38	—
			(0.04)	
R^2	0.9985	0.9989	0.9988	0.9990

The correlation is also high for current and lagged price.

Alternative dynamic specifications, housing services

Variable	(1)	(2)	(3)	(4)
LGDPI	1.03	—	—	0.33
	(0.01)			(0.15)
LGDPI(–1)	—	1.01	—	0.68
		(0.01)		(0.15)
LGDPI(–2)	—	—	0.98	—
			(0.01)	
LGPRHOUS	–0.48	—	—	–0.09
	(0.04)			(0.17)
LGPRHOUS(–1)	—	–0.43	—	–0.36
		(0.04)		(0.17)
LGPRHOUS(–2)	—	—	–0.38	—
			(0.04)	
R^2	0.9985	0.9989	0.9988	0.9990

Notice how the standard errors have increased. The fact that the coefficients seem plausible is probably just an accident.

Alternative dynamic specifications, housing services

Variable	(1)	(2)	(3)	(4)	(5)
LGDPI	1.03	—	—	0.33	0.29
	(0.01)			(0.15)	(0.14)
LGDPI(–1)	—	1.01	—	0.68	0.22
		(0.01)		(0.15)	(0.20)
LGDPI(–2)	—	—	0.98	—	0.49
			(0.01)		(0.13)
LGPRHOUS	–0.48	—	—	–0.09	–0.28
	(0.04)			(0.17)	(0.17)
LGPRHOUS(–1)	—	–0.43	—	–0.36	0.23
		(0.04)		(0.17)	(0.30)
LGPRHOUS(–2)	—	—	–0.38	—	–0.38
			(0.04)		(0.18)
R^2	0.9985	0.9989	0.9988	0.9990	0.9993

If we add income and price lagged two years, the results become even more erratic. For a category of expenditure such as housing, where one might expect long lags, this is clearly not a constructive approach to determining the lag structure.

Estimates of long-run income and price elasticities

Specification	(1)	(2)	(3)	(4)	(5)
Sum of income elasticities	1.03	1.01	0.98	1.01	1.00
Sum of price elasticities	–0.48	–0.43	–0.38	–0.45	–0.43

$$Y_t = \beta_1 + \beta_2 X_t + \beta_3 X_{t-1} + \beta_4 X_{t-2} + u_t$$

$$\breve{Y} = \beta_1 + \beta_2 \tilde{X} + \beta_3 \tilde{X} + \beta_4 \tilde{X} = \beta_1 + (\beta_2 + \beta_3 + \beta_4)\tilde{X}$$

Despite the problem of multicollinearity, we may be able to obtain relatively precise estimates of the long-run elasticities with respect to income and price.

Estimates of long-run income and price elasticities

Specification	(1)	(2)	(3)	(4)	(5)
Sum of income elasticities	1.03	1.01	0.98	1.01	1.00
Sum of price elasticities	–0.48	–0.43	–0.38	–0.45	–0.43

$$Y_t = \beta_1 + \beta_2 X_t + \beta_3 X_{t-1} + \beta_4 X_{t-2} + u_t$$

$$\breve{Y} = \beta_1 + \beta_2 \tilde{X} + \beta_3 \tilde{X} + \beta_4 \tilde{X} = \beta_1 + (\beta_2 + \beta_3 + \beta_4)\tilde{X}$$

The usual way of investigating the long-run relationship between Y and X is to perform an exercise in comparative statics. One first determines how equilibrium $\breve{Y}$ would be related to equilibrium $\tilde{X}$, if the process ever reached equilibrium.

Estimates of long-run income and price elasticities

Specification	(1)	(2)	(3)	(4)	(5)
Sum of income elasticities	1.03	1.01	0.98	1.01	1.00
Sum of price elasticities	–0.48	–0.43	–0.38	–0.45	–0.43

$$Y_t = \beta_1 + \beta_2 X_t + \beta_3 X_{t-1} + \beta_4 X_{t-2} + u_t$$

$$\breve{Y} = \beta_1 + \beta_2 \tilde{X} + \beta_3 \tilde{X} + \beta_4 \tilde{X} = \beta_1 + \left(\beta_2 + \beta_3 + \beta_4\right) \tilde{X}$$

One then evaluates the effect of a change in equilibrium $\tilde{X}$ on equilibrium $\breve{Y}$.

Estimates of long-run income and price elasticities

Specification	(1)	(2)	(3)	(4)	(5)
Sum of income elasticities	1.03	1.01	0.98	1.01	1.00
Sum of price elasticities	–0.48	–0.43	–0.38	–0.45	–0.43

$$Y_t = \beta_1 + \beta_2 X_t + \beta_3 X_{t-1} + \beta_4 X_{t-2} + u_t$$

$$\breve{Y} = \beta_1 + \beta_2 \tilde{X} + \beta_3 \tilde{X} + \beta_4 \tilde{X} = \beta_1 + \left(\beta_2 + \beta_3 + \beta_4\right)\tilde{X}$$

In the model with two lags shown, ($\beta_2 + \beta_3 + \beta_4$) is a measure of the long-run effect of *X*. We contrast this with the short-run effect, which is simply β_2, the impact of current X_t on Y_t.

