Лекция 4. Плоские электромагнитные волны в диспергирующих средах

1. Поперечные электромагнитные волны в газе осцилляторов
- Дисперсионное уравнение
- Поглощение волн

2. Электромагнитные волны в изотропной плазме:
- Поперечные электромагнитные волны
- Продольные волны

3. Плоские волны в анизотропных средах
- Дисперсионное уравнение для волн в анизотропных средах
- Двойное лучепреломление
- Волны в плазме в магнитном поле
- Вращение плоскости поляризации

$$\operatorname{div} \mathbf{D} = 0, \qquad \left(\mathbf{k} \cdot \mathbf{D}(\omega, \mathbf{k}) \right) = 0\,,$$

$$\operatorname{rot} \operatorname{rot} \mathbf{E} = -\frac{1}{c^2} \frac{\partial^2 \mathbf{D}}{\partial t^2}\,. \qquad \left[\mathbf{k} \cdot [\mathbf{k} \cdot \mathbf{E}(\omega, \mathbf{k})] \right] = -\frac{\omega^2}{c^2} \mathbf{D}(\omega, \mathbf{k})\,.$$

$$\varepsilon^l(\omega, \mathbf{k})(\mathbf{k} \cdot \mathbf{E}^l) = 0\,,$$

$$\left(k^2 - \frac{\omega^2}{c^2} \varepsilon^{tr}(\omega, \mathbf{k}) \right) \mathbf{E}^{tr} = \frac{\omega^2}{c^2} \varepsilon^l(\omega, \mathbf{k}) \mathbf{E}^l\,.$$

$$\varepsilon^l(\omega, \mathbf{k}) = 0$$

$$k^2 - \frac{\omega^2}{c^2} \varepsilon^{tr}(\omega, \mathbf{k}) = 0$$

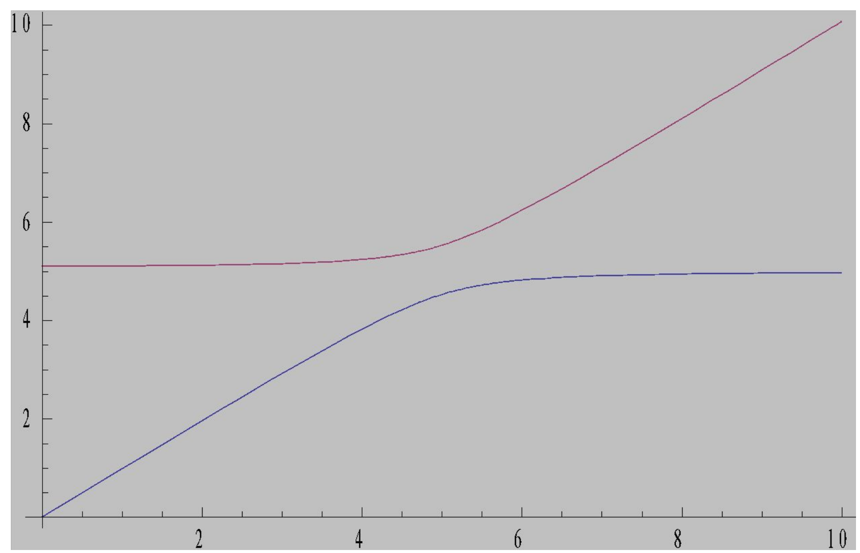
$$k^2 - \frac{\omega^2}{c^2} \left(1 - \frac{\omega_0^2}{\omega^2 - \omega_s^2 + i\nu\omega} \right) = 0$$

$$\omega_{1,2} = \sqrt{\frac{1}{2}(\omega_s^2 + k^2c^2 + \omega_0^2) \mp \frac{1}{2}\sqrt{(\omega_s^2 + k^2c^2 + \omega_0^2)^2 - 4\omega_s^2 k^2 c^2}}$$

