Лекция 9.
Плоские гармонические волны в неравновесных системах

1. Волны в одномерном пучке электронов
   а. Быстрая и медленная волны плотности заряда
   б. Энергии импульсы волн плотности заряда пучка

2. Пучковая неустойчивость в плазме
   а. Дисперсионное уравнение пучковой неустойчивости в плазме
   б. Нерезонансная и резонансная пучково-плазменные неустойчивости
   в. Начальная задача для волн в пучково-плазменной системе
   г. Дифференциальное уравнение пучково-плазменного взаимодействия

3. Неустойчивость плазмы с током
   а. Дисперсионное уравнение для частот волн в холодной плазме с током
   б. Нерезонансная и резонансная неустойчивости Бунемана

$$\frac{\partial j_{b1}}{\partial t} + u\frac{\partial j_{b1}}{\partial z} - \frac{\omega_{Lb}^2}{4\pi}E_z = 0,$$

$$\frac{\partial j_{b2}}{\partial t} + u\frac{\partial j_{b2}}{\partial z} + u\frac{\partial j_{b1}}{\partial z} = 0,$$

$$\frac{\partial E_z}{\partial t} + 4\pi j_{b1} + 4\pi j_{b2} = 0.$$

$$\mathbf{\Psi}(t,z) = \{ j_{b1}, j_{b2}, E_z \} = \{ j_1(\omega,k), j_2(\omega,k), e_z(\omega,k)\}\exp(-i\omega t + ikz)$$

$$-i(\omega - ku)j_1 - \left(\omega_{Lb}^2/4\pi\right)e_z = 0,$$

$$-i(\omega - ku)j_2 + ikuj_1 = 0,$$

$$-i\omega e_z + 4\pi j_1 + 4\pi j_2 = 0.$$

$$D(\omega,k) \equiv -(\omega - ku)^2 + \omega_{Lb}^2 = 0$$

$$\omega_1 = ku + \omega_{Lb}, \quad \omega_2 = ku - \omega_{Lb}$$

$$\mathbf{\Psi}(t,z) = \left\{ A_1\begin{pmatrix}1\\ -i4\pi/\omega_{Lb}\end{pmatrix}\exp(-i\omega_{Lb}t) + A_2\begin{pmatrix}1\\ i4\pi/\omega_{Lb}\end{pmatrix}\exp(+i\omega_{Lb}t)\right\}\exp[ik(z-ut)]$$

$$\left[\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial z}\right)^2 + \omega_{Lb}^2\right]A(t,z) = 0 \qquad \mathbf{\Psi}(t,z) = \{J_{b1}, E_z\} = \left\{A(t,z), \left(4\pi/\omega_{Lb}^2\right)\hat{L}A(t,z)\right\}$$

$$A(t,z) = f_1(z-ut)\exp(-i\omega_{Lb}t) + f_2(z-ut)\exp(i\omega_{Lb}t)$$

$$j(\omega,k) = j_1(\omega,k) + j_2(\omega,k) = i\frac{\omega_{Lb}^2}{4\pi}\frac{\omega}{(\omega-ku)^2}e_z(\omega,k)$$

$$\varepsilon^l(\omega,k) = 1 - \frac{\omega_{Lb}^2}{(\omega-ku)^2}$$

$$W_l(k) = \left(1 + \frac{\omega_{Lb}^2(\omega+ku)}{(\omega-ku)^3}\right)\frac{|e_z(k)|^2}{16\pi} \qquad W_l^{(1,2)}(k) = \pm\frac{\omega(k)}{\omega_{Lb}}\frac{|e_z(k)|^2}{8\pi}$$

$$W_E = \frac{E_z^2}{8\pi} \qquad W_K = \frac{m}{2}(n_{0b} + \tilde{n})(u + \tilde{u})^2 - \frac{m}{2}n_{0b}u^2 \qquad W = \frac{1}{T}\int_0^T (W_E + W_K)dt$$

$$P = m(n_{0b} + \tilde{n})(u + \tilde{u}) - mn_{0b}u = mu\tilde{n} + mn_{0b}\tilde{u} + m\tilde{n}\tilde{u}$$

$$P = \pm\frac{k}{\omega_{Lb}^2}\frac{|e_z(\omega,k)|^2}{8\pi} = \frac{k}{\omega}W$$

$$\frac{\partial j_{b1}}{\partial t} + u\frac{\partial j_{b1}}{\partial z} - \frac{\omega_{Lb}^2}{4\pi}E_z = 0,$$

$$\frac{\partial j_{b2}}{\partial t} + u\frac{\partial j_{b2}}{\partial z} + u\frac{\partial j_{b1}}{\partial z} = 0,$$

$$\frac{\partial j_e}{\partial t} - \frac{\omega_{Le}^2}{4\pi}E_z = 0,$$

$$\frac{\partial E_z}{\partial t} + 4\pi\, j_{b1} + 4\pi\, j_{b2} + 4\pi\, j_e = 0.$$

