Лекция 6.
Математические методы физики волновых явлений – применение

1. Поперечные электромагнитные волны в изотропном диэлектрике

2. Продольные электромагнитные волны в изотропной плазме

3. Электромагнитные волны в металлах
   и поперечные электромагнитные волны в плазме

4. Продольные ионнозвуковые волны в неизотермической изотропной плазме

5. Продольно-поперечные волны в анизотропной плазме

$$\frac{\partial E_x}{\partial t} + \frac{c}{\varepsilon_0}\frac{\partial B_y}{\partial z} = 0,$$

$$\frac{\partial B_y}{\partial t} + c\frac{\partial E_x}{\partial z} - k_\perp c E_z = 0,$$

$$\frac{\partial E_z}{\partial t} + \frac{1}{\varepsilon_0} k_\perp c B_y = 0.$$

$$\boxed{\mathbf{\Psi}(t,z) = \{E_x, B_y, E_z\} = \{e_x(\omega,k), b_y(\omega,k), e_z(\omega,k)\}\exp(-i\omega t + ikz),}$$

$$\longrightarrow$$

$$-i\omega\, e_x + ik(c/\varepsilon_0) b_y = 0,$$

$$ikc\, e_x - i\omega\, b_y - k_\perp c e_z = 0,$$

$$k_\perp (c/\varepsilon_0) b_y - i\omega\, e_z = 0,$$

$$D(\omega,k) \equiv -\omega^2 + k^2 c_0^2 + k_\perp^2 c_0^2 = 0$$

$$\omega_1 = \sqrt{k^2 c_0^2 + k_\perp^2 c_0^2}, \quad \omega_2 = -\sqrt{k^2 c_0^2 + k_\perp^2 c_0^2}$$

$$\mathbf{\Psi}(t,z) = \{E_x, B_y\} = \sum_{m=1}^{2} A_m \left\{1, \frac{\varepsilon_0 \omega_m}{kc}\right\} \exp[-i\omega_m(k)t + ikz]$$

$$\left(\frac{\partial^2}{\partial t^2} - c^2\frac{\partial^2}{\partial z^2} + k_\perp^2 c_0^2\right) A(t,z) = 0$$

$$\mathbf{\Psi}(t,z) = \{E_x(t,z), B_y(t,z)\} = \left\{A(t,z),\; -\frac{\varepsilon_0}{c}\int \frac{\partial A}{\partial t}(t,z)\,dz\right\}$$

$$\frac{\partial j_e}{\partial t} - \frac{\omega_{Le}^2}{4\pi} E_z = 0,$$

$$\frac{\partial E_z}{\partial t} + 4\pi\, j_e = 0,$$

$$D(\omega,k) \equiv -\omega^2 + \omega_{Le}^2 = 0$$

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{Le}^2\right) A(t,z) = 0$$

$$\frac{\partial j_e}{\partial t} - \frac{\omega_{Le}^2}{4\pi} E_z + \nu_e j_e = 0 \qquad D(\omega,k) \equiv -\omega^2 - i\nu_e\omega + \omega_{Le}^2 = 0$$

$$\left(\frac{\partial^2}{\partial t^2} + \nu_e\frac{\partial}{\partial t} + \omega_{Le}^2\right) A(t,z) = 0$$

$$\omega_1 = \omega'_{Le} - i\frac{\nu_e}{2}, \quad \omega_2 = -\omega'_{Le} - i\frac{\nu_e}{2}, \quad \omega'_{Le} = \sqrt{\omega_{Le}^2 - \frac{\nu_e^2}{4}} \quad \xrightarrow{2\omega_{Le} \ll \nu_e} \quad \omega_1 = -i\omega_{Le}\frac{\omega_{Le}}{\nu_e}, \quad \omega_2 = -i\nu_e$$

$$D(\omega,k) \equiv -i\nu_e\omega + \omega_{Le}^2 = 0 \qquad \left(\frac{\partial}{\partial t} + \omega_{Le}\frac{\omega_{Le}}{\nu_e}\right) A(t,z) = 0$$

$$\frac{\partial j_e}{\partial t} - \frac{\omega_{Le}^2}{4\pi} E_z + V_{\gamma e}^2 \frac{\partial \rho_e}{\partial z} = 0, \qquad \mathbf{\Psi}(t,z) = \{ j_e, E_z, \rho_e \} = \{ j(\omega,k), e_z(\omega,k), r(\omega,k) \} \exp(-i\omega t + ikz),$$

