Лекция 10
Волны в связанных системах

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4. Волны и взаимодействие волн в периодических структурах
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   б. Обобщение на случай волн произвольной природы
   в. Брэгговское отражение

$$\left(\frac{\partial^2}{\partial t^2} - c^2\frac{\partial^2}{\partial z^2}\right)A_E(t,z) = k_\perp c\frac{\partial A_P}{\partial t},$$

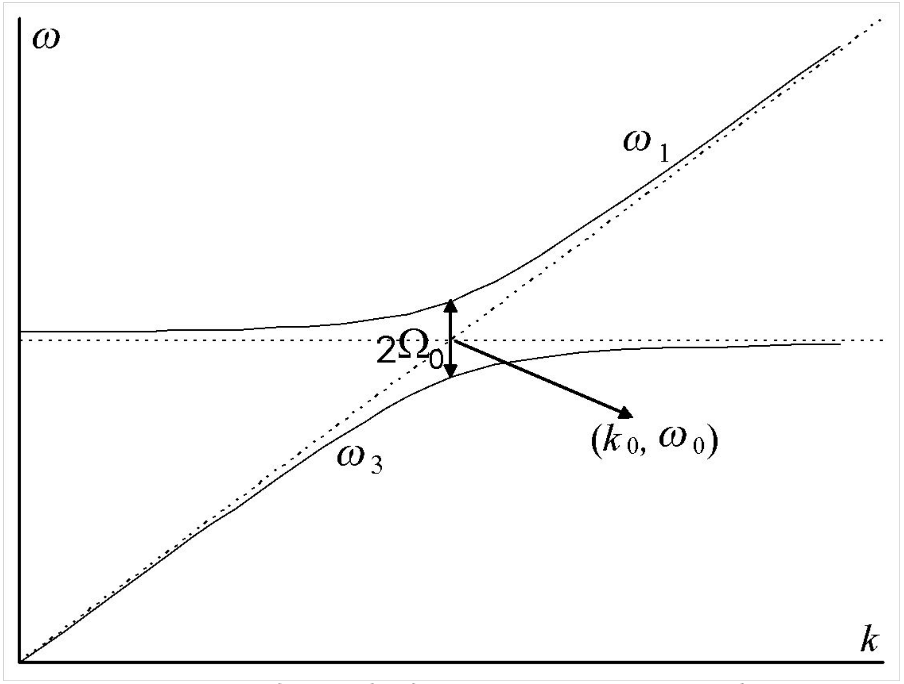
$$\left(\frac{\partial^2}{\partial t^2} + \omega_{Le}^2\right)A_P(t,z) = -k_\perp c\frac{\partial A_E}{\partial t}.$$

$$D_E(\omega,k) \equiv -\omega^2 + k^2c^2 = 0,$$
$$D_p(\omega,k) \equiv -\omega^2 + \omega_{Le}^2 = 0.$$

$$\omega_0 = \omega_p, \quad k_0 = \omega_{Le}/c$$

$$\omega = \omega_0 + \Omega, \quad |\Omega| << \omega_0$$

$$\Omega^2 = \frac{1}{4}k_\perp^2c^2 \equiv \Omega_0^2 \;\rightarrow\; \Omega_{1,2} = \pm\Omega_0$$



$$(\omega^2 - k^2c^2)(\omega - \Omega_e) = \omega\omega_{Le}^2$$

$$A_E(t,z) = \widetilde{A}_E(t)\exp(-i\omega_0 t + ik_0 z),$$
$$A_p(t,z) = \widetilde{A}_p(t)\exp(-i\omega_0 t + ik_0 z).$$

$$\frac{d\widetilde{A}_E}{dt} = \Omega_0\widetilde{A}_p, \quad \frac{d\widetilde{A}_p}{dt} = -\Omega_0\widetilde{A}_E.$$

$$\widetilde{A}_E(t=0) = A_{E0}, \quad \widetilde{A}_p(t=0) = A_{p0}$$

$$A(\Omega) = \int_0^\infty A(t)\exp(i\Omega t)dt \qquad A(t) = \frac{1}{2\pi}\int_{C(\Omega)} A(\Omega)\exp(-i\Omega t)d\Omega$$

