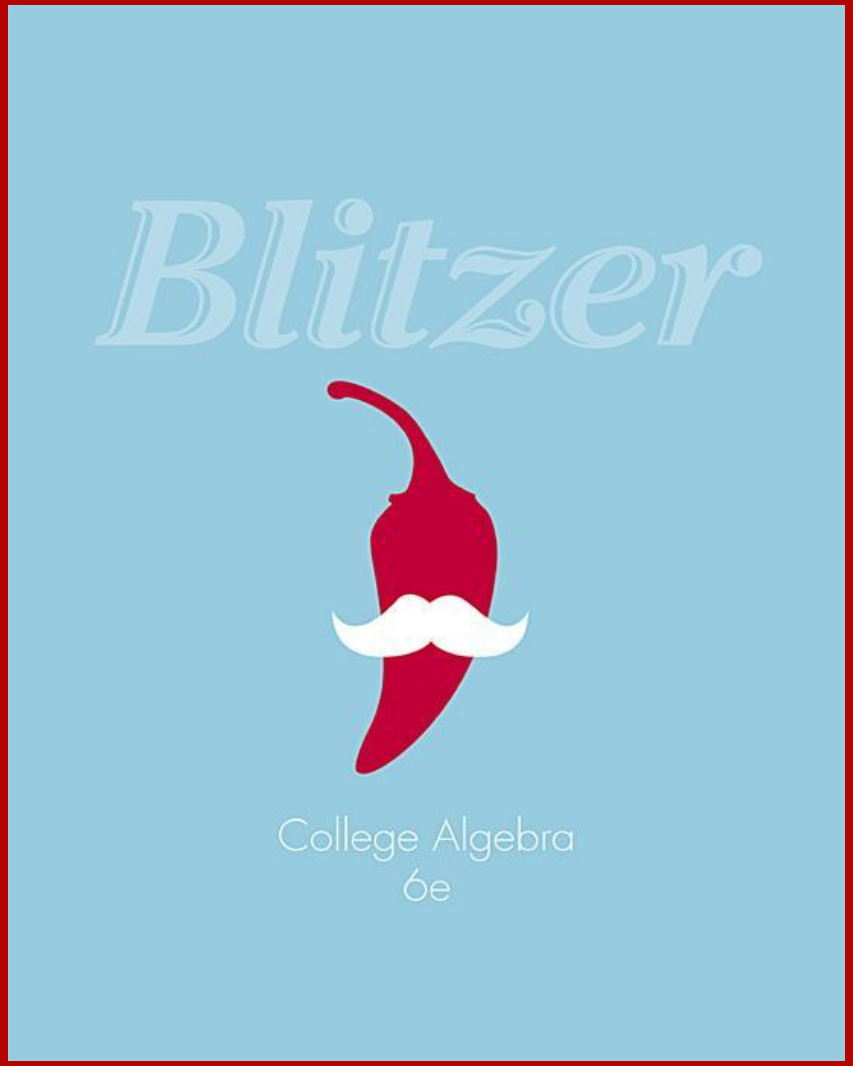
Chapter 2

Functions and Graphs

2.1 Basics of Functions and Their Graphs



Objectives:

- Find the domain and range of a relation.
- Determine whether a relation is a function.
- Determine whether an equation represents a function.
- Evaluate a function.
- Graph functions by plotting points.
- Use the vertical line test to identify functions.
- Obtain information about a function from its graph.
- Identify the domain and range of a function from its graph.
- Identify intercepts from a function's graph.

Definition of a Relation

A **relation** is any set of ordered pairs. The set of all first components of the ordered pairs is called the **domain** of the relation and the set of all second components is called the **range** of the relation.

Example: Finding the Domain and Range of a Relation

Find the domain and range of the relation:

{(0, 9.1), (10, 6.7), (20, 10.7), (30, 13.2), (40, 21.2)}

domain: {0, 10, 20, 30, 40}

range: {9.1, 6.7, 10.7, 13.2, 21.2}

Definition of a Function

A **function** is a correspondence from a first set, called the **domain**, to a second set, called the **range**, such that each element in the domain corresponds to *exactly one* element in the range.