$$\frac{\partial E_z}{\partial t} + 4\pi\ j_e = 0,$$

$$\frac{\partial \rho_e}{\partial t} + \frac{\partial j_e}{\partial z} = 0\,,$$

$$\longrightarrow$$

$$\left(\omega_{Le}^2 / 4\pi\right) e_z + i\omega\ j - iV_{\gamma e}^2 k r = 0,$$

$$-i\omega\, e_z + 4\pi\ j = 0,$$

$$kj - \omega\ r = 0,$$

$$D(\omega,k) \equiv -\omega^2 + k^2 V_{Te}^2 + \omega_p^2 = 0 \qquad \left(\frac{\partial^2}{\partial t^2} - V_{\gamma e}^2 \frac{\partial^2}{\partial z^2} + \omega_p^2 \right) A(t,z) = 0$$

$$\frac{\partial j_e}{\partial t} - \frac{\omega_{Le}^2}{4\pi} E_z + V_{\gamma e}^2 \frac{\partial \rho_e}{\partial z} + \nu_e j_e = 0 \qquad D(\omega,k) \equiv -\omega(\omega + i\nu_e) + k^2 V_{\gamma e}^2 + \omega_{Le}^2 = 0$$

$$\nu_e << \omega_{Le} \qquad\qquad \nu_e >> \omega_{Le}$$

$$\omega_{1,2} = \pm\sqrt{k^2 V_{\gamma e}^2 + \omega_{Le}^2} - i\frac{\nu_e}{2} \qquad \omega_1 = -i\nu_e^{-1}(k^2 V_{\gamma e}^2 + \omega_{Le}^2), \ \ \omega_2 = -i\nu_e$$

$$\boxed{\frac{\partial N_e}{\partial t} = D_e \frac{\partial^2 N_e}{\partial z^2}} \quad \boxed{D_e = \frac{V_{Te}^2}{\nu_e} = V_{Te} l_e, \ \ l_e = \tau_e V_{Te}, \quad \tau_e = \frac{1}{\nu_e}} \quad \left(\frac{\partial}{\partial t} - \frac{V_{\gamma e}^2}{\nu_e} \frac{\partial^2}{\partial z^2} + \frac{\omega_{Le}^2}{\nu_e} \right) A(t,z) = 0$$

$$\frac{\partial B_y}{\partial z} + \frac{4\pi\sigma_0\mu_0}{c} E_x = 0,$$

$$\frac{\partial B_y}{\partial t} + c\frac{\partial E_x}{\partial z} = 0,$$

$$\frac{\partial E_z}{\partial z} - \frac{4\pi}{\varepsilon_0}\rho = 0,$$

$$\frac{\partial \rho}{\partial t} + \sigma_0 \frac{\partial E_z}{\partial z} = 0,$$

$$\frac{\partial \rho}{\partial t} + \frac{4\pi\sigma_0}{\varepsilon_0}\rho = 0 \quad \rho(t,z) = \rho_0(z)\exp\left(-\frac{t}{\tau}\right), \quad \tau = \frac{\varepsilon_0}{4\pi\sigma_0}.$$

$$D(\omega,k)\equiv\omega+ik^2c^2\frac{1}{4\pi\sigma_0\mu_0}=0 \qquad k=\pm(1+i)\delta_{sk}^{-1},\quad \delta_{sk}=\frac{c}{\sqrt{2\pi\,\mu_0\sigma_0\omega}} \qquad \exp[(i-1)\,z/\delta_{sk}],\ z\geq 0$$

$$\left(\frac{\partial}{\partial t}-\frac{c^2}{4\pi\sigma\mu_0}\frac{\partial^2}{\partial z^2}\right)A(t,z)=0 \qquad A(t,z)=A_0\exp\left(-k^2c^2(4\pi\sigma_0\mu_0)^{-1}t-kz\right)$$

$$\frac{\partial E_x}{\partial t}+c\frac{\partial B_y}{\partial z}+4\pi\, j_{ex}=0,$$

$$\frac{\partial B_y}{\partial t}+c\frac{\partial E_x}{\partial z}=0, \qquad D(\omega,k)\equiv-\omega^2+k^2c^2+\omega_{Le}^2=0$$

$$\frac{\partial j_{ex}}{\partial t}-\frac{\omega_{Le}^2}{4\pi}E_x=0.$$

$$\frac{\partial j_{ex}}{\partial t}-\frac{\omega_{Le}^2}{4\pi}E_x+\nu_e j_{ex}=0 \qquad D(\omega,k)\equiv-\omega^2+k^2c^2+\omega_p^2\frac{\omega}{\omega+i\nu_{en}}=0$$