$$i\Omega\widetilde{A}_E(\Omega) + \Omega_0\widetilde{A}_p(\Omega) = -A_{E0},$$
$$i\Omega\widetilde{A}_p(\Omega) - \Omega_0\widetilde{A}_E(\Omega) = -A_{p0}.$$

$$\widetilde{A}_E(t) = A_{E0}\cos\Omega_0 t + A_{p0}\sin\Omega_0 t = |A_0|\sin(\Omega_0 t + \varphi_0),$$
$$\widetilde{A}_p(t) = A_{p0}\cos\Omega_0 t - A_{E0}\sin\Omega_0 t = |A_0|\cos(\Omega_0 t + \varphi_0),$$

$$D_1(\hat{\omega},\hat{k})A_1(t,z)=S_1(\hat{\omega},\hat{k})A_2(t,z) \qquad A_1(t,z)=\tilde{A}_1(t)\exp(-i\omega_0 t+ik_0 z)$$

$$D_2(\hat{\omega},\hat{k})A_2(t,z)=S_2(\hat{\omega},\hat{k})A_1(t,z) \qquad A_2(t,z)=\tilde{A}_2(t)\exp(-i\omega_0 t+ik_0 z)$$

$$D_{1,2}(\hat{\omega},k_0)=D_{1,2}(\omega_0+i\frac{d}{dt},k_0)\approx D_{1,2}(\omega_0,k_0)+i\frac{\partial D_{1,2}}{\partial\omega}\frac{d}{dt}=i\frac{\partial D_{1,2}}{\partial\omega}\frac{d}{dt},$$

$$S_{1,2}(\hat{\omega},k_0)=S_{1,2}(\omega_0+i\frac{d}{dt},k_0)\approx S_{1,2}(\omega_0,k_0).$$

$$\frac{d\tilde{A}_1(t)}{dt}=-iS_1(\omega_0,k_0)\left[\frac{\partial D_1(\omega_0,k_0)}{\partial\omega}\right]^{-1}\tilde{A}_2(t),$$

$$\frac{d\tilde{A}_2(t)}{dt}=-iS_2(\omega_0,k_0)\left[\frac{\partial D_2(\omega_0,k_0)}{\partial\omega}\right]^{-1}\tilde{A}_1(t).$$

$$\tilde{A}_1(t=0)=A_{10},\quad \tilde{A}_2(t=0)=A_{20}$$

$$\tilde{A}_{1,2}(t)=B_{1,2}\exp(-i\Omega t)$$

$$\Omega^2=S_1S_2\left(\frac{\partial D_1}{\partial\omega}\frac{\partial D_2}{\partial\omega}\right)^{-1}\equiv\Omega_0^2$$

$$B_2=\frac{1}{\Omega}S_2\left(\frac{\partial D_2}{\partial\omega}\right)^{-1}B_1$$

$$\tilde{A}_1(t)=a\exp(-i\Omega_0 t)+b\exp(i\Omega_0 t)$$

$$\tilde{A}_2(t)=S_2\,(\partial D_2/\partial\omega)^{-1}\Omega_0^{-1}\big(a\exp(-i\Omega_0 t)-b\exp(i\Omega_0 t)\big)$$

$$a=\frac{1}{2}\big(A_{10}+\Omega_0 S_2^{-1}(\partial D_2/\partial\omega)A_{20}\big),\quad b=\frac{1}{2}\big(A_{10}-\Omega_0 S_2^{-1}(\partial D_2/\partial\omega)A_{20}\big)$$

$$\left(\omega^2-\omega_{Le}^2\right)\left((\omega-ku)^2-\omega_{Lb}^2\right)-\omega_{Le}^2\omega_{Lb}^2=0$$

$$\left(\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial z}\right)^2+\omega_{Lb}^2\right)j_b=-\omega_{Lb}^2 j_e,$$

