Margita Vajsáblová

Mongeova projekcia

- polohové úlohy

Základné pojmy a obraz bodu v Mongeovej projekcii

Priemetne:

π – pôdorysňa, ${}^1s \perp \pi$,

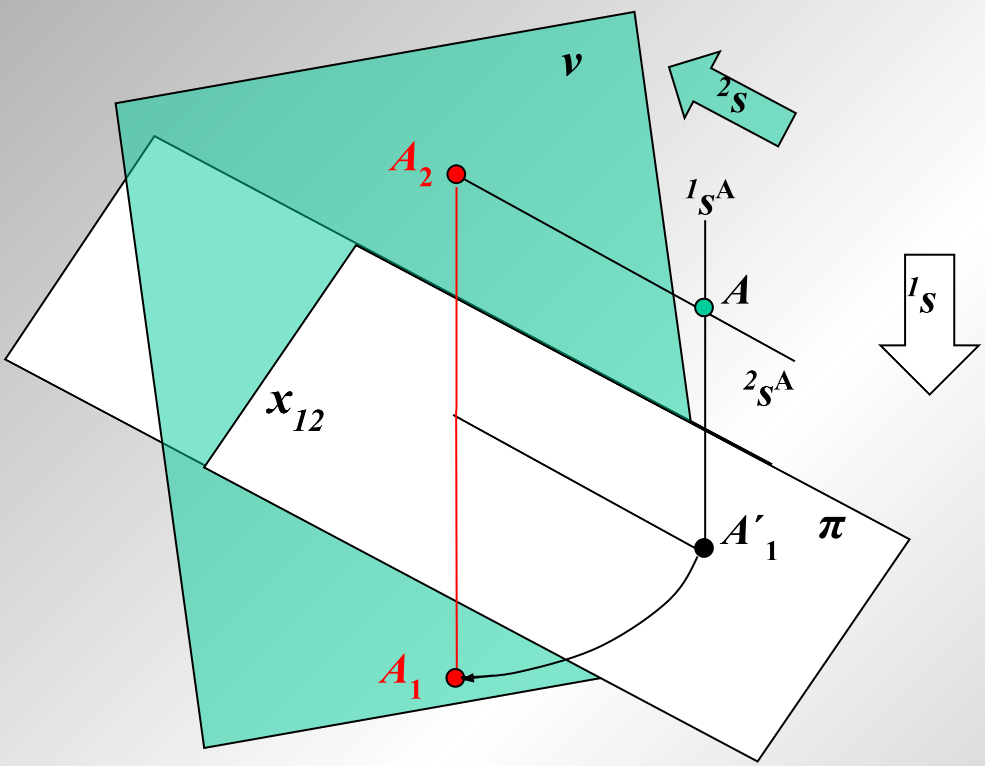
ν – nárysňa, ${}^2s \perp \nu$,

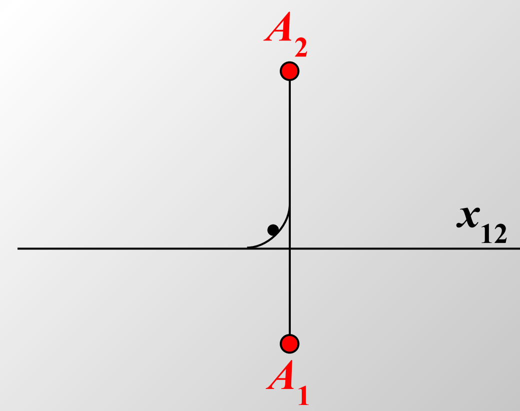
$\pi \perp \nu$, $\pi \cap \nu = x$, označujeme ju x_{12} – základnica.

Priemety bodu *A:*

$\pi \cap {}^1s^A = A'_1$ – pôdorys bodu A, ${}^1s^A$: $A \in {}^1s^A$, ${}^1s^A \perp \pi$,

$\nu \cap {}^2s^A = A_2$ – nárys bodu A, ${}^2s^A$: $A \in {}^2s^A$, ${}^2s^A \perp \nu$.





Združenie priemetní*:*

π otočíme do ν okolo x, A'_1 sa otočí do A_1,

A_1, A_2 – *združené priemety bodu A*,

platí $A_1A_2 \perp x_{12}$,

A_1A_2 – ordinála bodu A.

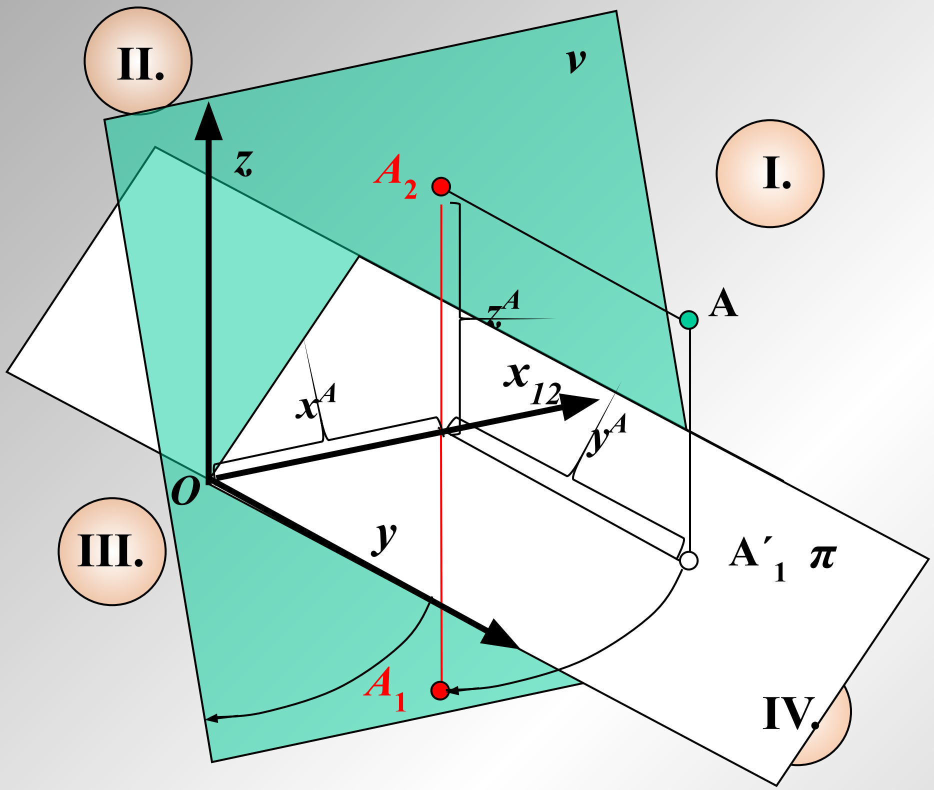
Definícia: Bijektívne zobrazenie, ktoré každému bodu $A \in E_3$ priradí združené priemety $[A_1, A_2]$, $A_1A_2 \perp x_{12}$, voláme ***kolmé premietanie na dve navzájom kolmé priemetne – Mongeova projekcia.***

Obraz bodu v Mongeovej projekcii

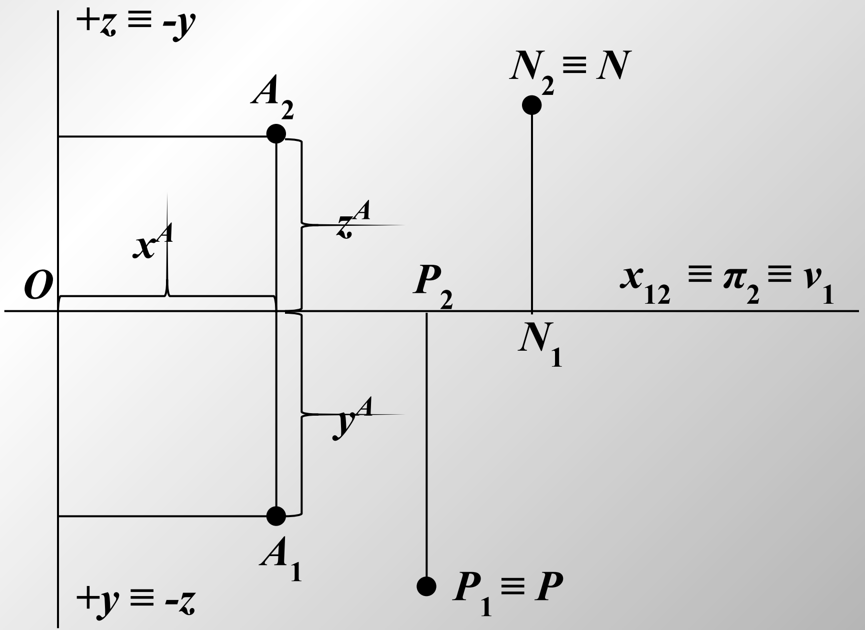
Pravouhlá súradnicová sústava:

$x, y \subset \pi$, $A_1 [x^A, y^A]$, kde x je základnica,

$x, z \subset \nu$, $A_2 [x^A, z^A]$,



V združení priemetní: $+z \equiv -y$, $+y \equiv -z$



Kvadranty: π a ν rozdeľujú E_3 na 4 kvadranty

I. kvadrant $y > 0$, $z > 0$, II. kvadrant $y < 0$, $z > 0$,

III. kvadrant $y < 0$, $z < 0$, IV. kvadrant $y > 0$, $z < 0$.

Body priemetní:

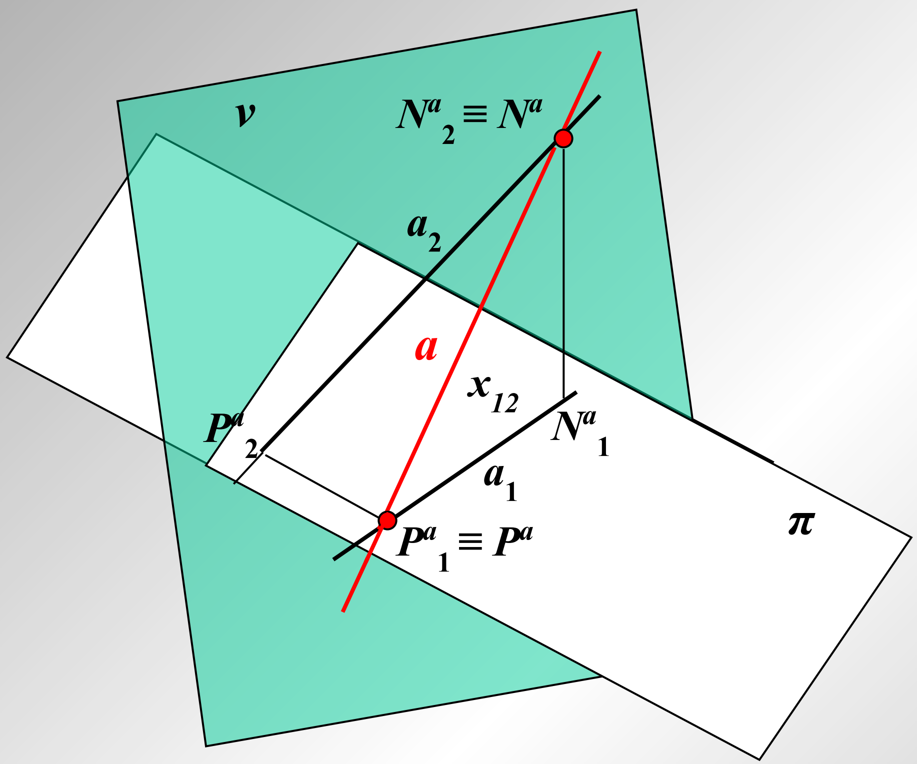
$P \in \pi \Rightarrow P_1 \equiv P,\ P_2 \in x_{12},\ z^P = 0$