$$\omega_0^2 \omega_s^2 << (\omega_s^2 - k^2c^2)^2$$

$$\omega_1 = kc - \frac{1}{2} \frac{\omega_0^2}{\omega_s^2 - k^2c^2} kc,$$

$$\omega_2 = \omega_s + \frac{\omega_0^2}{\omega_s^2 - k^2c^2} \omega_s\,.$$

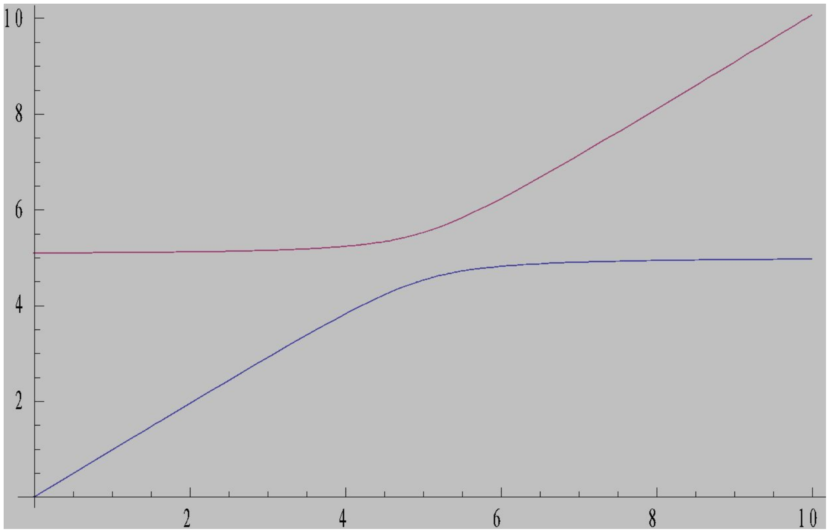


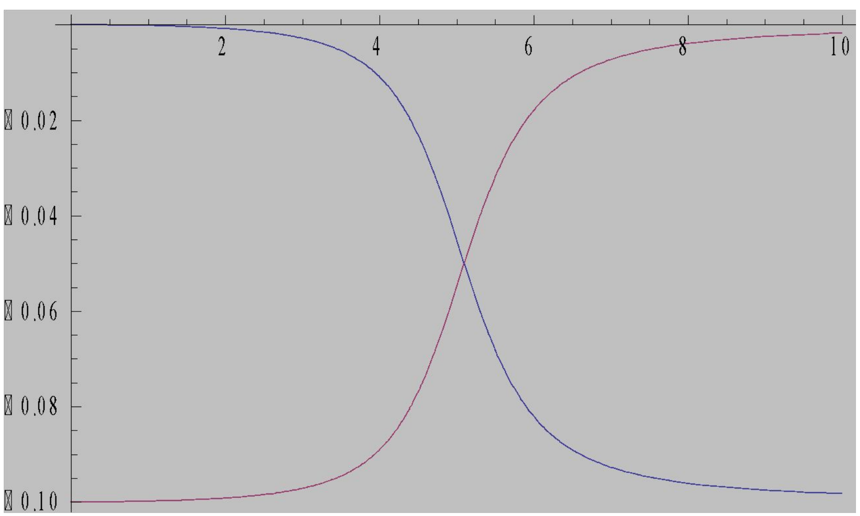
$$\nu kc \ll | \omega_s^2 - k^2c^2 |$$

$$\omega_1 = kc - \frac{1}{2}\frac{\omega_0^2}{\omega_s^2 - k^2c^2} kc - i\nu \frac{\omega_0^2 k^2 c^2}{(\omega_s^2 - k^2c^2)^2}$$