Example: Determining Whether a Relation is a Function

Determine whether the relation is a function:

$\{(1, 2), (3, 4), (6, 5), (8, 5)\}$

No two ordered pairs in the given relation have the same first component and different second components. Thus, the relation is a function.

Functions as Equations

If an equation is solved for y and more than one value of y can be obtained for a given x, then the equation does not define y as a function of x.

Example: Determining Whether an Equation Represents a Function

Determine whether the equation defines y as a function of x.

$$x^2 + y^2 = 1$$

$$y^2 = 1 - x^2$$

$$y = \pm\sqrt{1 - x^2}$$

The $\pm$ shows that for certain values of x, there are two values of y. For this reason, the equation does not define y as a function of x.

Function Notation

The special notation $f(x)$, read "f of x" or "f at x", represents the value of the function at the number x.

Example: Evaluating a Function

If $f(x) = x^2 - 2x + 7$, evaluate $f(-5)$.

$$f(x) = x^2 - 2x + 7,$$

$$f(-5) = (-5)^2 - 2(-5) + 7 = 25 + 10 + 7 = 42$$

Thus, $f(-5) = 42$.

Graphs of Functions

The **graph of a function** is the graph of its ordered pairs.

Example: Graphing Functions

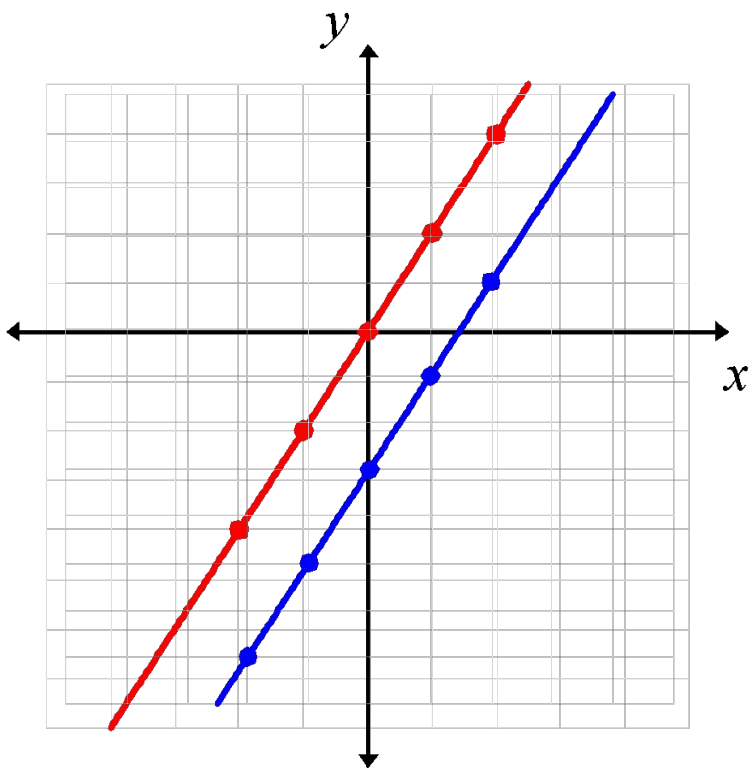
Graph the functions $f(x) = 2x$ and $g(x) = 2x - 3$ in the same rectangular coordinate system. Select integers for x, starting with -2 and ending with 2.

Example: Graphing Functions *(continued)*

We set up a partial table of coordinates for each function. We then plot the points and connect them.