$$\left(\frac{\partial^2}{\partial t^2}+\omega_{Le}^2\right)j_e=-\omega_{Le}^2 j_b.$$

$$\left(\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial z}\right)^2+\omega_{Lb}^2\right)A_b=-\omega_{Lb}^2 S_b(\hat{\omega},\hat{k})A_w,$$

$$D_w(\hat{\omega},\hat{k})A_w=\omega_w^2 S_w(\hat{\omega},\hat{k})A_b.$$

$$D_w(\omega,k)\left((\omega-ku)^2-\omega_{Lb}^2\right)-\omega_w^2\omega_{Lb}^2 S_w(\omega,k)S_b(\omega,k)=0$$

$$D_w(\omega,k)=0$$

$$D_b(\omega,k)\equiv(\omega-ku)^2-\omega_{Lb}^2=0$$

$$\omega_0/k_0\approx u<c$$

$$\omega=\omega_0+\Omega,\quad |\Omega|<<\omega_0$$

$$\Omega\left(\Omega^2\pm 2\Omega\omega_{Lb}\right)=\omega_w^2\omega_{Lb}^2 S_w(\omega_0,k_0)S_b(\omega_0,k_0)\left(\frac{\partial D_w(\omega_0,k_0)}{\partial\omega}\right)^{-1}\equiv\tilde{\Omega}\omega_{Lb}^2$$

$$|\Omega|>>\omega_{Lb}\ \rightarrow\ \Omega_m=\delta_m\left(\tilde{\Omega}\omega_{Lb}^2\right)^{1/3},\quad m=1,2,3$$

$$|\Omega|<<\omega_{Lb}\ \rightarrow\ \Omega_{1,2}=\pm i\left(\frac{1}{2}\tilde{\Omega}\omega_{Lb}\right)^{1/2}$$

$$\frac{d^2\tilde{A}_b}{dt^2}\mp 2i\omega_{Lb}\frac{d\tilde{A}_b}{dt}=-\omega_{Lb}^2 S_b(\omega_0,k_0)\tilde{A}_w,$$

$$\frac{d\tilde{A}_w}{dt}=-i\omega_w^2 S_w(\omega_0,k_0)\left|\frac{\partial D_w(\omega_0,k_0)}{\partial\omega}\right|^{-1}\tilde{A}_b.$$

$$\frac{d\tilde{A}_b}{dt}=\mp i\frac{1}{2}\omega_{Lb}S_b(\omega_0,k_0)\tilde{A}_w,$$

$$\frac{d\tilde{A}_w}{dt}=-i\omega_w^2 S_w(\omega_0,k_0)\left|\frac{\partial D_w(\omega_0,k_0)}{\partial\omega}\right|^{-1}\tilde{A}_b.$$

$$\tilde{A}_w=\frac{1}{2}\left(A_{w0}-i(\tilde{\Omega}S_b^{-1}\Omega_0^{-1})A_{b0}\right)\exp(\Omega_0 t)+\frac{1}{2}\left(A_{w0}+i(\tilde{\Omega}S_b^{-1}\Omega_0^{-1})A_{b0}\right)\exp(-\Omega_0 t)\qquad \Omega_0=\sqrt{(1/2)\tilde{\Omega}\omega_{Lb}}$$

$$\frac{\partial E_x}{\partial t}+\frac{c}{\varepsilon_0}\frac{\partial B_y}{\partial z}=0,\quad \frac{\partial B_y}{\partial t}+c\frac{\partial E_x}{\partial z}=0 \qquad \varepsilon_0(z)=\varepsilon+h\cos\chi z \qquad \mu=h/\varepsilon<<1$$

$$\mathbf{\Psi}(t,z)=\left\{E_x(t,z),B_y(t,z)\right\}=\left\{E_x(\omega,k,z),\ B_y(\omega,k,z)\right\}\exp(-i\omega t+ikz)$$

$$E_x(\omega,k,z)=\sum_{n=-\infty}^{\infty}E_n(\omega,k)\exp(in\chi z),\quad B_y(\omega,k,z)=\sum_{n=-\infty}^{\infty}B_n(\omega,k)\exp(in\chi z)$$