$$\omega_{1,2}=\pm\sqrt{k^2c^2+\omega_{Le}^2}-i\frac{\nu_e}{2}\frac{\omega_{Le}^2}{k^2c^2+\omega_{Le}^2} \qquad \omega_3=-i\nu_e\frac{k^2c^2}{k^2c^2+\omega_{Le}^2}\approx\begin{cases}-i\nu_e\,, & k^2c^2>>\omega_{Le}^2\\ -i\nu_e\,k^2c^2/\omega_{Le}^2\,, & k^2c^2<<\omega_{Le}^2\end{cases}$$

$$\left(\frac{\partial}{\partial t}-\nu_e\frac{c^2}{\omega_{Le}^2}\frac{\partial^2}{\partial z^2}\right)A(t,z)=0$$

$$\frac{\partial\rho_i}{\partial t}+e_i n_{0i}\frac{\partial V_i}{\partial z}=0,\quad \mathbf{\Psi}(t,z)=\{\rho_i,V_i,E_z,\rho_e\}=\{r_i(\omega,k),u_i(\omega,k),e_z(\omega,k),r_e(\omega,k)\}\exp(-i\omega t+ikz),$$

$$\frac{\partial V_i}{\partial t}-\frac{e_i}{M}E_z=0,$$

$$\frac{\partial E_z}{\partial z}-4\pi\rho_e-4\pi\rho_i=0,$$

$$\frac{\partial\rho_e}{\partial z}-V_{\gamma e}^{-2}\frac{e^2 n_{0e}}{m}E_z=0.$$

$$\omega r_i+en_{0e}ku_i=0,$$

$$i\omega u_i-Z(e/M)e_z=0,$$

$$4\pi r_i-ike_z+4\pi r_e=0,$$

$$V_{\gamma e}^{-2}(e^2n_{0e}/m)e_z-ikr_e=0,$$

$$D(\omega,k)\equiv\omega^2(k^2+r_{De}^{-2})-k^2\omega_{Li}^2=0\qquad \omega_{1,2}=\pm\omega_{Li}\frac{kr_{De}}{\sqrt{1+k^2r_{De}^2}},\quad V_{\Phi}^{(1,2)}=\pm\frac{\omega_{Li}r_{De}}{\sqrt{1+k^2r_{De}^2}}$$

$$\left(\frac{\partial^2}{\partial t^2}-V_S^2\frac{\partial^2}{\partial z^2}-r_{De}^2\frac{\partial^2}{\partial t^2}\frac{\partial^2}{\partial z^2}\right)A(t,z)=0$$

$$\frac{\partial V_i}{\partial t}-\frac{e_i}{M}E_z+\nu_iV_i=0\qquad D(\omega,k)\equiv\omega(\omega+i\nu_i)(k^2+r_{De}^{-2})-k^2\omega_{Li}^2=0$$

$$\nu_i\gg\omega_{Li}\qquad \omega_1=-i\frac{k^2r_{De}^2}{1+k^2r_{De}^2}\frac{\omega_{Li}^2}{\nu_i},\quad \omega_2=-i\nu_i\qquad \left(\frac{\partial}{\partial t}-r_{De}^2\frac{\omega_{Li}^2}{\nu_i}\frac{\partial^2}{\partial z^2}-r_{De}^2\frac{\partial}{\partial t}\frac{\partial^2}{\partial z^2}\right)A(t,z)=0$$

$$\left(\frac{\partial}{\partial t}-D_A\frac{\partial^2}{\partial z^2}\right)A(t,z)=0,\quad D_A=r_{De}^2\frac{\omega_{Li}^2}{\nu_i}\qquad D_A=\frac{(T_e+T_i)D_eD_i}{T_iD_e+T_eD_i}\quad D_{e,i}=\frac{V_{Te,i}^2}{\nu_{e,i}}$$

$$\frac{\partial E_x}{\partial t} + c\frac{\partial B_y}{\partial z} = 0,$$

$$\frac{\partial B_y}{\partial t} + c\frac{\partial E_x}{\partial z} - k_\perp c E_z = 0,$$

$$\frac{\partial J_p}{\partial t} - \frac{\omega_{Le}^2}{4\pi} E_z = 0,$$

$$\frac{\partial E_z}{\partial t} + 4\pi j_z + k_\perp c B_y = 0.$$

$$\overset{\sqcup\!\sqcup}{\Psi}(t,z) = \left\{ e_x(\omega,k), b_y(\omega,k), j(\omega,k), e_z(\omega,k) \right\} \exp(-i\omega t + ikz),$$