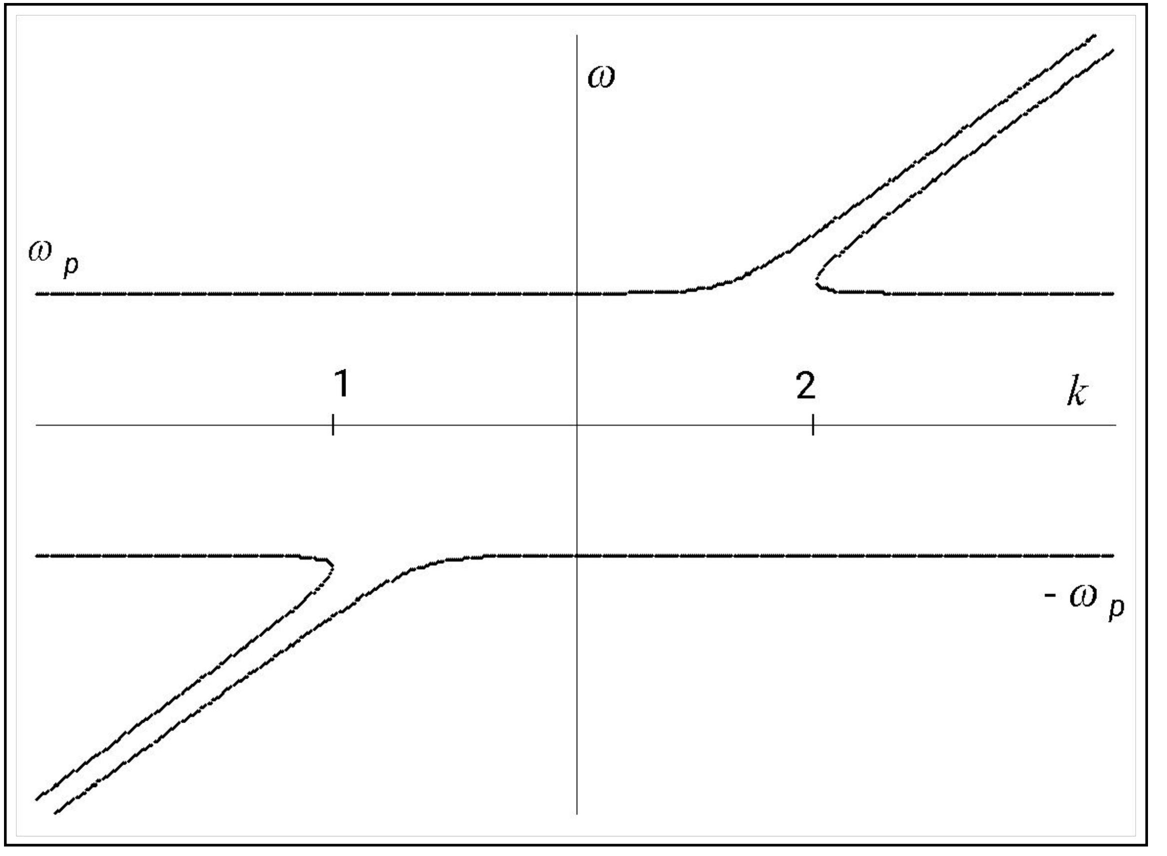
$$\omega_4 = -\omega_{Le}$$

$$\omega = ku + \delta\omega, \qquad |\delta\omega| << ku,\ \omega_{Le}.$$

$$\left(1 - \frac{\omega_{Le}^2}{k^2u^2}\right) + 2\left(\frac{\omega_{Le}}{ku}\right)^3 \frac{\delta\omega}{\omega_{Le}} - \frac{\omega_{Lb}^2}{\delta\omega^2} = 0$$