$$E_x(\omega,k,z)=E_{-1}\exp(-i\chi z)+E_0+E_1\exp(i\chi z),$$
$$B_x(\omega,k,z)=B_{-1}\exp(-i\chi z)+B_0+B_1\exp(i\chi z).$$

$$[\omega^2-(k-\chi)^2c_0^2]\,E_{-1}+\omega^2(\mu/2)E_0=0,$$
$$[\omega^2-k^2c_0^2]\,E_0+\omega^2(\mu/2)E_{-1}+\omega^2(\mu/2)E_1=0,$$
$$[\omega^2-(k+\chi)^2c_0^2]\,E_1+\omega^2(\mu/2)E_0=0,$$

$$D_{-1}D_0D_1-\omega^4\left(\frac{\mu}{2}\right)^2(D_{-1}+D_1)=0 \qquad D_n=\omega^2-(k+n\chi)^2c_0^2$$

$$\begin{matrix}D_0(\omega,k)\approx 0\\ D_{\pm 1}(\omega,k)\neq 0\end{matrix}\longrightarrow \quad \omega_{1,2}=\pm kc_0\left[1+\frac{1}{4}\left(\frac{\mu}{2}\right)^2\left(1-\frac{\chi^2}{4k^2}\right)^{-1}\right]$$

$$\psi(t,z)=\begin{Bmatrix}E_x(t,z)\\ B_y(t,z)\end{Bmatrix}=\sum_{m=1}^{2}A_m\begin{Bmatrix}\left[1-\frac{\mu}{2}\frac{k^2}{\chi(2k-\chi)}\exp(-i\chi z)+\frac{\mu}{2}\frac{k^2}{\chi(2k+\chi)}\exp(i\chi z)\right]\\ \frac{kc}{\omega_m}\left[1-\frac{\mu}{2}\frac{k(k-\chi)}{\chi(2k-\chi)}\exp(-i\chi z)+\frac{\mu}{2}\frac{k(k+\chi)}{\chi(2k+\chi)}\exp(i\chi z)\right]\end{Bmatrix}\exp(-i\omega_m t+ikz).$$

$$D_0(\omega,k) \equiv \omega^2 - k^2 c_0^2 = 0,$$
$$D_{-1}(\omega,k) \equiv \omega^2 - (k-\chi)^2 c_0^2 = 0.$$

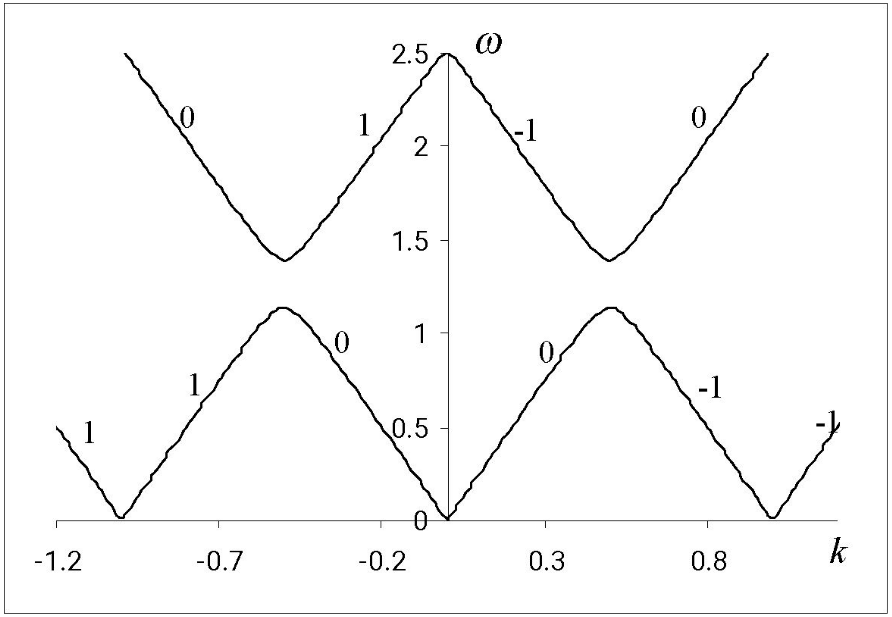
$$\omega_0 = \chi c_0 / 2, \quad k_0 = \chi / 2$$