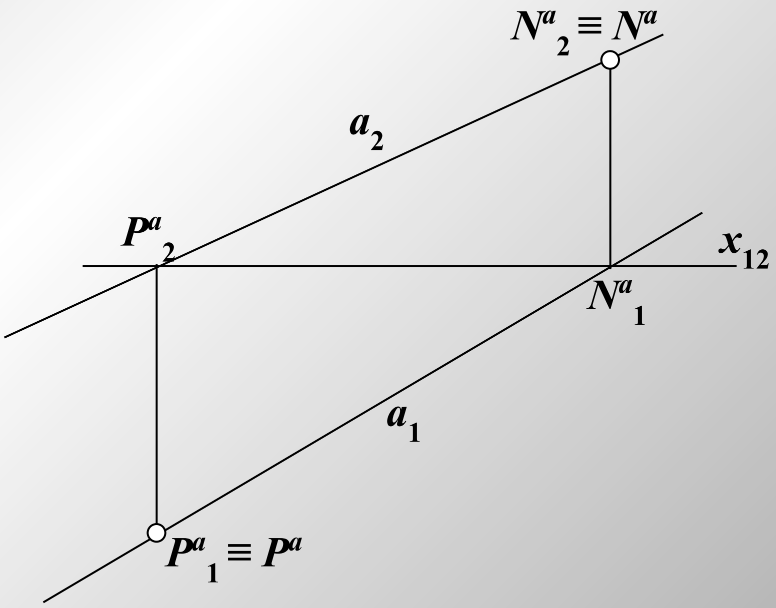
$N \in \nu \Rightarrow N_1 \in x_{12},\ N_2 \equiv N,\ y^N = 0$

Obraz priamky v Mongeovej projekcii

Stopníky priamky: $a \cap \pi = P^a$ – pôdorysný stopník priamky a,

$a \cap \nu = N^a$ – nárysný stopník priamky a.





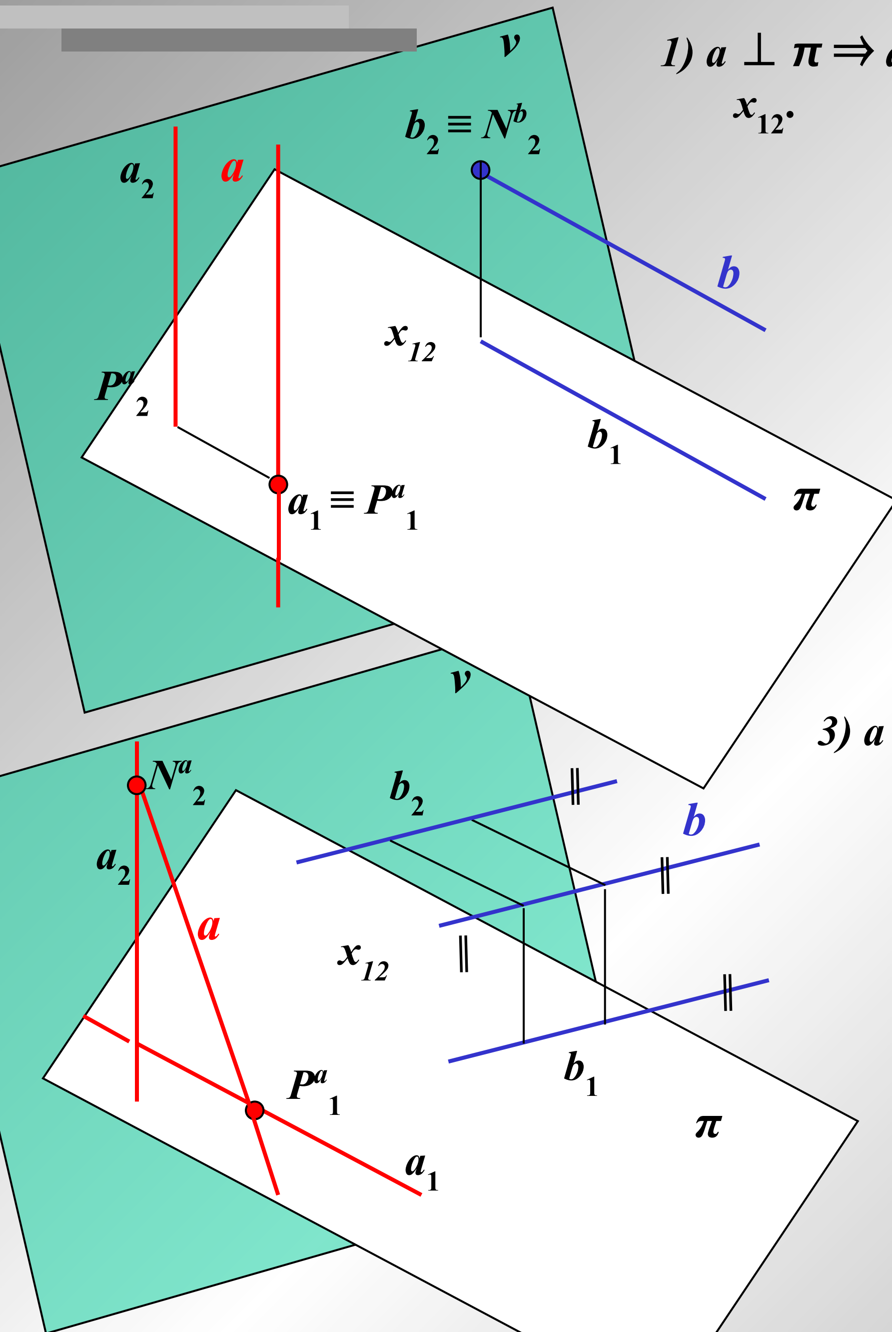
Konštrukcia pôdorysného stopníka:

$a_2 \cap x_{12} = P^a{}_2,\ P_1 \in a_1$

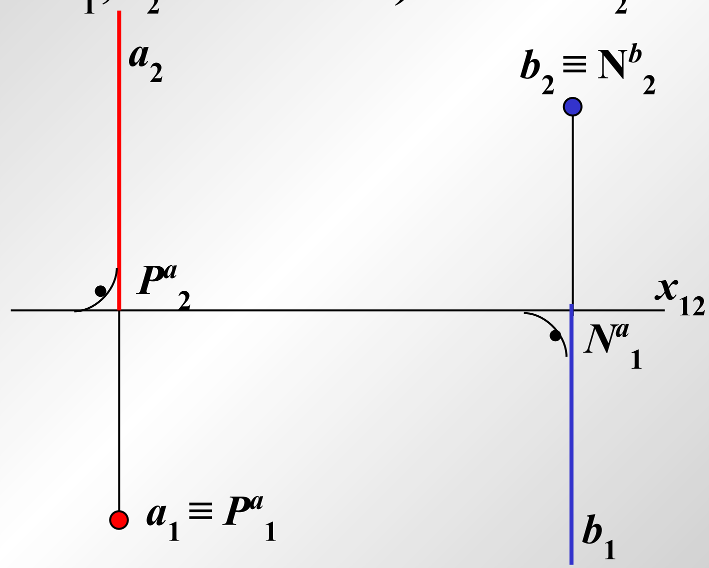
Konštrukcia nárysného stopníka:

$a_1 \cap x_{12} = N^a{}_1,\ N_2 \in a_2$.

Obraz priamok v Mongeovej projekcii



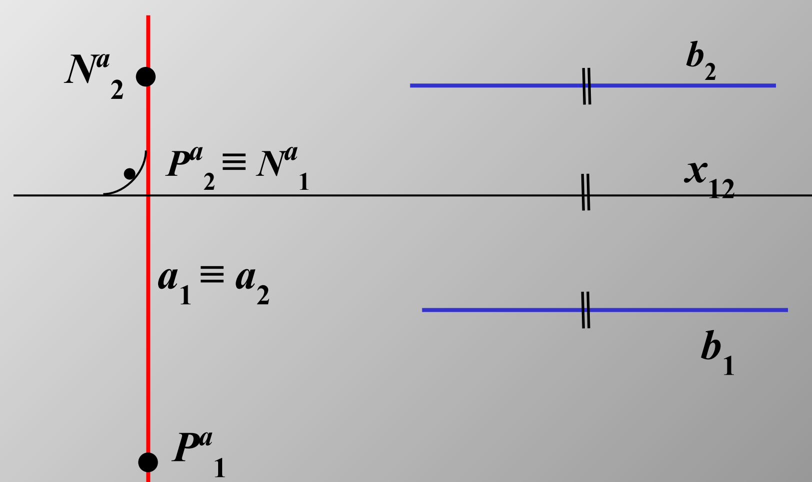
1) $a \perp \pi \Rightarrow a_1 \equiv P^a{}_1,\ a_2 \perp x_{12}.$