$f(x) = 2x$

x	$y = f(x)$
–2	–4
–1	–2
0	0
1	2
2	4



$g(x) = 2x - 3$

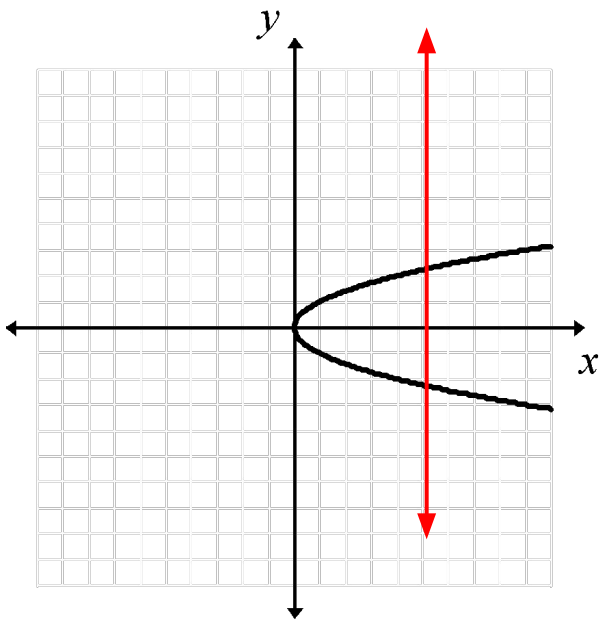
x	$y = f(x)$
–2	–7
–1	–5
0	–3
1	–1
2	1

The Vertical Line Test for Functions

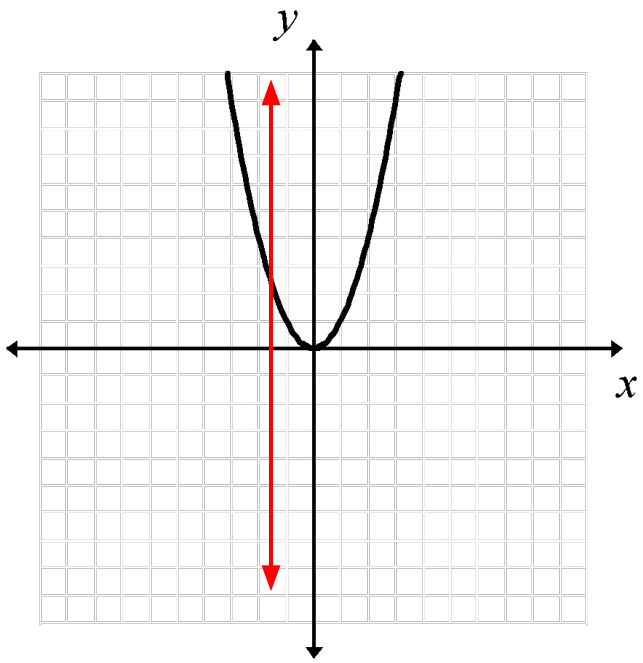
If any vertical line intersects a graph in more than one point, the graph does not define y as a function of x.

Example: Using the Vertical Line Test

Use the vertical line test to identify graphs in which y is a function of x.