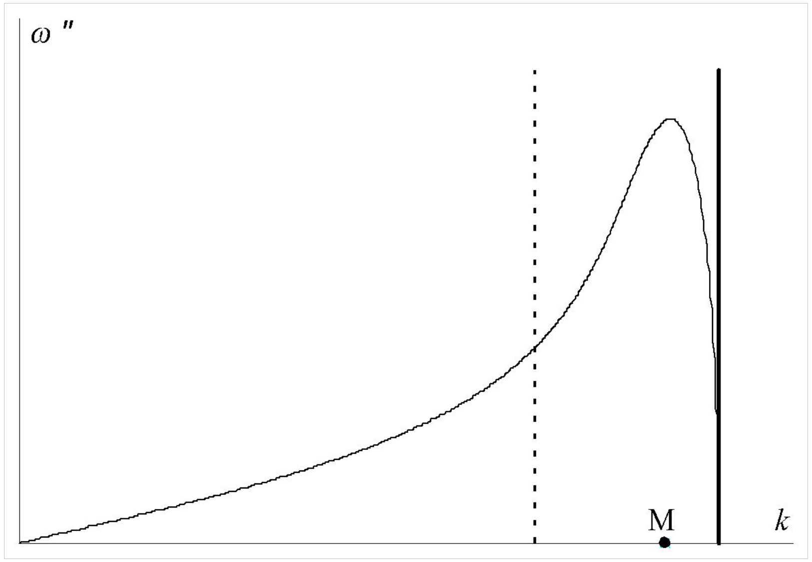
$$D(\omega,k) \equiv \left(\omega^2 - \omega_{Le}^2\right)\left(\omega - ku\right)^2 - \omega^2\omega_{Lb}^2 = 0$$

$$\omega_{Lb}^2 << \omega_{Le}^2$$



$$\begin{cases} \omega_{1,2} = ku \pm \begin{cases} i\dfrac{\omega_{Lb}}{\sqrt{\omega_{Le}^2 - k^2u^2}}ku, & ku < \omega_{Le}, \\ \dfrac{\omega_{Lb}}{\sqrt{k^2u^2 - \omega_{Le}^2}}ku, & ku > \omega_{Le}, \end{cases} \\ \omega_3 = \omega_{Le}. \end{cases}$$

$$\begin{cases} \omega_{1,2} = ku + \left(-\dfrac{1}{2} \pm i\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{\omega_{Lb}^2}{2\omega_{Le}^2}\right)^{1/3}\omega_{Le}, \\ \omega_3 = ku + \left(\dfrac{\omega_{Lb}^2}{2\omega_{Le}^2}\right)^{1/3}\omega_{Le}. \end{cases}$$



$$\mathbf{\Psi}(t,z)=\begin{pmatrix} j_{b1}\\ j_{b2}\\ j_e\\ E_z\end{pmatrix}=\sum_{m=1}^{4}A_m\begin{pmatrix} j_1\\ j_2\\ j\\ e_z\end{pmatrix}_{(m)}\exp[-i\omega_m(k)t+ikz]$$

$$\{\,j_1,j_2,j,e_z\,\}_{(m)}=\left\{1,\ ku\Delta_m^{-1},\ \omega_m^{-1}\frac{\omega_{Le}^2}{\omega_{Lb}^2}\Delta_m,\ -i\frac{4\pi}{\omega_{Lb}^2}\Delta_m\right\}$$

$$\Delta_m=\omega_m-ku$$

$$j_{b1}(t,z)=j_{10}\exp[ik(z-ut)]\begin{cases}\mathbf{ch}(|\Delta|t), & ku<\omega_{Le},\\ \cos(|\Delta|t), & ku>\omega_{Le}.\end{cases}\qquad j_{b1}(t,z)=\frac{1}{3}j_{10}\exp[ik(z-ut)]\sum_{m=1}^{3}\exp(-i\delta_m\Delta_0t),$$

$$j_{b2}(t,z)=j_{10}\frac{ku}{\Delta}\exp[ik(z-ut)]\begin{cases}\mathbf{sh}(|\Delta|t), & ku<\omega_{Le},\\ \sin(|\Delta|t), & ku>\omega_{Le}.\end{cases}\qquad j_{b2}(t,z)=\frac{1}{3}j_{10}\frac{ku}{\Delta_0}\exp[ik(z-ut)]\sum_{m=1}^{3}\delta_m^*\exp(-i\delta_m\Delta_0t).$$

