

BBA182 Applied Statistics Week 4 (2) Measures of variation

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Average distance to the mean:

Standard deviation

Most commonly used measure of variability

Measures the standard (average) distance of all data points from the mean.



Using Microsoft Excel

Descriptive Statistics can be obtained from Microsoft® Excel

Select:

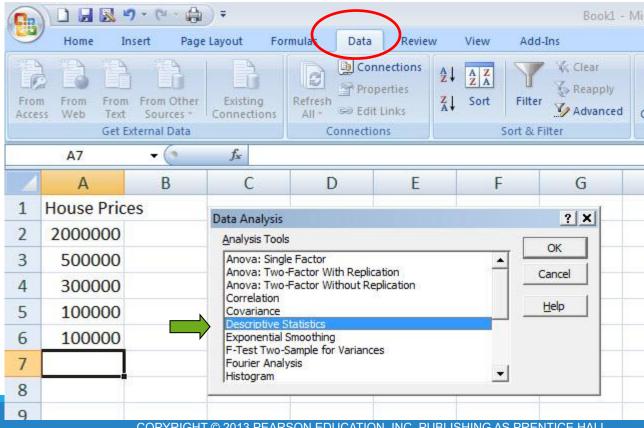
data / data analysis / descriptive statistics

Enter details in dialog box



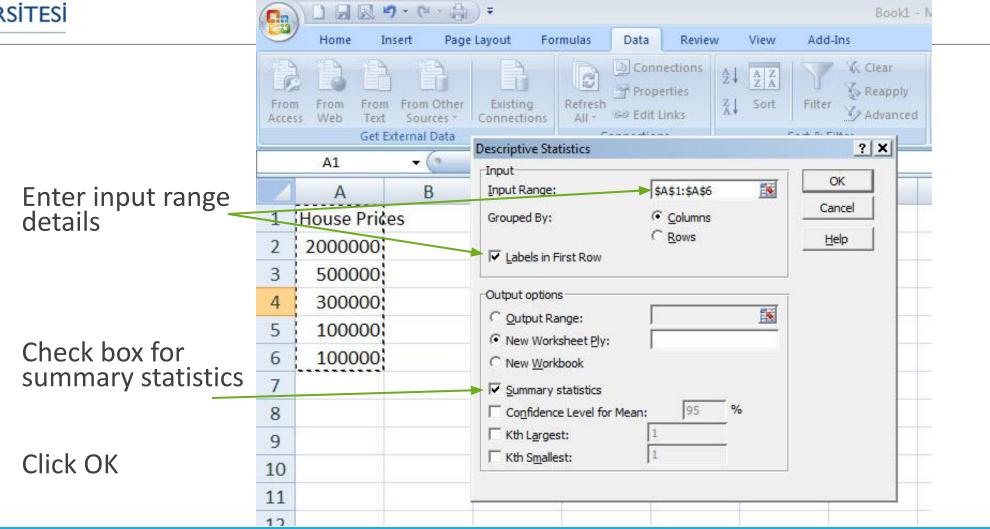
sing Excel to find Descriptive Statistics

Select data / data analysis / descriptive statistics





Using Excel to find Descriptive Statistics





Excel output

Microsoft Excel descriptive statistics output, using the house price data:

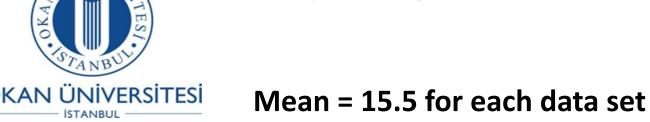
House Prices:

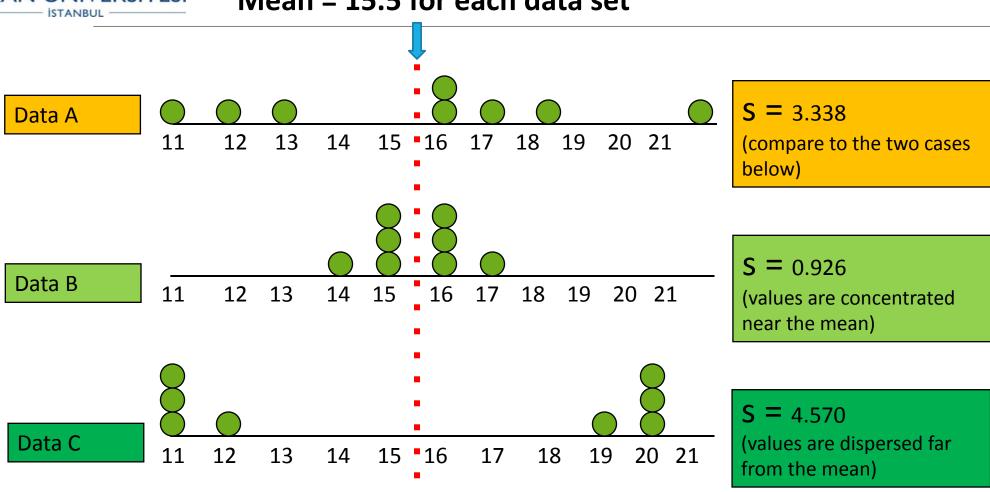
\$2,000,000 500,000 300,000 100,000

| | А | В |
|----|--------------------|-------------|
| 1 | House Prices | |
| 2 | | |
| 3 | Mean | 600000 |
| 4 | Standard Error | 357770.8764 |
| 5 | Median | 300000 |
| 6 | Mode | 100000 |
| 7 | Standard Deviation | 800000 |
| 8 | Sample Variance | 6.4E+11 |
| 9 | Kurtosis | 4.130126953 |
| 10 | Skewness | 2.006835938 |
| 11 | Range | 1900000 |
| 12 | Minimum | 100000 |
| 13 | Maximum | 2000000 |
| 14 | Sum | 3000000 |
| 15 | Count | 5 |
| 16 | | |



Comparing Standard Deviations of 3 different data







Comparing Standard Deviations of 2 data sets

Without calculating, which of the two data sets do you *expect* to have the highest variation and standard deviation? Why?



Describing distributions – what to pay attention to!

Pay attention to:

- its' shape (symmetric, right or left skewed)
- its' center (mean, median, mode)
- Its' spread (variance, standard deviation)



Effect of the size of the standard on the shape of a distribution

The standard deviation affects the shape of a distribution:

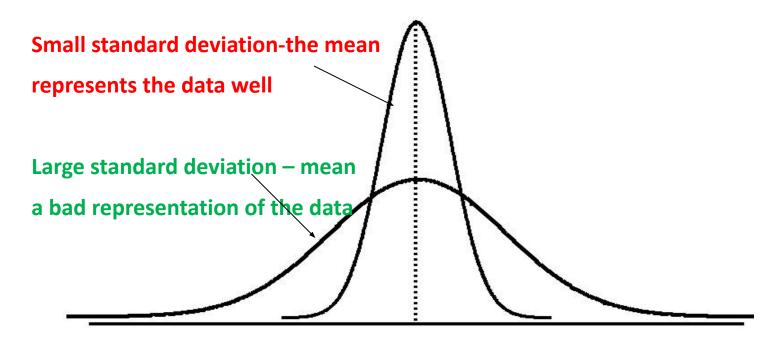
When there are small distances between the data points, most of the scores in the data set will be close to the mean and the resulting standard deviation will be small. The distribution will be narrow.

When there are large distances between data points, the scores will be further away from the mean and the standard deviation is larger. The distribution will be wide.

As illustrated in the following slide:



Effect of the size of the standard deviation on the shape of a distribution





camples of applications of the standard deviation in business

Logistics:

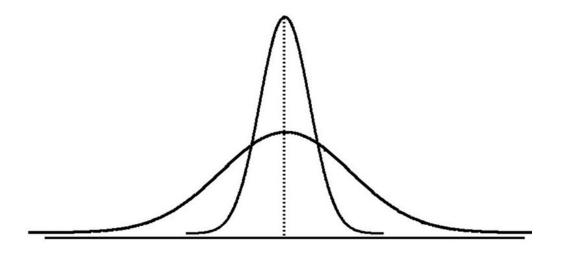
Measurement of timeliness/reliability/consistency

Financial sector:

Measurement of risk (difference between actual rate of return and the expected rate of return)

Production:

Quality control management. Measurement of consistency and reliability of manufacturing processes





Standard deviation a measure for risk in Finance

Comparing 2 different assets, asset A and asset B with the same mean:

| Table 2.5 Rates | of Return: A | Asset A and | Asset B |
|-----------------|--------------|-------------|---------|
|-----------------|--------------|-------------|---------|

| | Asset A | Asset B |
|--------------------------------------|---------|---------|
| Mean Rate of Return | 12.2% | 12.2% |
| Standard Deviation in Rate of Return | 0.63 | 3.12 |