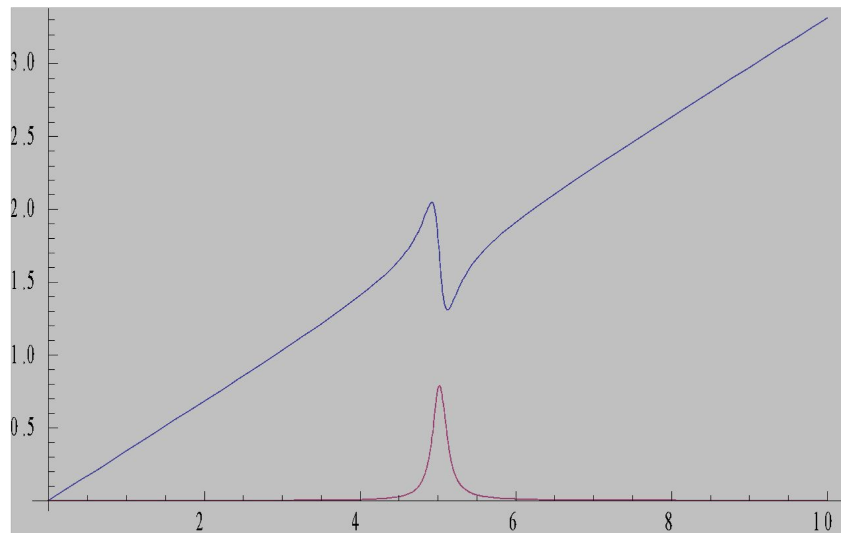
$$\nu \ll \omega_s$$

$$\omega_2 = \omega_s + \frac{\omega_0^2}{\omega_s^2 - k^2c^2}\omega_s - \frac{1}{2} i\nu$$





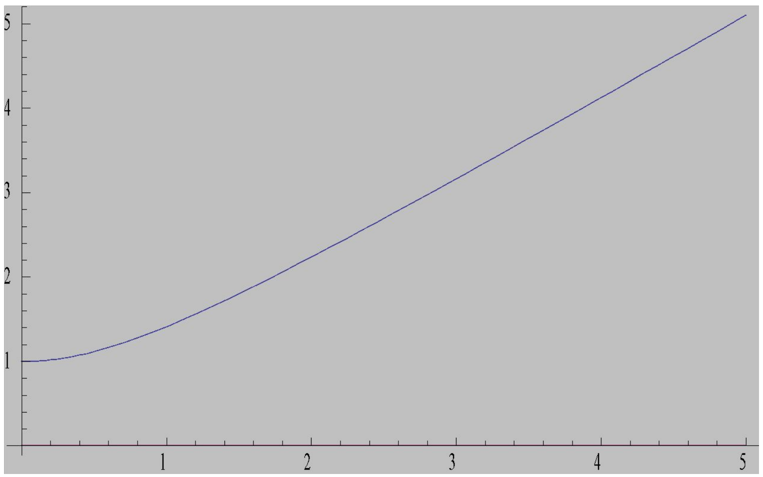
$$k(\omega) = \frac{\omega}{c}\sqrt{1 - \frac{\omega_0^2}{\omega^2 - \omega_s^2 + i\nu\omega}}$$

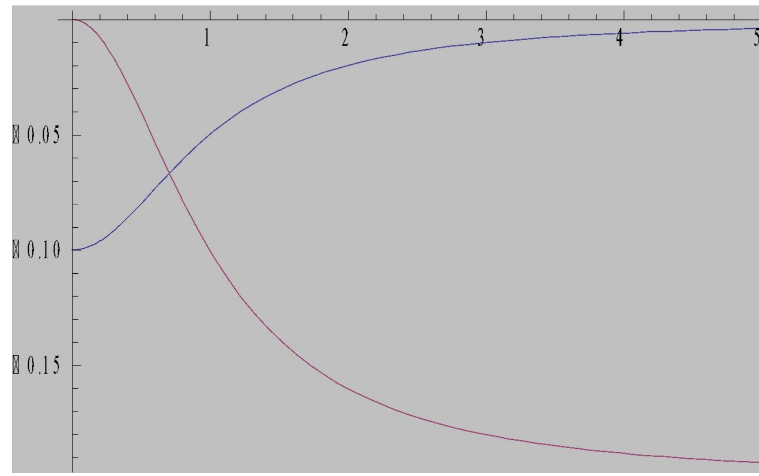


$$k(\omega_s) = \frac{\omega_s}{c}\sqrt{1 + i\frac{\omega_0^2}{\nu\omega_s}}$$

$$\omega_{Li} << \omega_{Le}: \qquad k^2 - \frac{\omega^2}{c^2}\left(1 - \frac{\omega_{Le}^2}{\omega(\omega + i\nu_e)}\right)$$

$$\nu_e << \omega_{Le}: \qquad \omega_{1,2} = \sqrt{k^2c^2 + \omega_{Le}^2} - i\frac{\nu_e}{2}\frac{\omega_{Le}^2}{k^2c^2 + \omega_{Le}^2}; \qquad \omega_3 = -i\nu_e\frac{k^2c^2}{k^2c^2 + \omega_{Le}^2}$$