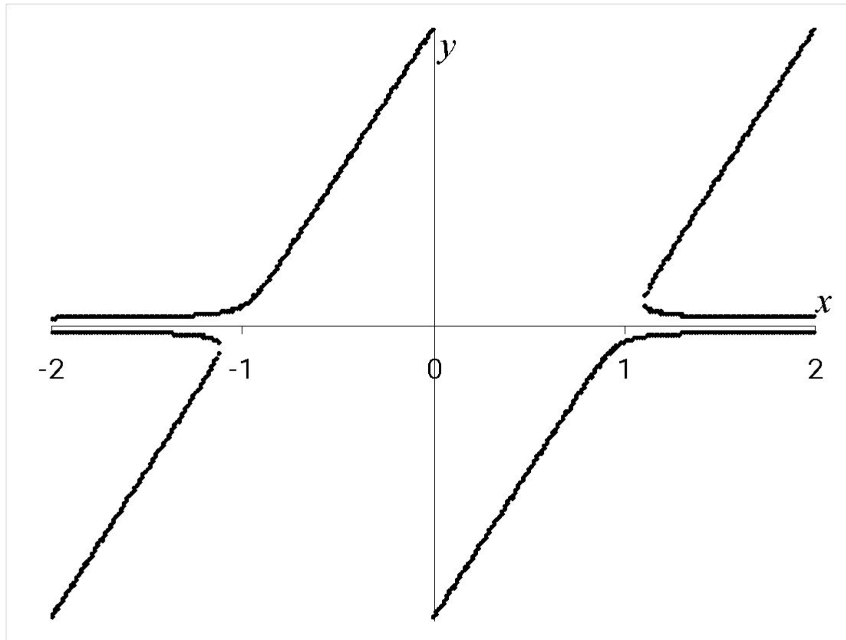
$$\Delta=i\frac{\omega_{Lb}}{\sqrt{\omega_{Le}^2-k^2u^2}}ku\qquad \Delta_0=\left(\frac{\omega_{Lb}^2}{2\omega_{Le}^2}\right)^{1/3}\omega_{Le}\qquad \delta_1=\frac{-1+i\sqrt{3}}{2},\quad \delta_2=\frac{-1-i\sqrt{3}}{2},\quad \delta_3=1.$$

$$\left[\left(\frac{\partial^2}{\partial t^2}+\omega_{Le}^2\right)\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial z}\right)^2+\omega_{Lb}^2\frac{\partial^2}{\partial t^2}\right]A(t,z)=0$$

$$\mathbf{\Psi}(t,z)=\{j_{b1},j_{b2},j_e,E_z\}=$$
$$=\left\{\omega_{Lb}^2\hat{L}A,\ -\omega_{Lb}^2u\frac{\partial A}{\partial z},\ \omega_{Le}^2\int\hat{L}^2A\,dt,\ 4\pi\hat{L}^2A\right\}.$$

$$D(\omega,k) \equiv (\omega^2 - \omega_{Li}^2)(\omega - ku)^2 - \omega^2\omega_{Le}^2 = 0$$

$$\varepsilon^l(\omega,k) = 1 - \frac{\omega_{Li}^2}{\omega^2} - \frac{\omega_{Le}^2}{(\omega - ku)^2} \qquad \varepsilon^l(\omega,k) = 1 - \frac{\omega_{Le}^2}{\omega^2} - \frac{\omega_{Lb}^2}{(\omega - ku)^2}$$



$$\omega_{1,2} = \left(\frac{1}{2} \pm i\frac{\sqrt{3}}{2}\right)\left(\frac{\omega_{Li}^2}{2\omega_{Le}^2}\right)^{1/3}\omega_{Le},$$

$$\omega_3 = -\left(\frac{\omega_{Li}^2}{2\omega_{Le}^2}\right)^{1/3}\omega_{Le}.$$

$$\omega_{1,2} = \pm\begin{cases} i\omega_{Li}\dfrac{ku}{\sqrt{\omega_{Le}^2 - k^2u^2}}, & ku < \omega_{Le}, \\ \omega_{Li}\dfrac{ku}{\sqrt{k^2u^2 - \omega_{Le}^2}}, & ku > \omega_{Le}. \end{cases}$$