Estimates of long-run income and price elasticities

Specification	(1)	(2)	(3)	(4)	(5)
Sum of income elasticities	1.031.010.981.011.00				
Sum of price elasticities	–0.48	–0.43	–0.38	–0.45	–0.43

$$Y_t = \beta_1 + \beta_2 X_t + \beta_3 X_{t-1} + \beta_4 X_{t-2} + u_t$$

$$\breve{Y} = \beta_1 + \beta_2 \tilde{X} + \beta_3 \tilde{X} + \beta_4 \tilde{X} = \beta_1 + \left(\beta_2 + \beta_3 + \beta_4\right)\tilde{X}$$

We can calculate the long-run effect from the point estimates of β_2, β_3, and β_4 in the original specification. The estimate of the sum may be quite stable, even though the estimates of the individual coefficients may be subject to multicollinearity.

Estimates of long-run income and price elasticities

Specification	(1)	(2)	(3)	(4)	(5)
Sum of income elasticities	1.031.010.981.011.00				
Sum of price elasticities	–0.48	–0.43	–0.38	–0.45	–0.43

$$Y_t = \beta_1 + \beta_2 X_t + \beta_3 X_{t-1} + \beta_4 X_{t-2} + u_t$$

$$\tilde{Y} = \beta_1 + \beta_2 \tilde{X} + \beta_3 \tilde{X} + \beta_4 \tilde{X} = \beta_1 + \left(\beta_2 + \beta_3 + \beta_4\right) \tilde{X}$$

The table presents an example of this. It gives the sum of the income and price elasticities for the five specifications of the logarithmic housing demand function considered earlier. The estimates of the long-run elasticities are very similar.

Estimates of long-run income and price elasticities

Specification	(1)	(2)	(3)	(4)	(5)
Sum of income elasticities	1.031.010.981.011.00				
Sum of price elasticities	–0.48	–0.43	–0.38	–0.45	–0.43

$$Y_t = \beta_1 + \beta_2 X_t + \beta_3 X_{t-1} + \beta_4 X_{t-2} + u_t$$

$$\tilde{Y} = \beta_1 + \beta_2 \tilde{X} + \beta_3 \tilde{X} + \beta_4 \tilde{X} = \beta_1 + (\beta_2 + \beta_3 + \beta_4) \tilde{X}$$

$$Y_t = \beta_1 + (\beta_2 + \beta_3 + \beta_4) X_t - \beta_3 (X_t - X_{t-1}) - \beta_4 (X_t - X_{t-2}) + u_t$$

If we are estimating long-run effects, we need standard errors as well as point estimates. The most straightforward way of obtaining the standard error is to reparameterize the model. In the case of the present model, we could rewrite it as shown.

Estimates of long-run income and price elasticities

Specification	(1)	(2)	(3)	(4)	(5)
Sum of income elasticities	1.03	1.01	0.98	1.01	1.00
Sum of price elasticities	–0.48	–0.43	–0.38	–0.45	–0.43

$$Y_t = \beta_1 + \beta_2 X_t + \beta_3 X_{t-1} + \beta_4 X_{t-2} + u_t$$

$$\tilde{Y} = \beta_1 + \beta_2 \tilde{X} + \beta_3 \tilde{X} + \beta_4 \tilde{X} = \beta_1 + (\beta_2 + \beta_3 + \beta_4)\tilde{X}$$

$$Y_t = \beta_1 + (\beta_2 + \beta_3 + \beta_4) X_t - \beta_3 (X_t - X_{t-1}) - \beta_4 (X_t - X_{t-2}) + u_t$$

The point estimate of the coefficient of X_t will be the sum of the point estimates of β_2, β_3, and β_4 in the original specification and so the standard error of that coefficient is the standard error of the estimate of the long-run effect.

TIME SERIES MODELS: STATIC MODELS AND MODELS WITH LAGS

Estimates of long-run income and price elasticities

Specification	(1)	(2)	(3)	(4)	(5)
Sum of income elasticities	1.03	1.01	0.98	1.01	1.00
Sum of price elasticities	–0.48	–0.43	–0.38	–0.45	–0.43

$$Y_t = \beta_1 + \beta_2 X_t + \beta_3 X_{t-1} + \beta_4 X_{t-2} + u_t$$

$$\breve{Y} = \beta_1 + \beta_2 \tilde{X} + \beta_3 \tilde{X} + \beta_4 \tilde{X} = \beta_1 + \left(\beta_2 + \beta_3 + \beta_4\right)\tilde{X}$$

$$Y_t = \beta_1 + \left(\beta_2 + \beta_3 + \beta_4\right)X_t - \beta_3\left(X_t - X_{t-1}\right) - \beta_4\left(X_t - X_{t-2}\right) + u_t$$

Since X_t may well not be highly correlated with $(X_t - X_{t-1})$ or $(X_t - X_{t-2})$, there may not be a problem of multicollinearity and the standard error may be relatively small.