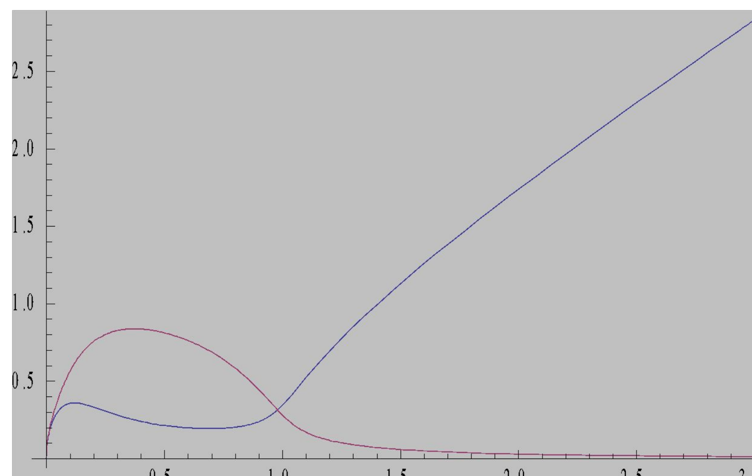




$$k = \frac{\omega}{c}\sqrt{1 - \frac{\omega_{Le}^2}{\omega(\omega + i\nu_e)}}$$

$$\omega^2 < \omega_{Le}^2: \quad \exp[-(\omega/c)\sqrt{\omega_{Le}^2/\omega^2 - 1}\ z], \ \ z > 0$$

$$|\omega| << \nu_e: \ k = \frac{i-1}{\sqrt{2}}\frac{\omega_{Le}}{c}\sqrt{\frac{\omega}{\nu_e}} = (i-1)\sqrt{\frac{2\pi\sigma(0)\omega}{c^2}}$$



$$1-\frac{\omega_{Li}^2}{\omega(\omega+i\nu_i)-k^2V_{\gamma i}^2}-\frac{\omega_{Le}^2}{\omega(\omega+i\nu_e)-k^2V_{\gamma e}^2}$$

$$|\omega|>\omega_{Le}:\ \omega_{1,2}=\pm\sqrt{\omega_{Le}^2+k^2V_{\gamma e}^2-\frac{\nu_e^2}{4}}-i\frac{\nu_e}{2}\ \Big|\ \nu_e^2>>\omega_{Le}^2+k^2V_{\gamma e}^2:\omega_1=-i\nu_e^{-1}(k^2V_{\gamma e}^2+\omega_{Le}^2),\ \omega_2=-i\nu_e$$

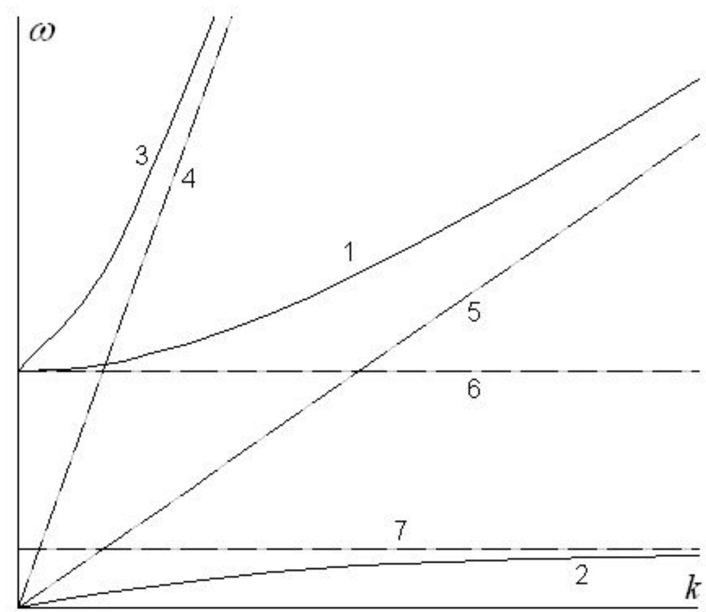
$$kV_{\gamma e}>>|\omega|>>kV_{\gamma i}:\ \ 1+\frac{\omega_{Le}^2}{k^2V_{\gamma e}^2}-\frac{\omega_{Li}^2}{\omega(\omega+i\nu_i)}=0$$