$$-i\omega\, e_x + ikc\, b_y = 0,$$

$$ikc\, e_x - i\omega\, b_y - k_\perp c e_z = 0,$$

$$-i\omega\, j - \left(\omega_{Le}^2 / 4\pi\right) e_z = 0,$$

$$k_\perp c b_y + 4\pi\, j - i\omega e_z = 0,$$

$$D(\omega,k) \equiv -\left(k_\perp^2 c^2 \omega^2 + (k^2c^2 - \omega^2)(\omega^2 - \omega_{Le}^2)\right) = 0$$

$$\omega_1 = \Omega_1, \quad \omega_2 = -\Omega_1, \quad \omega_3 = \Omega_2, \quad \omega_4 = -\Omega_2;$$

$$\Omega_{1,2}^2 = \frac{1}{2}\left\{ \left(k^2 + k_\perp^2\right)c^2 + \omega_p^2 \pm \sqrt{\left[\left(k^2 + k_\perp^2\right)c^2 + \omega_p^2\right]^2 - 4k^2c^2\omega_p^2} \right\}.$$

$$\Psi(t,z) = \left\{E_x, B_y, j_z, E_z\right\} = \sum_{m=1}^{4} A_m \left\{ 1,\ \frac{\omega_m}{kc},\ \frac{1}{4\pi}\omega_m \frac{k_\perp}{k}\frac{\omega_{Le}^2}{\omega_m^2 - \omega_{Le}^2},\ -i\frac{k_\perp}{k}\frac{\omega_m^2}{\omega_m^2 - \omega_{Le}^2} \right\} \exp\left[-i\omega_m(k)t + ikz\right]$$

$$A_1 \begin{pmatrix} 1 \\ \dfrac{\Omega_1}{kc} \\ \dfrac{1}{4\pi}\dfrac{k_\perp \omega_{Le}^2}{k\Omega_1\varepsilon_1} \\ -i\dfrac{k_\perp}{k\varepsilon_1} \end{pmatrix} + A_2 \begin{pmatrix} 1 \\ -\dfrac{\Omega_1}{kc} \\ -\dfrac{1}{4\pi}\dfrac{k_\perp \omega_{Le}^2}{k\Omega_1\varepsilon_1} \\ -i\dfrac{k_\perp}{k\varepsilon_1} \end{pmatrix} + A_3 \begin{pmatrix} 1 \\ \dfrac{\Omega_2}{kc} \\ \dfrac{1}{4\pi}\dfrac{k_\perp \omega_{Le}^2}{k\Omega_2\varepsilon_2} \\ -i\dfrac{k_\perp}{k\varepsilon_2} \end{pmatrix} + A_4 \begin{pmatrix} 1 \\ -\dfrac{\Omega_2}{kc} \\ -\dfrac{1}{4\pi}\dfrac{k_\perp \omega_{Le}^2}{k\Omega_2\varepsilon_2} \\ -i\dfrac{k_\perp}{k\varepsilon_2} \end{pmatrix} = \begin{pmatrix} E_{x0} \\ B_{y0} \\ J_{p0} \\ E_{z0} \end{pmatrix}.$$

$$D(\omega,k)\equiv\left(k^2c^2-\omega^2\right)\left(\omega^2-\omega_{Le}^2\right)+k_\perp^2c^2\omega^2=0$$

$$\left[\left(\frac{\partial^2}{\partial t^2}-c^2\frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial^2}{\partial t^2}+\omega_{Le}^2\right)+k_\perp^2c^2\frac{\partial^2}{\partial t^2}\right]A(t,z)=0$$

$$D(\omega,k)\equiv\omega^2(k^2+k_\perp^2)-k^2\omega_{Le}^2=0 \qquad \omega_1=\omega_{Le}\frac{k}{\sqrt{k^2+k_\perp^2}},\qquad \omega_2=-\omega_{Le}\frac{k}{\sqrt{k^2+k_\perp^2}}$$

$$\left(\frac{\partial^2}{\partial t^2}-\frac{\omega_p^2}{k_\perp^2}\frac{\partial^2}{\partial z^2}-k_\perp^{-2}\frac{\partial^2}{\partial t^2}\frac{\partial^2}{\partial z^2}\right)A(t,z)=0$$