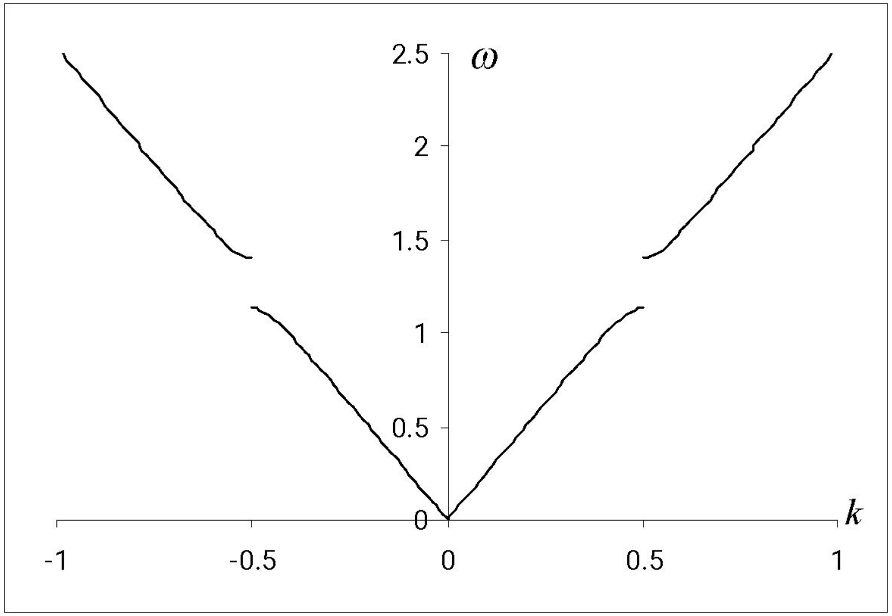
$$D_{-1} D_0 - \omega^4 (\mu/2)^2 = 0$$

$$\downarrow$$

$$\Omega^2 = \frac{1}{4}\omega_0^2 (\mu/2)^2 \equiv \Omega_0^2 \ \rightarrow \ \Omega_{1,2} = \pm\Omega_0$$

$$E_0(t,z) = \widetilde{A}_0(t)\exp\left(i\frac{\chi}{2}(z - c_0 t)\right),$$
$$E_{-1}(t,z) = \widetilde{A}_{-1}(t)\exp\left(-i\frac{\chi}{2}(z + c_0 t)\right),$$
$$\widetilde{A}_0(t) = B_0 \sin(\Omega_0 t + \varphi_0), \quad \widetilde{A}_{-1}(t) = B_0 \cos(\Omega_0 t + \varphi_0).$$





$$D(\hat{\omega},\hat{k})A(t,z)+\mu\eta(z)Q(\hat{\omega})A(t,z)=0 \qquad \eta(z)=\sum_{m=-\infty}^{\infty}\theta_m \exp(im\chi z)$$

$$A(t,z)=\exp(-i\omega t+ikz)\sum_{n=-\infty}^{\infty}A_n\exp(in\chi z) \qquad D(\omega,k+n\chi)A_n+\mu Q(\omega)\sum_{m=-\infty}^{\infty}\theta_m A_{n-m}=0\,, \quad n=0,\pm1,\ldots$$

$$\left\| \begin{array}{cccccc} \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & D_{-2} & \mu'\theta_{-1} & \mu'\theta_{-2} & \mu'\theta_{-3} & \mu'\theta_{-4}\cdots \\ \cdots & \mu'\theta_1 & D_{-1} & \mu'\theta_{-1} & \mu'\theta_{-2} & \mu'\theta_{-3}\cdots \\ \cdots & \mu'\theta_2 & \mu'\theta_1 & D_0 & \mu'\theta_{-1} & \mu'\theta_{-2}\cdots \\ \cdots & \mu'\theta_3 & \mu'\theta_2 & \mu'\theta_1 & D_1 & \mu'\theta_{-1}\cdots \\ \cdots & \mu'\theta_4 & \mu'\theta_3 & \mu\theta_2 & \mu'\theta_1 & D_2\ \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{array} \right\| = 0$$