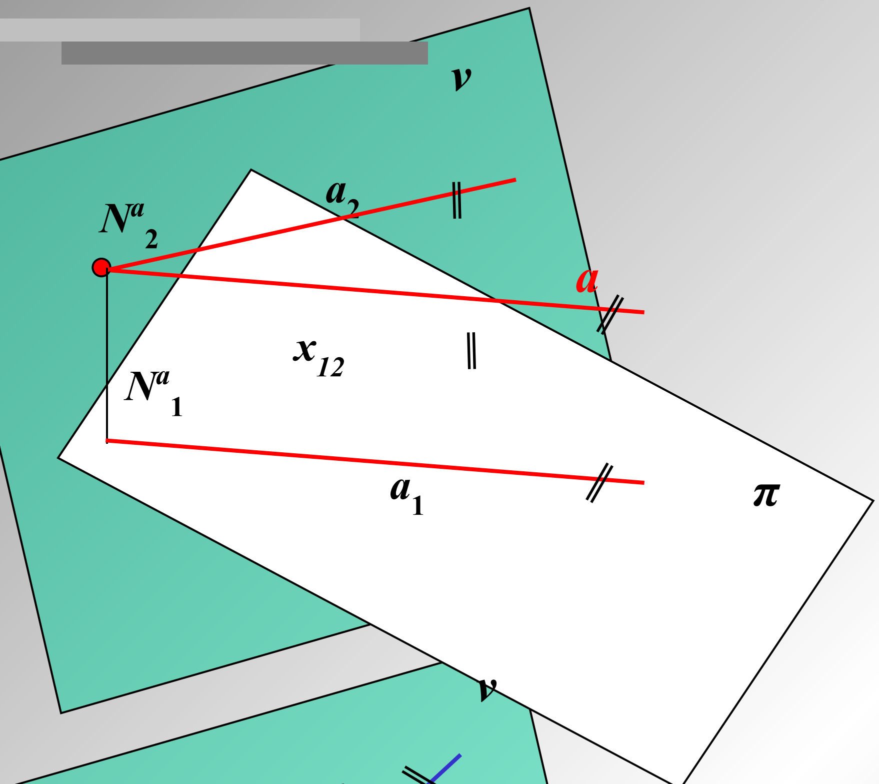
2) $b \perp \nu \Rightarrow b_2 \equiv N^b{}_2,\ b_1 \perp x_1.$

3) $a \perp x \Rightarrow a_1 \equiv a_2 \perp x_{12}.$

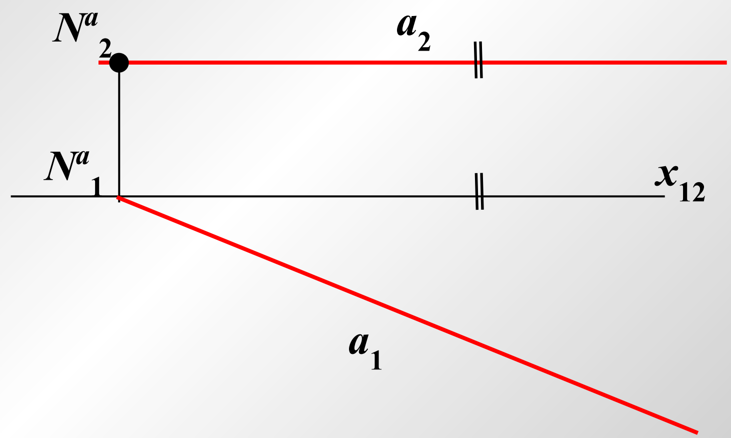
4) $b \parallel x \Rightarrow b_1 \parallel b_2 \parallel x_{12}.$

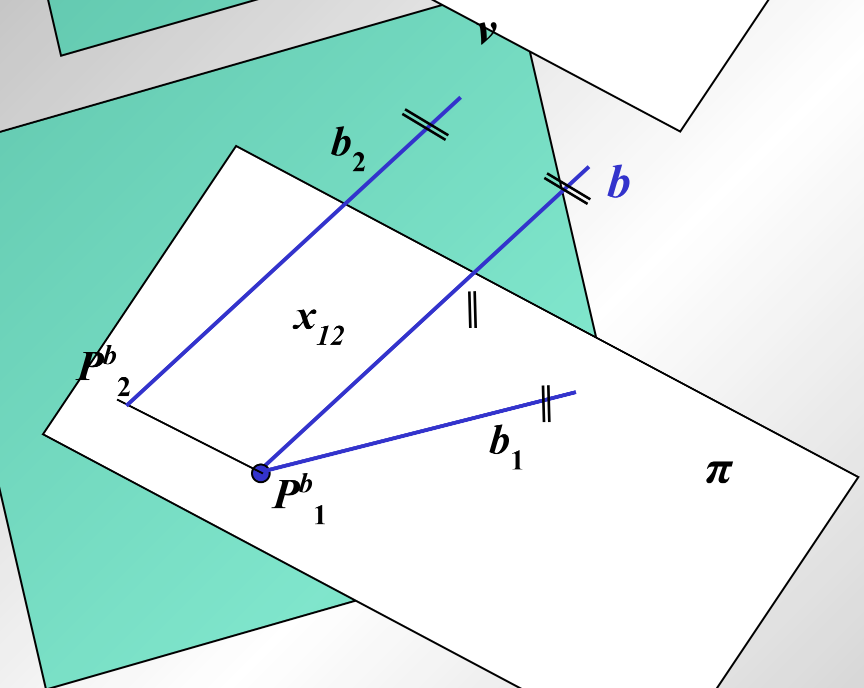


Obraz priamok v Mongeovej projekcii

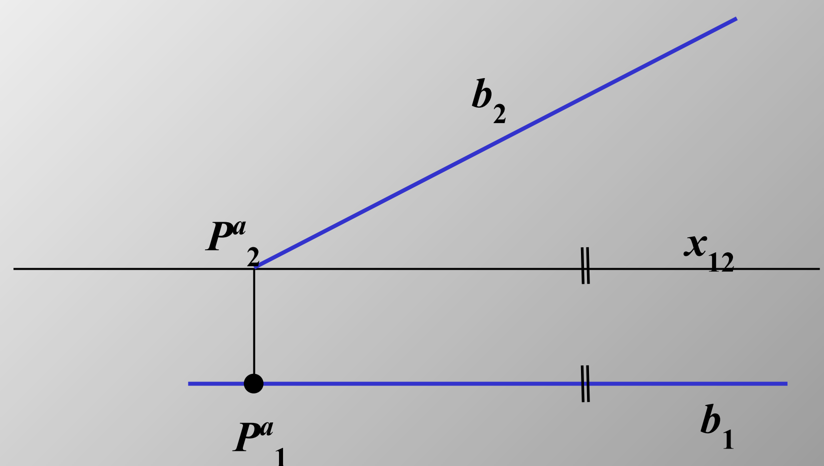


5) $a \parallel \pi \Rightarrow a_2 \parallel x_{12}$





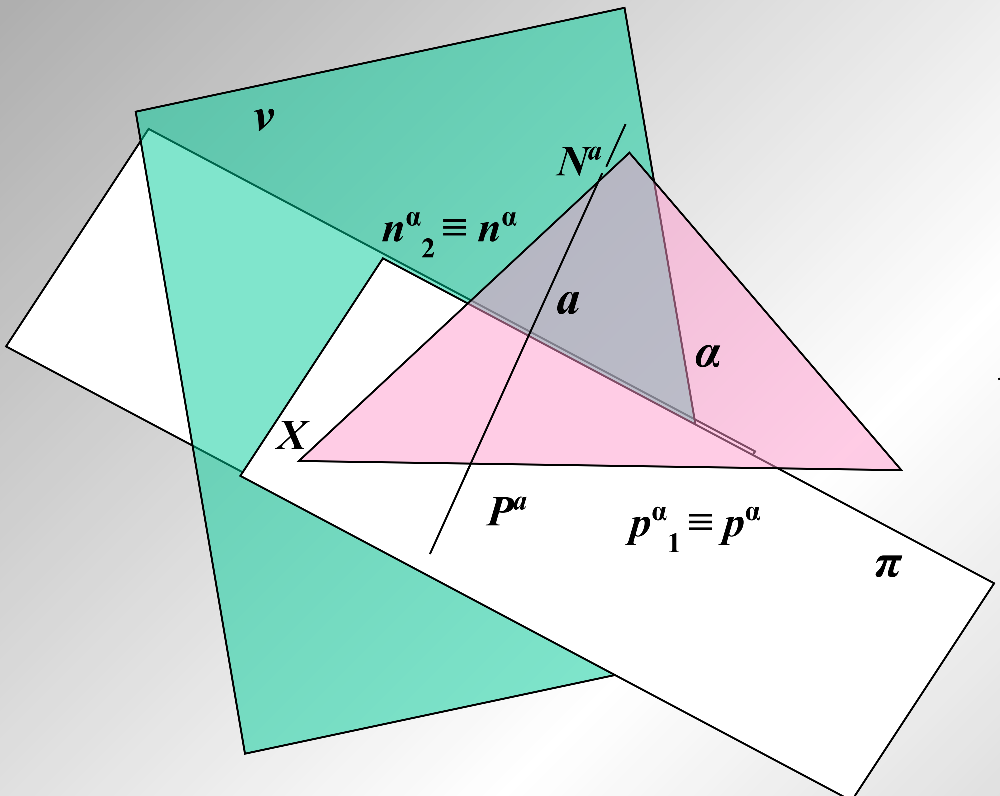
6) $b \parallel \nu \Rightarrow b_1 \parallel x_{12}$

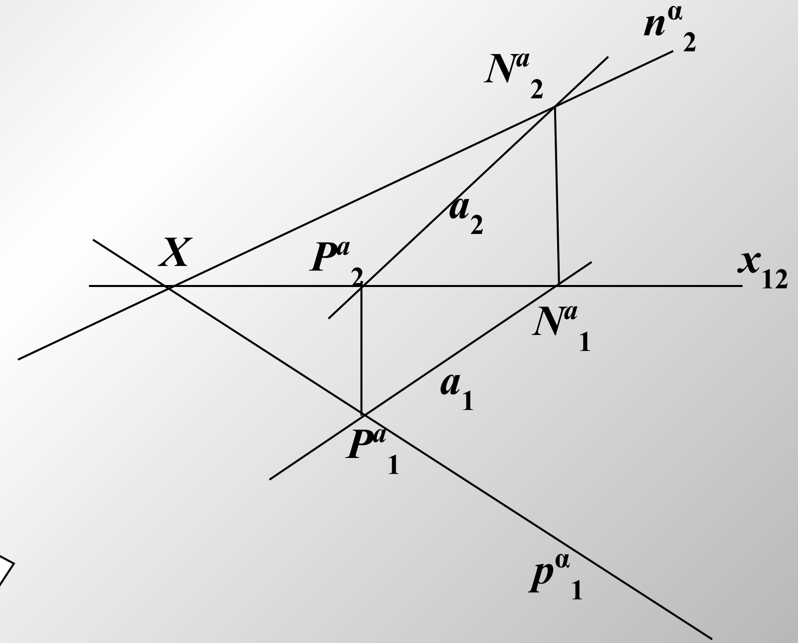


Obraz roviny v Mongeovej projekcii

Stopy roviny: $\alpha \cap \pi = p^{\alpha}$ – pôdorysná stopa roviny α,

$\alpha \cap \nu = n^{\alpha}$ – nárysná stopa roviny α. Ak existuje $X = p^{\alpha} \cap n^{\alpha}$, potom $X \in x$.