$$\omega_{1,2}=\pm\sqrt{\omega_{is}^2(k)-\frac{\nu_i^2}{4}}-i\frac{\nu_i}{2}$$

$$\omega_{is}(k)=\omega_{Li}\frac{kV_{\gamma e}}{\sqrt{\omega_{Le}^2+k^2V_{\gamma e}^2}}$$

$$\nu_i^2>>\omega_{is}^2(k):\ \omega_1=-i\nu_i^{-1}\omega_{is}^2(k),\ \omega_2=-i\nu_i$$

$$\omega\to 0:\quad k^2+1/r_D^2=0,\ r_D=\left(\frac{\omega_{Li}^2}{V_{\gamma i}^2}+\frac{\omega_{Le}^2}{V_{\gamma e}^2}\right)^{-1/2}$$



$$k_i \varepsilon_{ij}(\omega, \mathbf{k}) E_j(\omega, \mathbf{k}) = 0,$$

$$\left[k^2 \delta_{ij} - k_i k_j - \frac{\omega^2}{c^2} \varepsilon_{ij}(\omega, \mathbf{k}) \right] E_j(\omega, \mathbf{k}) = 0.$$

$$\det \left[k^2 \delta_{ij} - k_i k_j - \frac{\omega^2}{c^2} \varepsilon_{ij}(\omega, \mathbf{k}) \right]$$

$$\mathbf{k} = \{0, 0, k_z\}$$

$$\left(k_z^2 - \frac{\omega^2}{c^2} \varepsilon_{xx} \right) E_x - \frac{\omega^2}{c^2} \varepsilon_{xy} E_y = \frac{\omega^2}{c^2} \varepsilon_{xz} E_z,$$

$$\left(k_z^2 - \frac{\omega^2}{c^2} \varepsilon_{yy} \right) E_y - \frac{\omega^2}{c^2} \varepsilon_{yx} E_x = \frac{\omega^2}{c^2} \varepsilon_{yz} E_z,$$

$$\varepsilon_{zz} E_z = -\varepsilon_{zx} E_x - \varepsilon_{zy} E_y.$$

$$\varepsilon'_{ij} = \varepsilon'_{ji}, \quad \varepsilon''_{ij} = 0$$

$$D_x = \varepsilon_{xx} E_x, \; D_y = \varepsilon_{yy} E_y, \; D_z = \varepsilon_{zz} E_z$$

$$\left(k_z^2 + k_y^2 - \frac{\omega^2}{c^2} \varepsilon_{xx} \right) \left(k_z^2 + k_x^2 - \frac{\omega^2}{c^2} \varepsilon_{yy} \right) \left(k_x^2 + k_y^2 - \frac{\omega^2}{c^2} \varepsilon_{zz} \right) -$$

$$- k_z^2 k_y^2 \left(k_z^2 + k_y^2 - \frac{\omega^2}{c^2} \varepsilon_{xx} \right) - k_z^2 k_x^2 \left(k_z^2 + k_x^2 - \frac{\omega^2}{c^2} \varepsilon_{yy} \right) -$$

$$- k_x^2 k_y^2 \left(k_x^2 + k_y^2 - \frac{\omega^2}{c^2} \varepsilon_{zz} \right) - 2 k_x^2 k_y^2 k_z^2 = 0 .$$