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Standard deviation a measure for consistency in quality

control

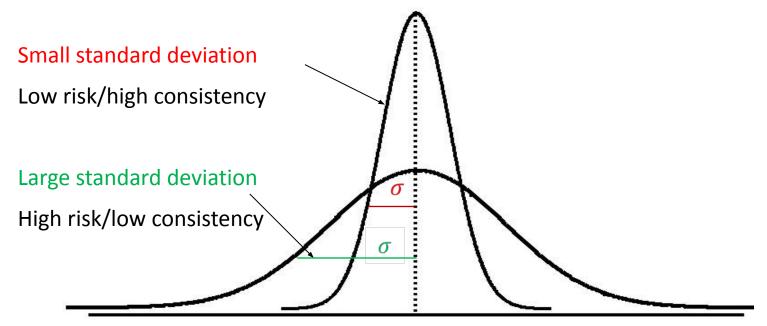
(Consistency in Turkish: Tutarlılık)

Comparing two manufacturing processes for number of defects in a sample, with similar means of defects:

| Process 1: | | Process 2 | |
|--------------------|----------|--------------------|----------|
| | | | |
| Mean | 10.4 | Mean | 10 |
| Median | 11 | Median | 10 |
| Mode | #N/A | Mode | #N/A |
| Standard Deviation | 4.393177 | Standard Deviation | 1.581139 |
| Sample Variance | 19.3 | Sample Variance | 2.5 |
| Count | 5 | Count | 5 |



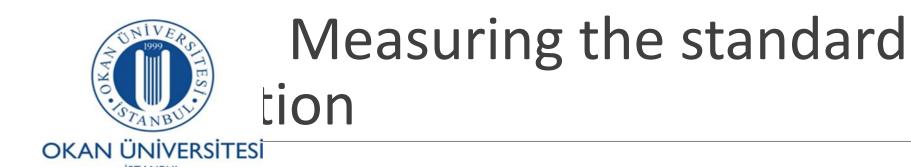
Measuring standard deviation



OKAN ÜNİVERSİTES ASURİNG STANDARD DE STAND

What does a standard deviation of 0 indicate?

What shape will the distribution have?



Example of a data set with a standard deviation of 0:

53 53 53 53 53



Advantages of Variance and Standard Deviation

Each single value in the data set is used in the calculation

$$\sigma^2 = \frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}}$$

Values far from the mean are given extra weight, such as outliers (because deviations from the mean are squared)



Effect of outliers on Variance and standard deviation

A large outlier (negative or positive) will increase the variance and standard deviation



Comparing the consistency of two types of Golf clubs

Golf equipment manufacturers are constantly seeking ways to improve their products. Suppose that the R&D department has developed a new golf iron (7-iron) to improve the consistency of its users.

A test golfer was asked to hit 150 shots using a 7-iron, 75 of which were hit with his current club and 75 with the newly developed 7-iron.

The distances were then measured and recorded.



Which iron is more consistent? The current or the newly developed? Excel output:

| Current | | Innovation | |
|--------------------|--------------|--------------------|--------------|
| | | | |
| Mean | 150.5466667 | Mean | 150.1466667 |
| Standard Error | 0.668814558 | Standard Error | 0.357011284 |
| Median | 151 | Median | 150 |
| Mode | 150 | Mode | 149 |
| Standard Deviation | 5.792103976 | Standard Deviation | 3.091808416 |
| Sample Variance | 33.54846847 | Sample Variance | 9.559279279 |
| Kurtosis | 0.126739586 | Kurtosis | -0.885417995 |
| Skewness | -0.429888289 | Skewness | 0.177337733 |
| Range | 28 | Range | 12 |
| Minimum | 134 | Minimum | 144 |
| Maximum | 162 | Maximum | 156 |
| Sum | 11291 | Sum | 11261 |
| Count | 75 | Count | 75 |



The standard deviation of the distances of the current iron is 5.79 meters whereas that of the newly developed 7-iron is 3.09 meters.

Based on this sample, the newly developed iron is more **consistent** (there is less variation in the distances shot with the innovative golf club).

Because the mean distances are similar it would appear that the new 7-iron is indeed superior.



OKAN ÜNİVERSİTESİ Coefficient of Variation (CV)

In situations where the means are almost the same, it is appropriate to use the standard deviations to see which process is the most consistent.

In situations where the means are **different** we need to calculate the **coefficient of variation** to compare the consistency or riskiness.

The *coefficient of variation* expresses the standard deviation as a percentage of the mean.