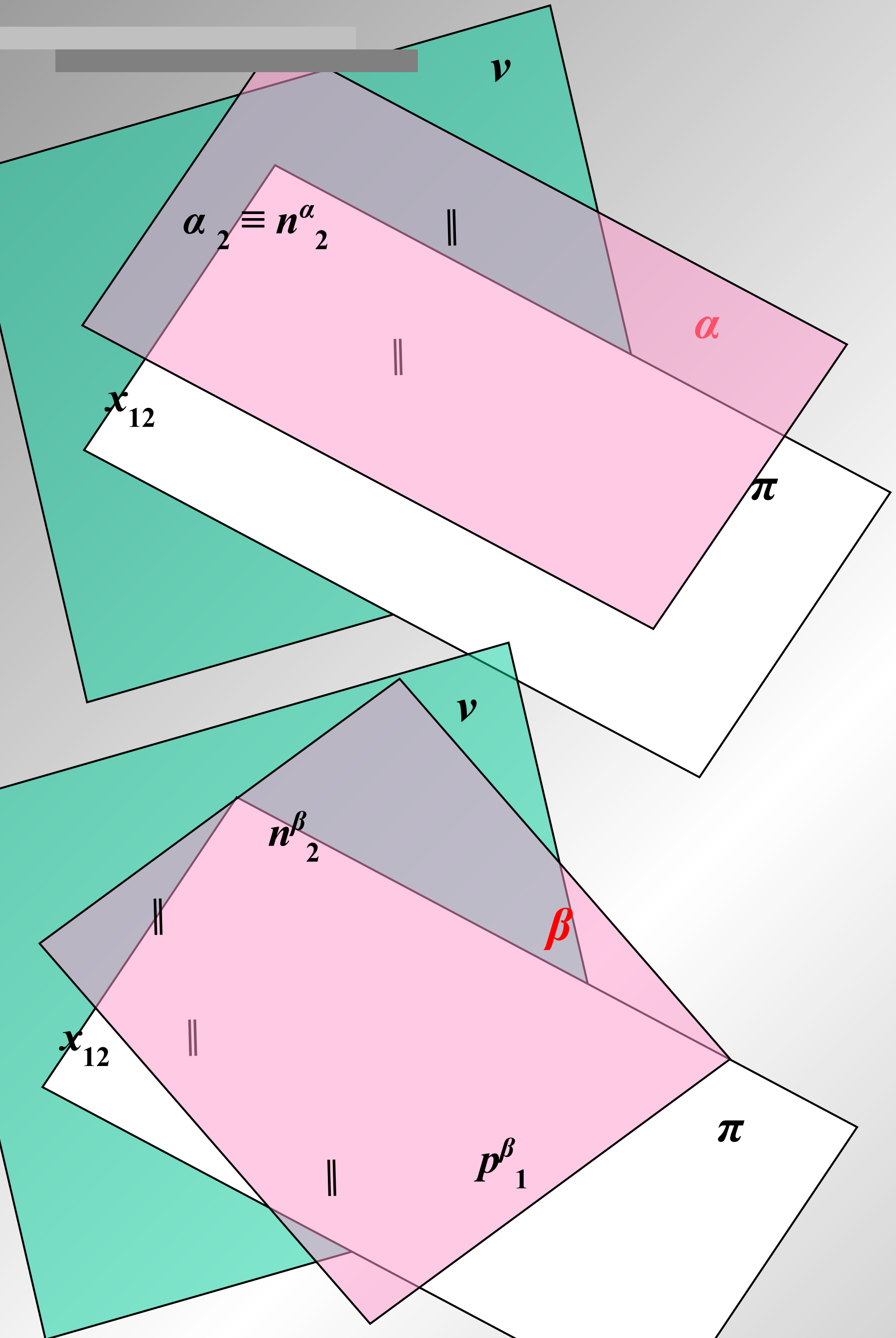




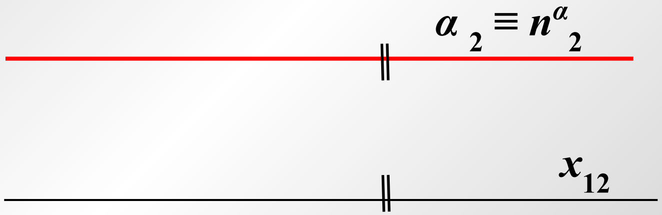
Ak priamka leží v rovine a má stopníky, potom jej pôdorysný stopník leží na pôdorysnej a nárysný na nárysnej stope roviny:

$$P^{a}_{1} \in p^{\alpha}_{1}, N^{a}_{2} \in n^{\alpha}_{2}$$

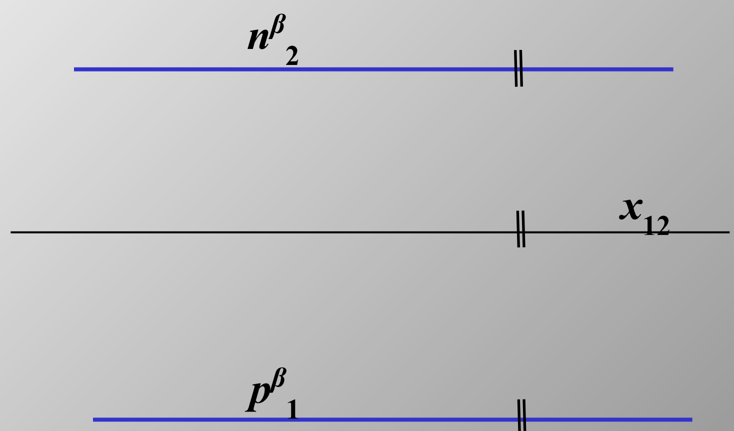
Roviny v Mongeovej projekcii



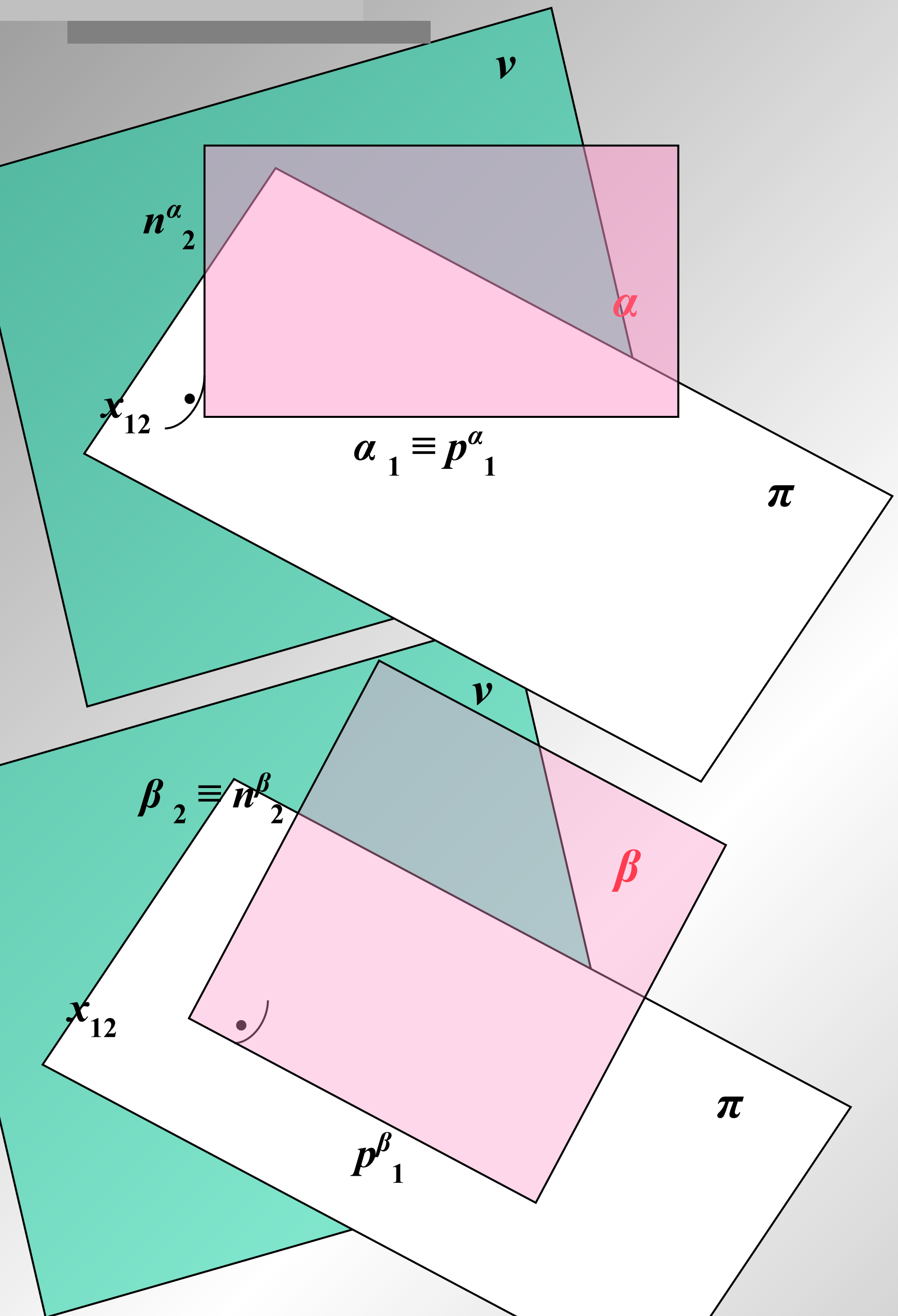
1) $\alpha \parallel \pi \Rightarrow \alpha_2 \equiv n^\alpha_2 \parallel x_{12}$



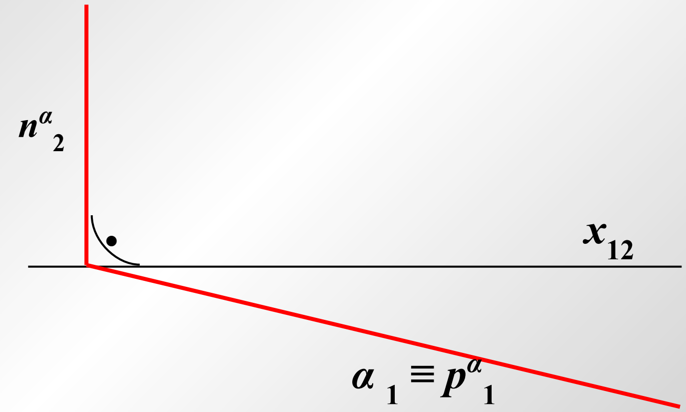
2) $\beta \parallel x \Rightarrow p^\beta_1 \parallel n^\beta_2 \parallel x_{12}$