$$k_y = 0 \qquad \left(k_z^2 + k_x^2 - \frac{\omega^2}{c^2}\varepsilon_{yy}\right)\left(\varepsilon_{zz}k_z^2 + \varepsilon_{xx}k_x^2 - \frac{\omega^2}{c^2}\varepsilon_{xx}\varepsilon_{zz}\right) = 0\,.$$

$$k_{z1} = \sqrt{\frac{\omega^2}{c^2}\varepsilon_{yy} - k_x^2}\,, \qquad k_{z2} = \sqrt{\frac{\omega^2}{c^2}\varepsilon_{xx} - k_x^2\frac{\varepsilon_{xx}}{\varepsilon_{zz}}}$$

$$\tan\alpha = k_x c \Big/ \sqrt{\omega^2 - k_x^2 c^2}$$

$$\tan\beta_{1,2} = k_x / k_{z1,2}$$

$$A_1 \exp(-i\omega t + i\mathbf{k}_1 \cdot \mathbf{r}) + A_2 \exp(-i\omega t + i\mathbf{k}_2 \cdot \mathbf{r})$$

$$\mathbf{k} = \{0,0,k_z\}$$

$$\varepsilon_{xx} = \varepsilon_{yy} = 1 - \frac{\omega_{Le}^2}{\omega^2 - \Omega_e^2} \equiv \varepsilon_\perp(\omega)\,,$$

$$\varepsilon_{xy} = -\varepsilon_{yx} = -i\frac{\omega_{Le}^2\Omega_e}{\omega\,(\omega^2 - \Omega_e^2)} \equiv ig(\omega)\,,$$

$$\varepsilon_{zz} = 1 - \frac{\omega_{Le}^2}{\omega^2} \equiv \varepsilon_\parallel(\omega). \qquad \varepsilon_{ij} = \varepsilon_{ji}^*$$

$$\left(k_z^2 - \frac{\omega^2}{c^2}\varepsilon_\perp(\omega)\right)E_x - i\frac{\omega^2}{c^2}g(\omega)E_y = 0\,,$$

$$\left(k_z^2 - \frac{\omega^2}{c^2}\varepsilon_\perp(\omega)\right)E_y + i\frac{\omega^2}{c^2}g(\omega)E_x = 0\,,$$

$$\varepsilon_\parallel(\omega)E_z = 0\,.$$

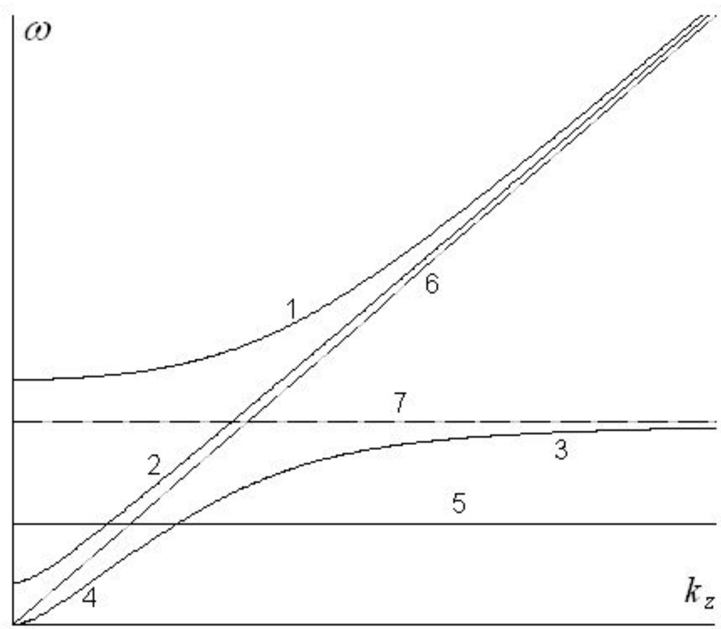
$$k_z^2 - \frac{\omega^2}{c^2}\varepsilon_\perp(\omega) = \pm\frac{\omega^2}{c^2}g(\omega) \rightarrow (\omega^2 - k_z^2c^2)(\omega \mp \Omega_e) = \omega\omega_{Le}^2$$

$$E_x / E_y = \pm i \quad \rightarrow \quad E_x(t,z) = \pm A\sin(\omega t - k_z z + \phi),\; E_y(t,z) = A\cos(\omega t - k_z z + \phi)$$