Coefficient of Variation (CV)

Measures relative variation within a dataset

Always in percentage (%) 0 - 100

A low CV translates into low variation within the same data set, a high CV into high variation

Population coefficient of variation (CV):

$$CV = \left(\frac{\sigma}{\mu}\right) \cdot 100\%$$

Sample coefficient of variation (CV):

$$CV = \left(\frac{s}{\overline{x}}\right) \cdot 100\%$$



Coefficient of Variation (CV)

Comparing 2 different production processes with different means:

| Process 1 | Process 2 |
|--------------------------------------|-----------------------------|
| 18 | 18 |
| 19 | 35 |
| 15 | 12 |
| 18 | 19 |
| <u>17</u> | <u>16</u> |
| 87/ 5 = \overline{X} = 17.4 | $100/5 = \overline{X} = 20$ |
| s = 1.51 | s = 8.80 |

Coefficient of Variation (CV)

Process 1:

$$87/5 = \overline{X} = 17.4$$

$$s = 1.51$$

$CV = \left(\frac{s}{\overline{x}}\right) \cdot 100\% = \frac{1.51}{17.4} \times 100 \% = 0.086 \times 100\% = 8.68\%$

Process 2:

$$100/5 = \overline{X} = 20$$

$$s = 8.80$$

$$CV = \frac{8.8}{20} = 0.44 \times 100\% = 44 \%$$



Comparing Coefficient of Variation

Stock A:

- Average price last year = \$ 4.00
- •Standard deviation = \$ 2.00

Stock B:

- Average price last year = \$80.00
- Standard deviation = \$ 8.00

Note: The standard deviation for stock A is lower than the standard deviation for stock B.



Comparing Coefficient of Variation

Stock A:

- Average price last year = \$ 4.00
- Standard deviation = \$ 2.00

$$CV_A = \left(\frac{s}{\overline{x}}\right) \cdot 100\% = \frac{\$2.00}{\$4.00} \cdot 100\% = \frac{50\%}{\$4.00}$$

Stock B:

- Average price last year = \$80.00
- Standard deviation = \$ 8.00

$$CV_B = \left(\frac{s}{\overline{x}}\right) \cdot 100\% = \frac{\$8.00}{\$80.00} \cdot 100\% \neq 10\%$$

Note: The standard deviation for stock A is lower than the standard deviation for stock B.

Comparing Coefficient of Variation, (CV) OKAN ÜNIVERSITESI

The standard deviation of stock A, is \$2, and that of stock B, is \$8, we would believe that stock B is more volatile or risky.

However, the average closing price for stock A is \$ 4, and \$ 80 for stock B.

The CV of stock A is higher, 50%, meaning that the market value of the stock fluctuates more from period to period than does that of stock B, 10%.

Therefore, a lower CV indicates lower riskiness in finance and higher precision or consistency in a production process.



When to use Standard deviation and coefficient of variation, when comparing two data sets

Use Standard deviation, SD, as a measure of risk/ consistency/reliability when comparing two or more objects:

Means are identical or very close

Use Coefficient of variation, CV, as a measure of risk/ consistency/reliability when comparing two or more objects:

Means are different

The *coefficient of variation, CV*, expresses the standard deviation as a percentage of it's mean. Is measured between 0 - 100 %.



Standard deviation and coefficient of variation – measures of variation

The standard deviation is the average distance of all the scores within a distribution around the mean.

The coefficient of variation is the standard deviation **relative** (in percent) to its' mean.

We can use the coefficient of variation to determine the relative variance within one particular process.



Application of coefficient of variation, CV

With the following information about investment A:

| | Investment A |
|-----------------------|--------------|
| Average annual return | 10.4 |
| Standard deviation | 4.393 |

Can we say what risk it carries? Is this a high or low risk?



Application of coefficient of variation

(continued)

With the coefficient of variation we can analyze the relative variation (in percent) around the mean:

| | Investment A |
|---------------------------------|--------------|
| Average annual return | 10.4 |
| Standard deviation | 4.393 |
| Coefficient of variation | 0.422403846 |

The coefficient of variation tells us that for investment A the sample standard deviation is 42.2 % from the mean.

What is the median?

What does the Range measure?

What does IQR measure?

How do we illustrate categorical data?

Why do we collect a sample from the population?

What are data?

What types of data do we work with in statistics?



Class quizz

Comparing the variation/ spread in two different processes: Standard deviation and Coefficient of variation:

(1) In which situation will we use the standard deviation as the measure of variation?

(2) In which **situations** will we need to use the Coefficient of variation?