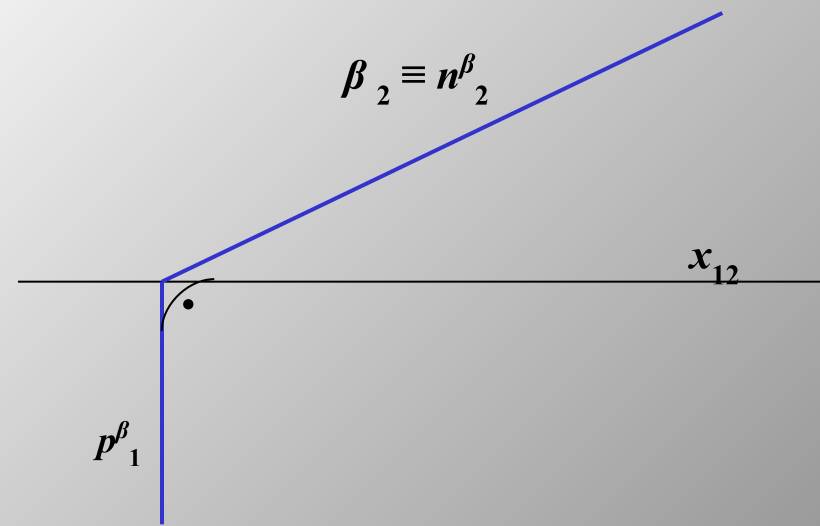
Roviny v Mongeovej projekcii



3) $\alpha \perp \pi \Rightarrow \alpha_1 \equiv p^{\alpha}_1,\ n^{\alpha}_2 \perp x_{12}$



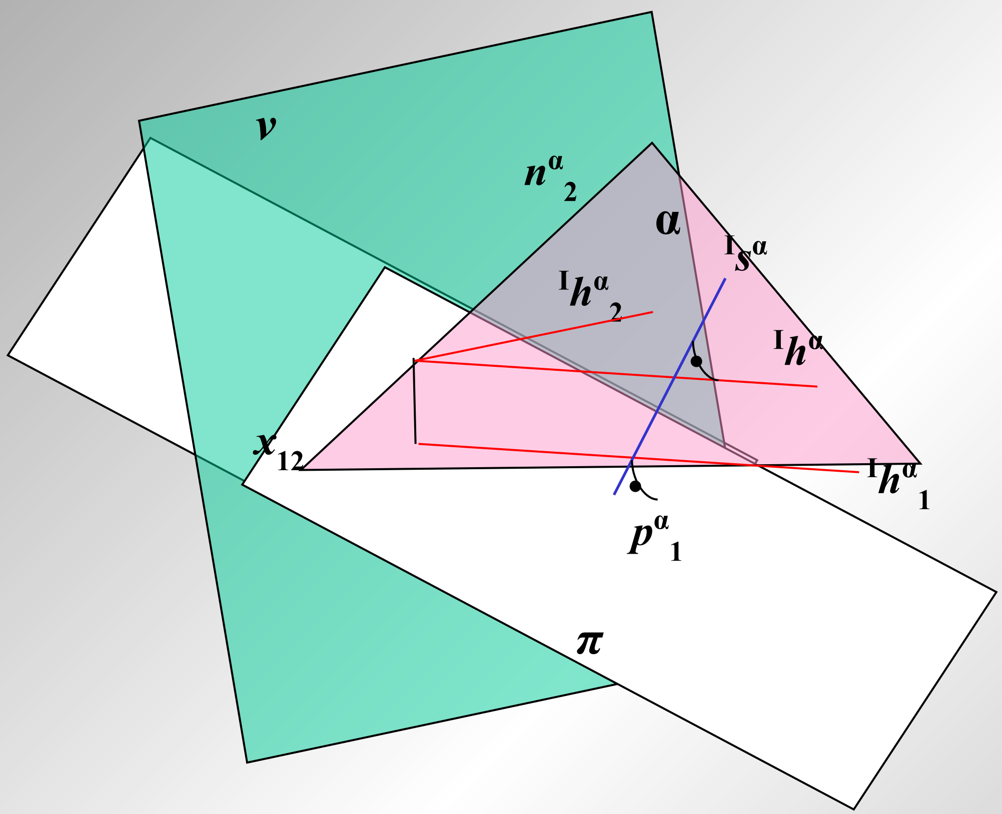
4) $\beta \perp \nu \Rightarrow \beta_2 \equiv n^{\beta}_2,\ p^{\beta}_1 \perp x_{12}$

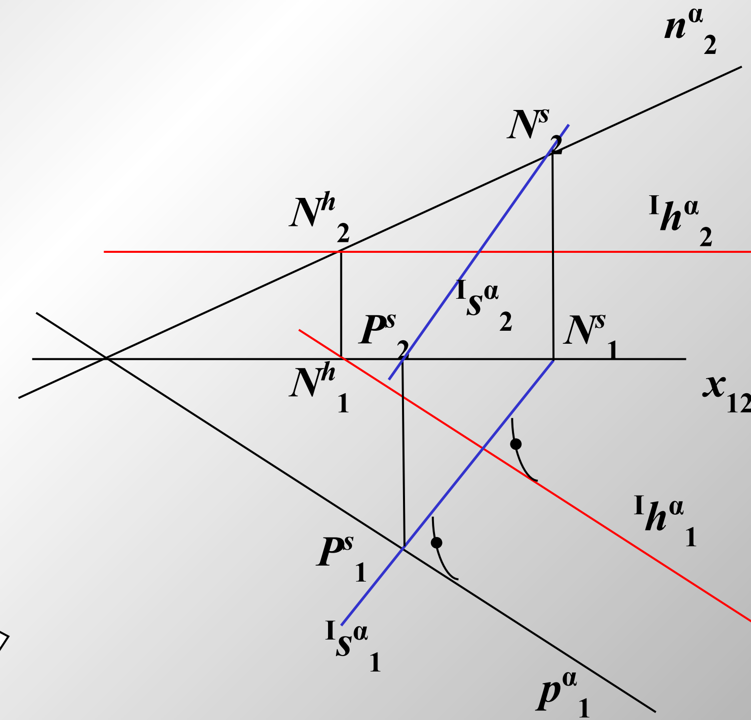


Hlavné a spádové priamky roviny v Mongeovej projekcii

Hlavné priamky I. osnovy roviny α: ${}^{I}h^{\alpha} \parallel \pi$,

${}^{I}h^{\alpha}_{1} \parallel p^{\alpha}_{1}$, ${}^{I}h^{\alpha}_{2} \parallel x_{12}$.





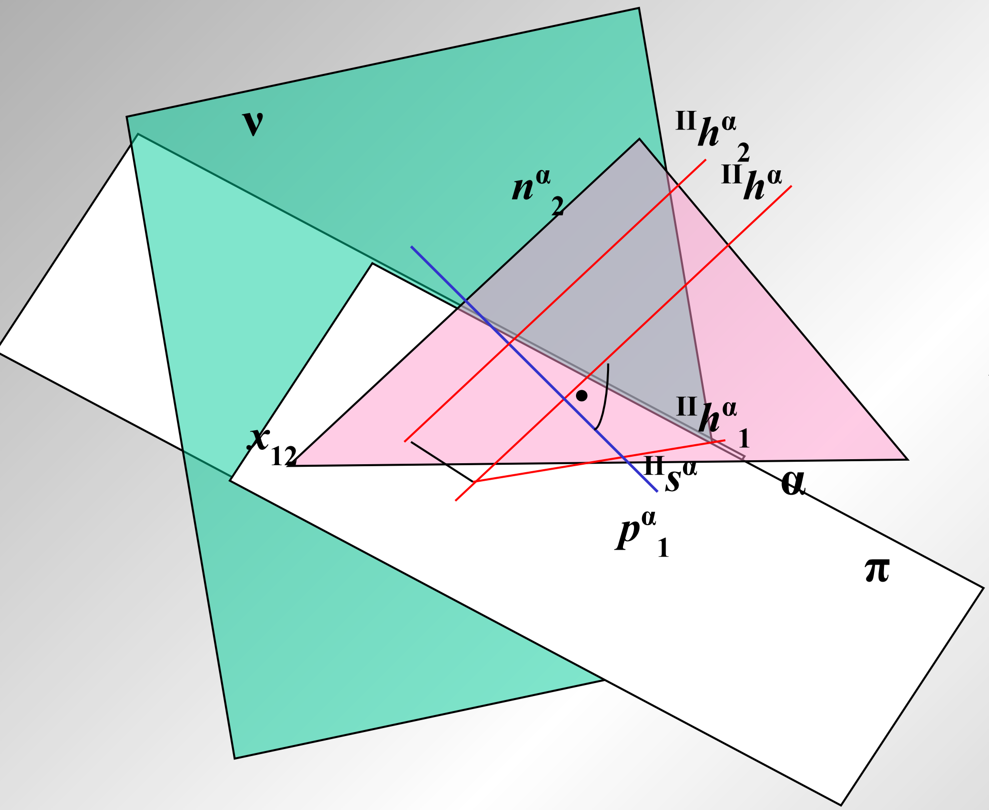
Spádové priamky I. osnovy roviny α: ${}^{I}s^{\alpha} \perp {}^{I}h^{\alpha}\,(p^{\alpha})$,

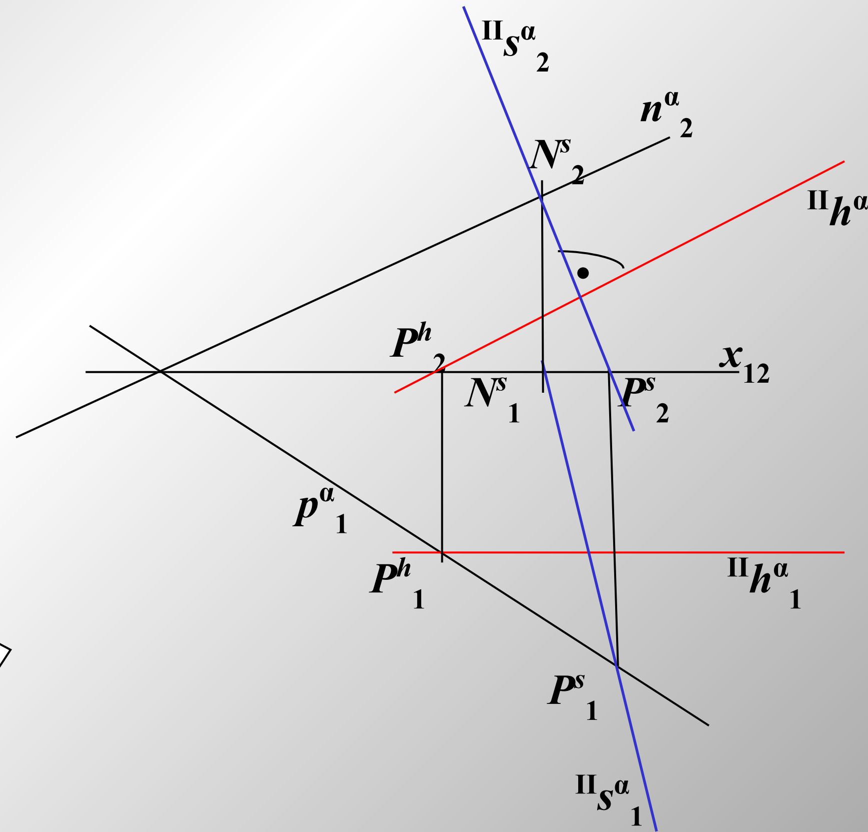
${}^{I}s^{\alpha}_{1} \perp p^{\alpha}_{1}$, ${}^{I}s^{\alpha}_{2} = P^{s}_{2}N^{s}_{2}$.