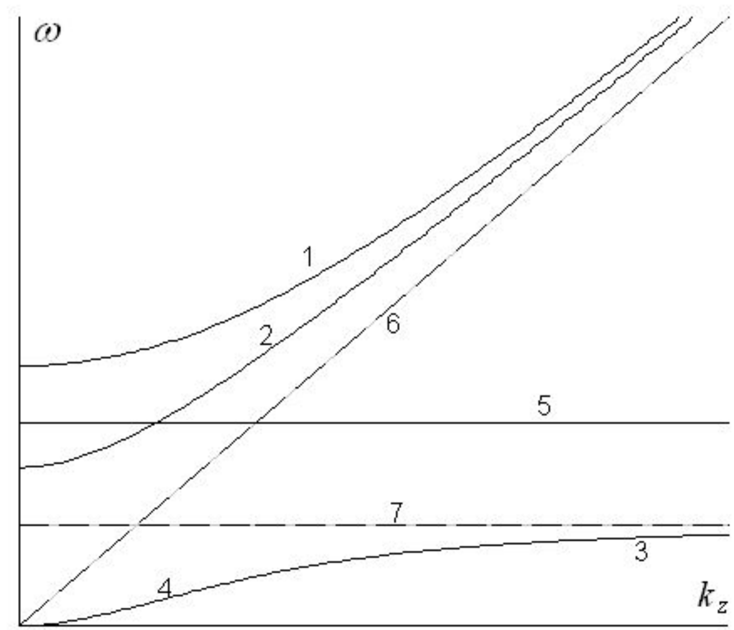
$$\omega = \begin{cases} \sqrt{k_z^2 c^2 + \omega_{Le}^2 \dfrac{k_z c}{k_z c \mp \Omega_e}}\,, & \omega_{Le}^2 \ll k_z c\,|\,k_z c \mp \Omega_e\,| \\ \sqrt{\omega_{Le}^2 + \Omega_e^2/4} \pm \Omega_e/2\,, & k_z c \ll \omega \end{cases}$$

$$\omega = \Omega_e + \Omega_e \frac{\omega_{Le}^2}{\Omega_e^2 - k_z^2 c^2}\,, \quad \omega_{Le}^2 \ll |\,\Omega_e^2 - k_z^2 c^2\,|$$

$$\omega = k_z^2 c^2 \frac{\Omega_e}{\omega_{Le}^2}\,, \; k_z^2 c^2 \ll \omega_{Le}^2$$