TIME SERIES MODELS: STATIC MODELS AND MODELS WITH LAGS

Dependent Variable: LGHOUS
Method: Least Squares
Sample(adjusted): 1961 2003
Included observations: 43 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.046768	0.133685	0.349839	0.7285
LGDPI	1.000341	0.006997	142.9579	0.0000
X1	-0.221466	0.196109	-1.129302	0.2662
X2	-0.491028	0.134374	-3.654181	0.0008
LGPRHOUS	-0.425357	0.033583	-12.66570	0.0000
P1	-0.233308	0.298365	-0.781955	0.4394
P2	0.378626	0.175710	2.154833	0.0379

R-squared	0.999265	Mean dependent var	6.398513
Adjusted R-squared	0.999143	S.D. dependent var	0.406394

$$Y_t = \beta_1 + (\beta_2 + \beta_3 + \beta_4)X_t - \beta_3(X_t - X_{t-1}) - \beta_4(X_t - X_{t-2}) + u_t$$

The output shows the result of fitting the reparameterized model for housing with two lags (Specification (5) in the table). ***X1* = *LGDPI* – *LGDPI*(–1), *X2* = *LGDPI* – *LGDPI*(–2), *P1* = *LGPRHOUS* – *LGPRHOUS*(–1), and *P2* = *LGPRHOUS* – *LGPRHOUS*(–2).**

Dependent Variable: LGHOUS
Method: Least Squares
Sample(adjusted): 1961 2003
Included observations: 43 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.046768	0.133685	0.349839	0.7285
LGDPI	1.000341	0.006997	142.9579	0.0000
X1	-0.221466	0.196109	-1.129302	0.2662
X2	-0.491028	0.134374	-3.654181	0.0008
LGPRHOUS	-0.425357	0.033583	-12.66570	0.0000
P1	-0.233308	0.298365	-0.781955	0.4394
P2	0.378626	0.175710	2.154833	0.0379

R-squared	0.999265	Mean dependent var	6.398513
Adjusted R-squared	0.999143	S.D. dependent var	0.406394

$$Y_t = \beta_1 + (\beta_2 + \beta_3 + \beta_4)X_t - \beta_3(X_t - X_{t-1}) - \beta_4(X_t - X_{t-2}) + u_t$$

As expected, the point estimates of the coefficients of *LGDPI* and *LGPRHOUS*, 1.00 and –0.43, are the sum of the point estimates of the coefficients of the current and lagged terms in Specification (5).

```
============================================================
Dependent Variable: LGHOUS
Method: Least Squares
Sample(adjusted): 1961 2003
Included observations: 43 after adjusting endpoints
============================================================
     Variable      Coefficient Std. Error t-Statistic  Prob.
============================================================
         C            0.046768   0.133685   0.349839   0.7285
       LGDPI          1.000341   0.006997   142.9579   0.0000
        X1           -0.221466   0.196109  -1.129302   0.2662
        X2           -0.491028   0.134374  -3.654181   0.0008
     LGPRHOUS        -0.425357   0.033583  -12.66570   0.0000
        P1           -0.233308   0.298365  -0.781955   0.4394
        P2            0.378626   0.175710   2.154833   0.0379
============================================================
R-squared             0.999265    Mean dependent var 6.398513
Adjusted R-squared    0.999143    S.D. dependent var 0.406394
S.E. of regression    0.011899    Akaike info criter 5.876807
Durbin-Watson stat    [illegible]    Prob(F-statistic)  0.000000
============================================================
```

$$Y_t = \beta_1 + (\beta_2 + \beta_3 + \beta_4)X_t - \beta_3(X_t - X_{t-1}) - \beta_4(X_t - X_{t-2}) + u_t$$

Also as expected, the standard errors, 0.01 and 0.03, are much lower than those of the individual coefficients in Specification (5).

The content of this slideshow comes from Section 11.3 of C. Dougherty, *Introduction to Econometrics*, fifth edition 2016, Oxford University Press. Additional (free) resources for both students and instructors may be downloaded from the OUP Online Resource Centre http://www.oup.com/uk/orc/bin/9780199567089/.

Individuals studying econometrics on their own who feel that they might benefit from participation in a formal course should consider the London School of Economics summer school course
EC212 Introduction to Econometrics
http://www2.lse.ac.uk/study/summerSchools/summerSchool/Home.aspx
or the University of London International Programmes distance learning course
20 Elements of Econometrics
www.londoninternational.ac.uk/lse.