Hlavné a spádové priamky roviny v Mongeovej projekcii

Hlavné priamky II. osnovy roviny α: $^{II}h^{\alpha} \parallel \nu$

$$^{II}h^{\alpha}_{1} \parallel x_{12},\ ^{II}h^{\alpha}_{2} \parallel n^{\alpha}_{2}$$



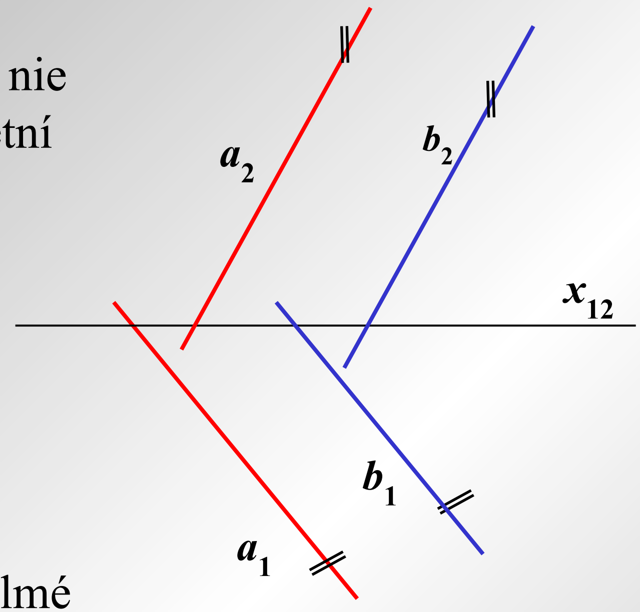


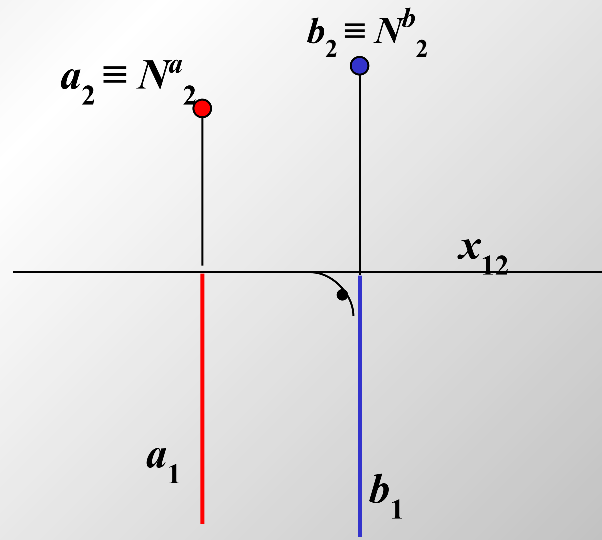
Spádové priamky II. osnovy roviny α: $^{II}s^{\alpha} \perp {}^{II}h^{\alpha}\,(n^{\alpha})$

$$^{II}s^{\alpha}_{2} \perp n^{\alpha}_{2},\ ^{II}s^{\alpha}_{1} = P^{s}_{1} N^{s}_{1}$$

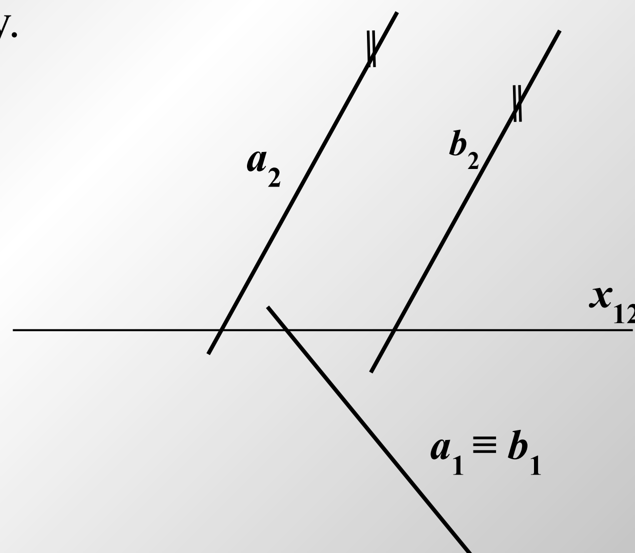
Vzájomná poloha 2 priamok v Mongeovej projekcii

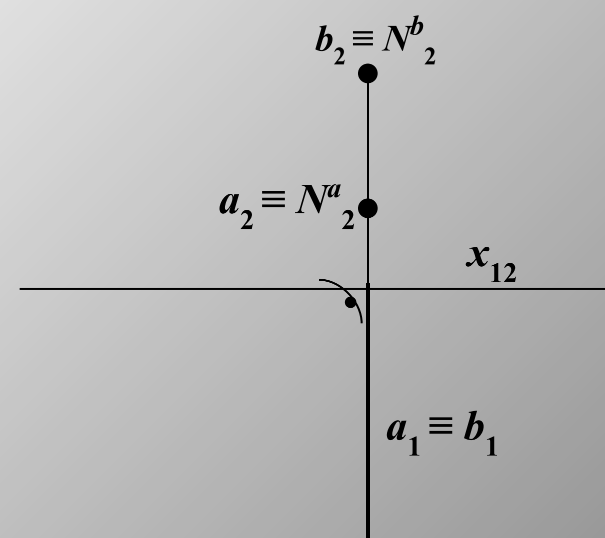
1) ***Rovnobežné priamky a, b,*** ak nie sú kolmé na žiadnu z priemetní a $a \parallel b \Rightarrow a_1 \parallel b_1, a_2 \parallel b_2$.





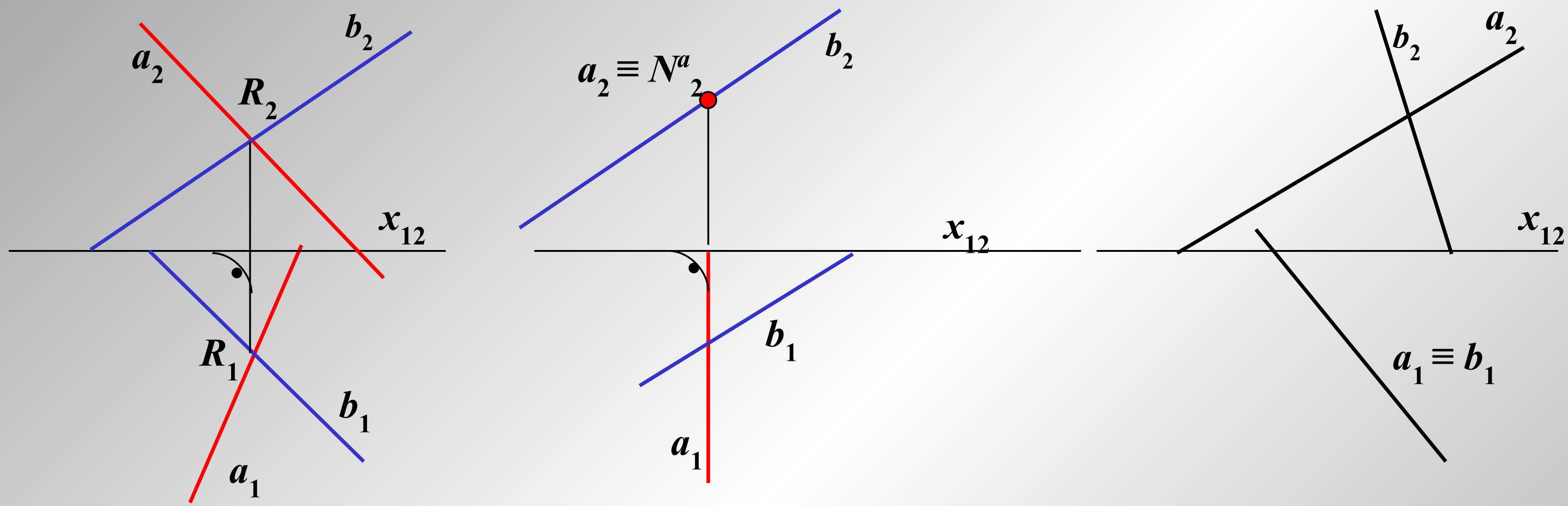
Ak sú rovnobežné priamky kolmé na niektorú z priemetní, ich priemetom v nej sú 2 body.