$\Omega_e > \omega_{Le}$



$\Omega_e < \omega_{Le}$

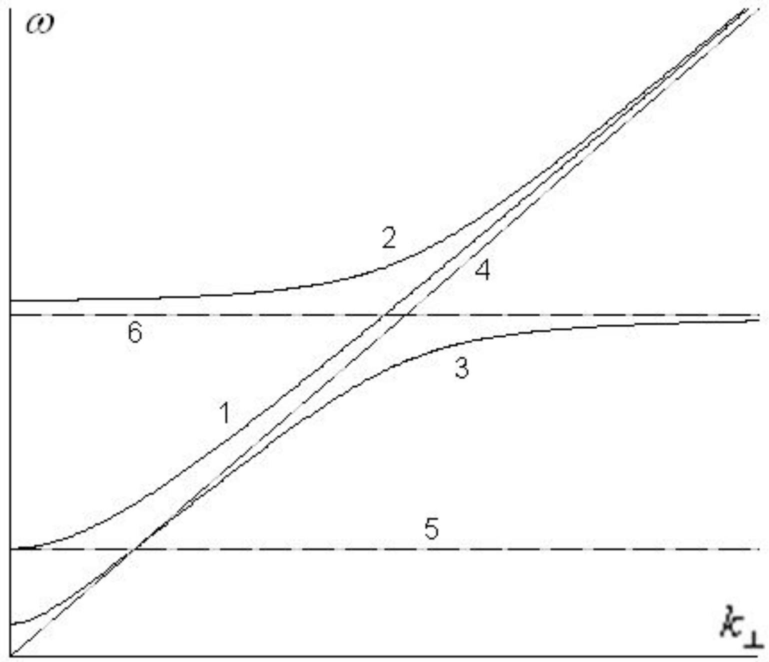
$$\mathbf{k} = \{k_x, 0, 0\}$$

$$\varepsilon_\perp(\omega)E_x + ig(\omega)E_y = 0\,,$$

$$\left(k_\perp^2 - \frac{\omega^2}{c^2}\varepsilon_\perp(\omega)\right)E_y = -i\frac{\omega^2}{c^2}g(\omega)E_x\,,$$

$$\left(k_\perp^2 - \frac{\omega^2}{c^2}\varepsilon_\parallel(\omega)\right)E_z = 0\,. \quad \longrightarrow \quad \omega = \sqrt{k_x^2c^2 + \omega_{Le}^2}$$

$$k_x^2 - \frac{\omega^2}{c^2}\frac{\varepsilon_\perp^2(\omega) - g^2(\omega)}{\varepsilon_\perp(\omega)} = 0 \rightarrow (\omega^2 - \omega_{Le}^2)^2 - \omega^2\Omega_e^2 = k_x^2c^2(\omega^2 - \Omega_g^2)$$



$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_\perp & ig & 0 \\ -ig & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix} \qquad k_z^2 - \frac{\omega^2}{c^2}(\varepsilon_\perp \pm g) = 0$$

$$E_x / E_y = \pm i$$

$$E_x(t,z) = \pm A \sin(\omega t - k_z z + \phi)$$

$$E_y(t,z) = A \cos(\omega t - k_z z + \phi)$$

$$k_{z1}(\omega) = \frac{\omega}{c}\sqrt{\varepsilon_\perp + g},$$

$$k_{z2}(\omega) = \frac{\omega}{c}\sqrt{\varepsilon_\perp - g}.$$

$$E_{x1}(t,z) = A_1 \sin(\omega t - k_{z1} z + \phi_1),$$

$$E_{y1}(t,z) = A_1 \cos(\omega t - k_{z1} z + \phi_1),$$

$$E_{x2}(t,z) = -A_2 \sin(\omega t - k_{z2} z + \phi_2),$$

$$E_{y2}(t,z) = A_2 \cos(\omega t - k_{z2} z + \phi_2).$$

$$E_{x1}(t,0) + E_{x1}(t,0) = 0,$$

$$E_{y1}(t,0) + E_{y1}(t,0) = A_0 \cos \omega t,$$

$$E_x(t,z) = E_{x1}(t,z) + E_{x2}(t,z) = A_0 \cos\left(\omega t - \frac{k_{z1}+k_{z2}}{2} z\right)\sin\left(\frac{k_{z1}-k_{z2}}{2} z\right),$$

$$E_y(t,z) = E_{y1}(t,z) + E_{y2}(t,z) = A_0 \cos\left(\omega t - \frac{k_{z1}+k_{z2}}{2} z\right)\cos\left(\frac{k_{z1}-k_{z2}}{2} z\right).$$

$$\theta = \frac{1}{2}(k_{z1} - k_{z2})z = \frac{\omega}{c} g\left(\sqrt{\varepsilon_\perp + g} + \sqrt{\varepsilon_\perp - g}\right)^{-1} z \qquad \theta = \frac{1}{2}\frac{\omega}{c} g z$$

$$\theta = -\frac{\Omega_e}{c}\frac{\omega_{Le}^2}{\omega^2 - \Omega_e^2}\left(\sqrt{1 - \frac{\omega_{Le}^2}{\omega(\omega - \Omega_e)}} + \sqrt{1 - \frac{\omega_{Le}^2}{\omega(\omega + \Omega_e)}}\right)^{-1} z \qquad \theta = -\frac{1}{2}\frac{\Omega_e}{c}\frac{\omega_{Le}^2}{\omega^2} z$$