$$D_n = D(\omega,k+n\chi)$$

$$D_0 = D(\omega,k)$$

$$\mu' = \mu Q(\omega)$$

$$D_{-2}D_{-1}D_0D_1D_2-\mu^2Q^2(\omega)\{|\theta_1|^2\,[D_0(D_1D_2+D_{-1}D_{-2})+D_{-2}D_2(D_{-1}+D_1)]+$$
$$+|\theta_2|^2\,[D_{-2}D_0D_2+D_{-1}D_1(D_{-2}+D_2)]+|\theta_3|^2\,D_0(D_1D_{-2}+D_{-1}D_2)+|\theta_4|^2\,D_{-1}D_0D_1\}=0$$

$$D_0=D_{\pm1}=0 \qquad D_0D_{\pm1}-\mu^2Q^2(\omega_0)\,|\theta_1|^2=0 \qquad \Delta_1=2\Omega=2\mu\left|\theta_1 Q(\omega_0)\left(\frac{\partial D_0(\omega_0,k_0)}{\partial\omega_0}\right)^{-1}\right|$$

$$D_{-2}D_{-1}D_0D_1D_2-\mu^2Q^2(\omega)\,|\theta_1|^2\,[D_0(D_{-1}D_{-2}+D_1D_2)+D_{-2}D_2(D_{-1}+D_1)]+$$
$$+\mu^4Q^4(\omega)\,|\theta_1|^4\,(D_0+D_{-2}+D_2)=0$$

$$\boxed{\Delta_2=\frac{1}{4\sqrt{3}}\mu^2\omega_0} \qquad \Delta_2=2\mu^2\,|\theta_2|^2\,Q^2(\omega_0)\left(\sqrt{|D_{-1}(\omega_0,k_0)D_1(\omega_0,k_0)|}\left|\frac{\partial D_0(\omega_0,k_0)}{\partial\omega_0}\right|\right)^{-1}$$

$$k = k_0 + \kappa,\ |\kappa| << k_0 = \chi/2 \qquad \kappa = \pm i\delta_B,\ \delta_B = \mu\left|\theta_1 Q(\omega_0)\left(\frac{\partial D(\omega_0,k_0)}{\partial k_0}\right)^{-1}\right|$$

$$A(t,z) = A_+(z)\exp(-i\omega t + ikz) + A_-(z)\exp(-i\omega t - ikz)$$

$$\frac{dA_+}{dz}\exp(ikz) - \frac{dA_-}{dz}\exp(-ikz) + i\mu Q(\omega)\left(\frac{\partial D}{\partial k}\right)^{-1}[\theta_1\exp(i\chi z) + \theta_1^*\exp(-i\chi z)][A_+\exp(ikz) + A_-\exp(-ikz)] = 0.$$

$$\frac{dA_+}{dz} + i\mu Q(\omega_0)\left(\frac{\partial D(\omega_0,k_0)}{\partial k_0}\right)^{-1}\theta_1 A_- = 0,$$

$$\frac{dA_-}{dz} - i\mu Q(\omega_0)\left(\frac{\partial D(\omega_0,k_0)}{\partial k_0}\right)^{-1}\theta_1^* A_+ = 0.$$

$$A_+(z) = B\exp(\delta_B z) + C\exp(-\delta_B z),$$

$$A_-(z) = \varsigma\,[B\exp(\delta_B z) - C\exp(-\delta_B z)],$$

$$A(t,z) = C\exp(-\delta_B z)\exp(-i\omega_0 t + ik_0 z) - \varsigma\, C\exp(-\delta_B z)\exp(-i\omega_0 t - ik_0 z)$$