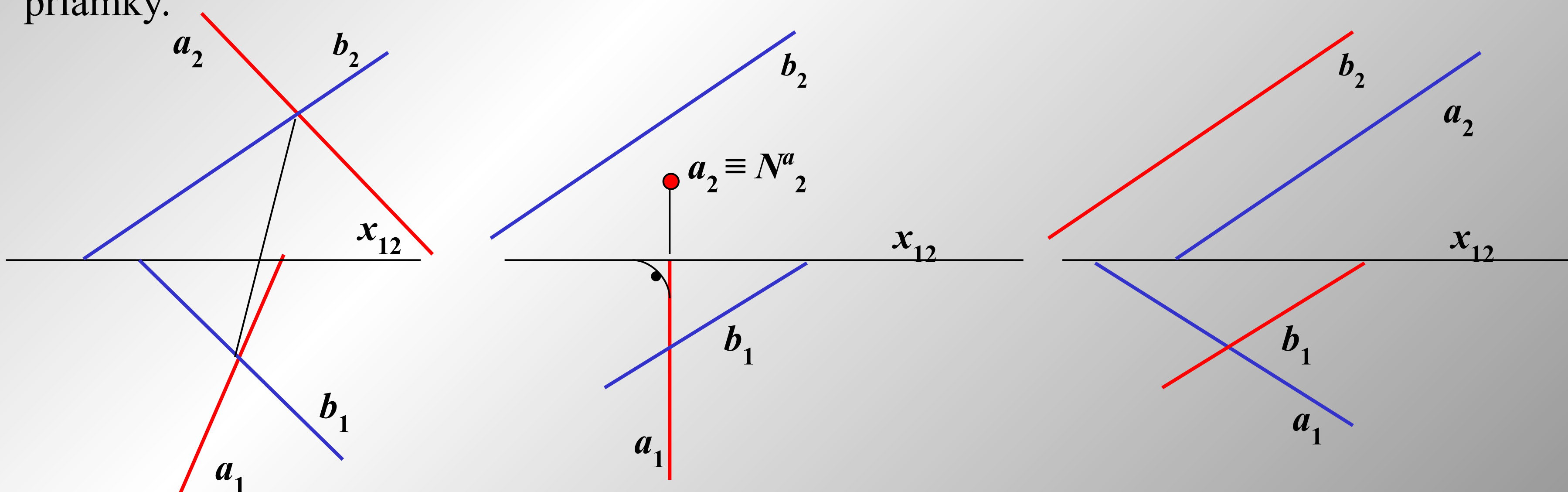


Vzájomná poloha 2 priamok v Mongeovej projekcii

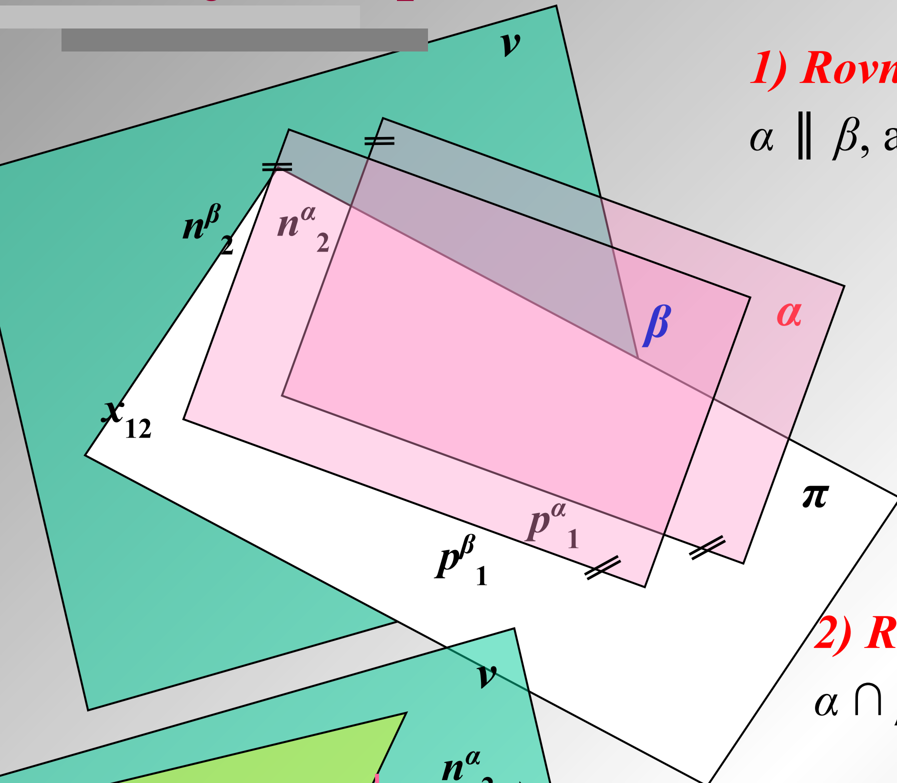
2) *Rôznobežné priamky a, b:* $a \cap b = R \Rightarrow a_1 \cap b_1 = R_1$, $a_2 \cap b_2 = R_2$, potom $R_1 R_2 \perp x_{12}$.



3) *Mimobežné priamky a, b:* neplatia predchádzajúce pravidlá pre rovnobežné, ani rôznobežné priamky.

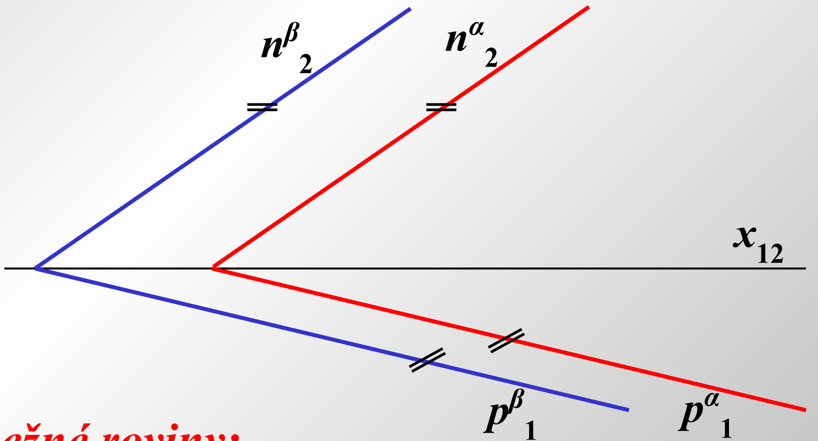


Vzájomná poloha 2 rovín v Mongeovej projekcii



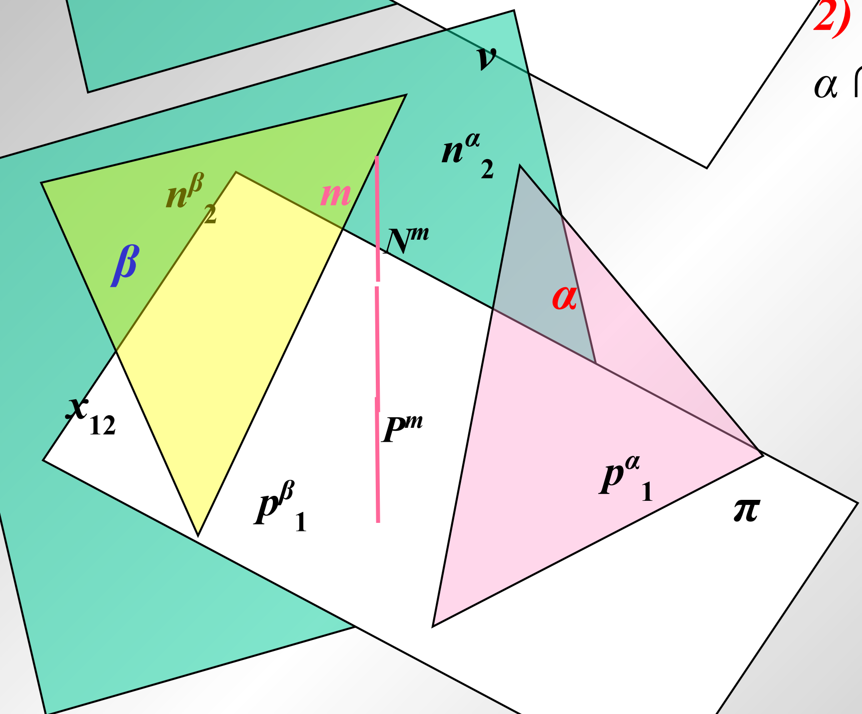
1) Rovnobežné roviny:

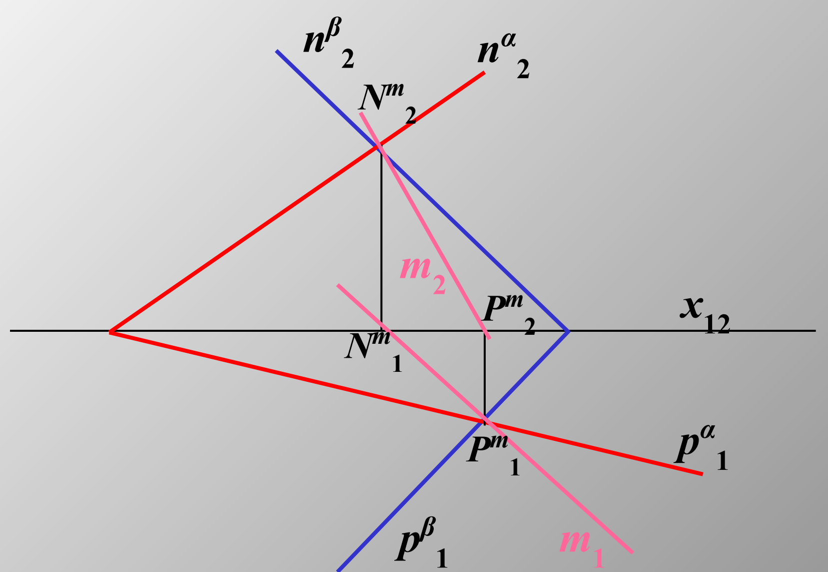
$\alpha \parallel \beta$, ak existujú ich stopy $\Rightarrow p^{\alpha}_{1} \parallel p^{\beta}_{1},\ n^{\alpha}_{2} \parallel n^{\beta}_{2}$



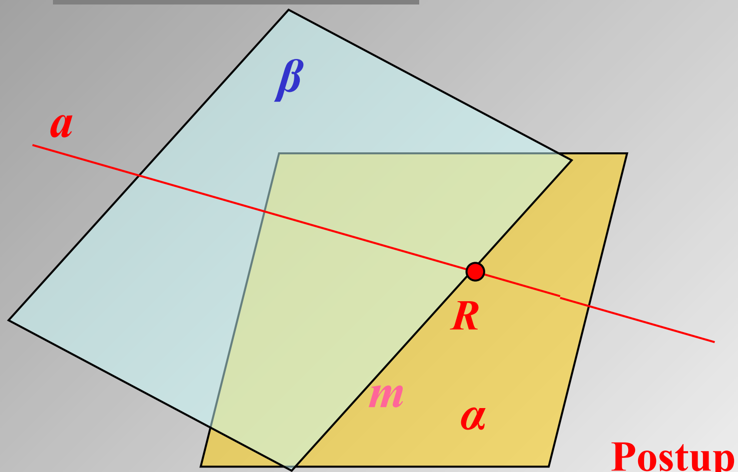
2) Rôznobežné roviny:

$\alpha \cap \beta = m \Rightarrow P^{m} = p^{\alpha} \cap\ p^{\beta},\ N^{m} = n^{\alpha}\ \cap n^{\beta}_{2}$.





Vzájomná poloha priamky a roviny v Mongeovej projekcii



Všeobecný postup $a \cap \alpha$:

1. **Priamkou a preložíme ľubovoľnú rovinu β: $a \subset \beta$.**
2. **Nech m je priesečnica rovín α a β: $\alpha \cap \beta = m$.**
3. **Podľa vzájomnej polohy priamok a a m určíme vzájomnú polohu priamky a a roviny α:**
   - a, $a \equiv m \Rightarrow a \subset \alpha$
   - b, $a \parallel m \Rightarrow a \parallel \alpha$
   - c, $a \cap m = R \Rightarrow R = a \cap \alpha$

Postup v Mongeovej projekcii, dané je $a[a_1, a_1]$, $\alpha(p^\alpha, n^\alpha)$, určte $a \cap \alpha$:

1. β: $a \subset \beta$, $\beta \perp \pi$

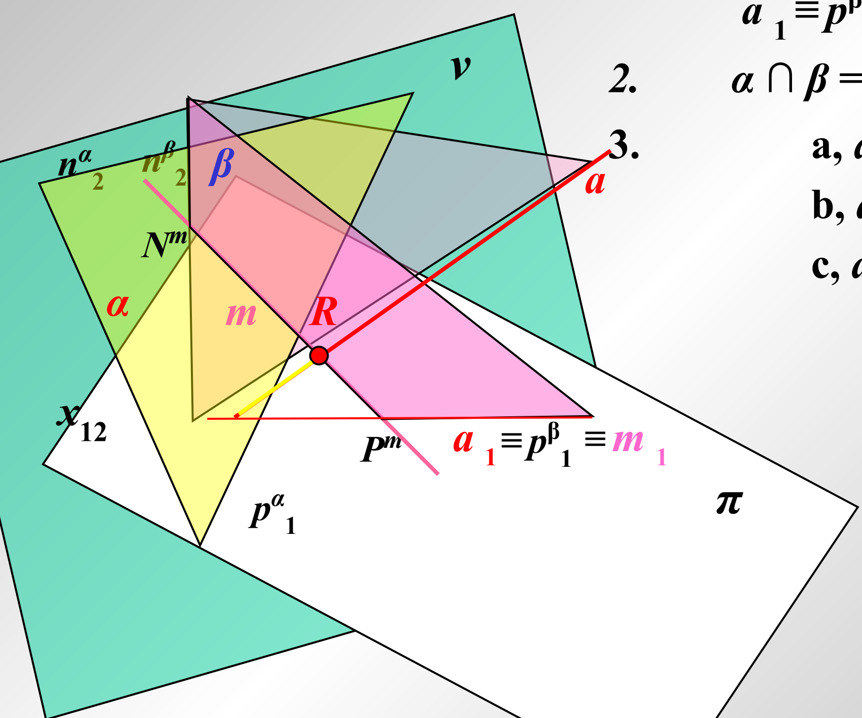
   $a_1 \equiv p^\beta_1$, $n^\beta_2 \perp x_{12}$

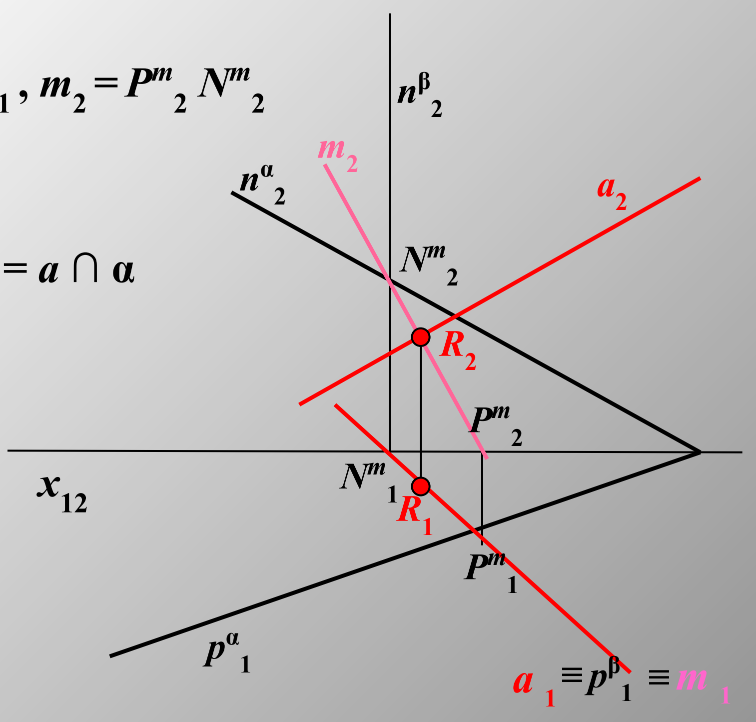
2. $\alpha \cap \beta = m$: $a_1 \equiv p^\beta_1 \equiv m_1$, $m_2 = P^m_2 N^m_2$

3. a, $a_2 \equiv m_2 \Rightarrow a \subset \alpha$

   b, $a_2 \parallel m_2 \Rightarrow a \parallel \alpha$

   c, $a_2 \cap m_2 = R_2 \Rightarrow R = a \cap \alpha$

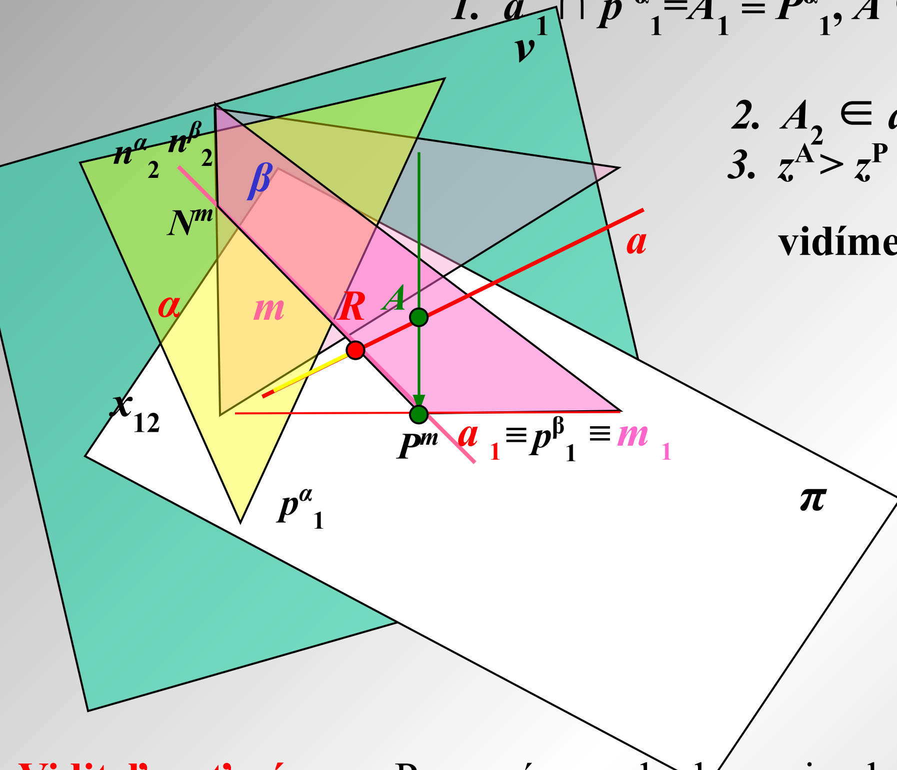




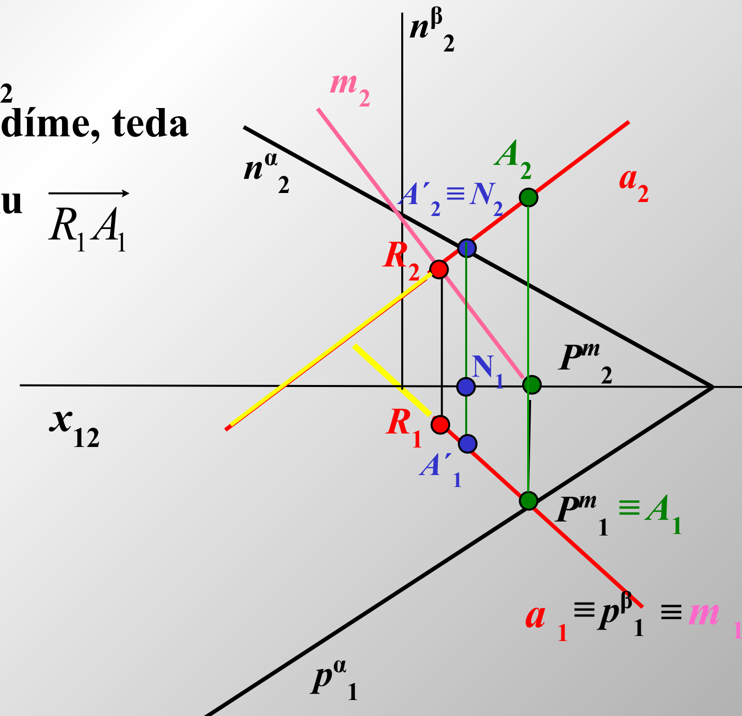
Viditeľnosť priamky vzhľadom na rovinu v Mongeovej projekcii

Viditeľnosť pôdorysu: Porovnávame bod na priamke a v rovine, ktorých pôdorysy sú totožné a viditeľný je ten, ktorý má väčšiu *z*-tovú súradnicu:

1. $a_1 \cap p^{\alpha}_1 = A_1 \equiv P^{\alpha}_1$, $A \in a$, $P^{\alpha} \in p^{\alpha}$



2. $A_2 \in a_2$, $P^{\alpha}_2 \in x_{12}$
3. $z^A > z^P \Rightarrow$ **bod *A* vidíme, teda vidíme polpriamku** $\overrightarrow{R_1A_1}$



Viditeľnosť nárysu: Porovnávame bod na priamke a v rovine, ktorých nárysy sú totožné a viditeľný je ten, ktorý má väčšiu *y*-ovú súradnicu:

1. $a_2 \cap n^{\alpha}_2 = A'_2 \equiv N_2$, $A' \in a$, $N \in n^{\alpha}$,
2. $A'_1 \in a_1$, $N_1 \in x_{12}$,
3. $y^{A'} > z^{N} \Rightarrow$ **bod *A* vidíme, teda vidíme polpriamku** $\overrightarrow{R_2A'_2}$.