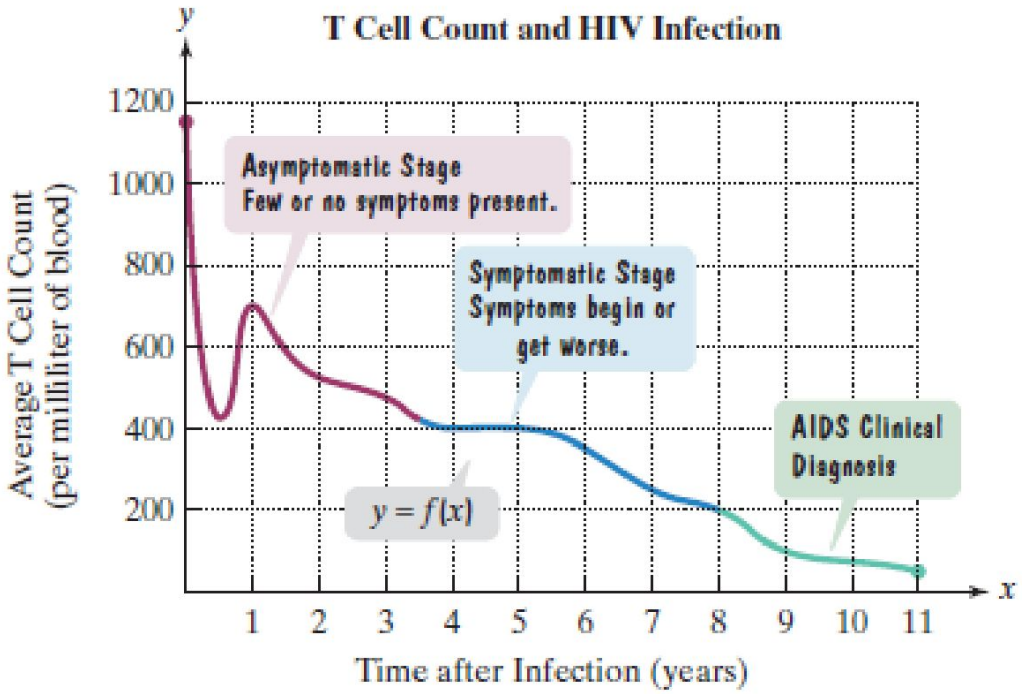
not a function



function

Example: Analyzing the Graph of a Function

Use the graph to find $f(5)$



$f(5)=400$

For what value of x is $f(x) = 100$?
$f(9) = 125$, so $x = 9$.

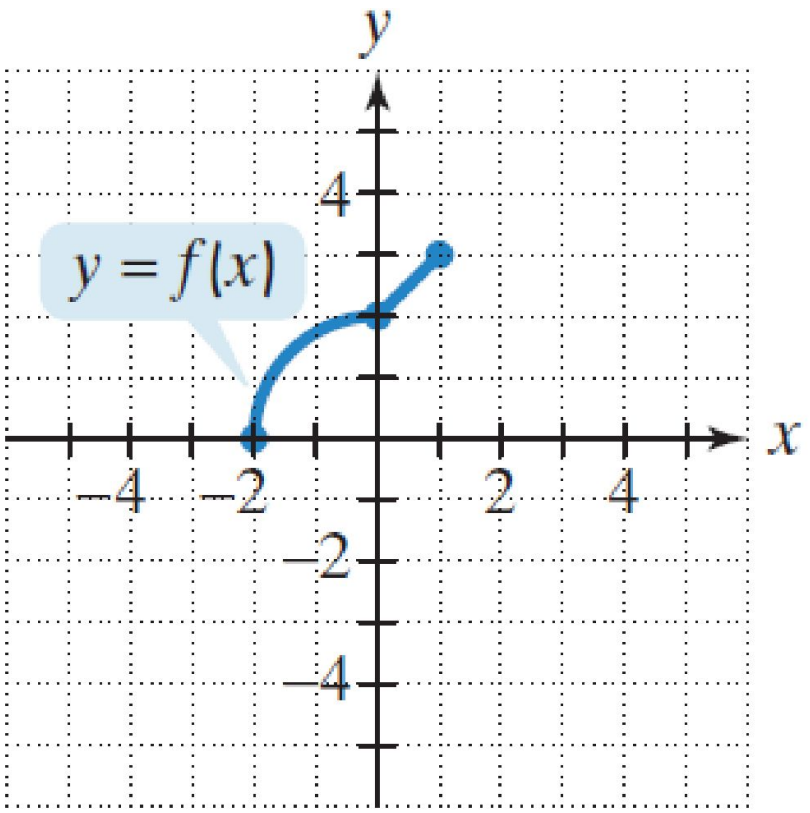
Identifying Domain and Range from a Function's Graph

To find the domain of a function from it's graph, look for all the inputs on the x-axis that correspond to points on the graph.

To find the range of a function from it's graph, look for all the outputs on the y-axis that correspond to points on the graph.

Example: Identifying the Domain and Range of a Function from Its Graph

Use the graph of the function to identify its domain and its range.



Domain $\{x \mid -2 \le x \le 1\}$

$[-2,1]$

Range $\{y \mid 0 \le y \le 3\}$

$[0,3]$

Example: Identifying the Domain and Range of a Function from Its Graph

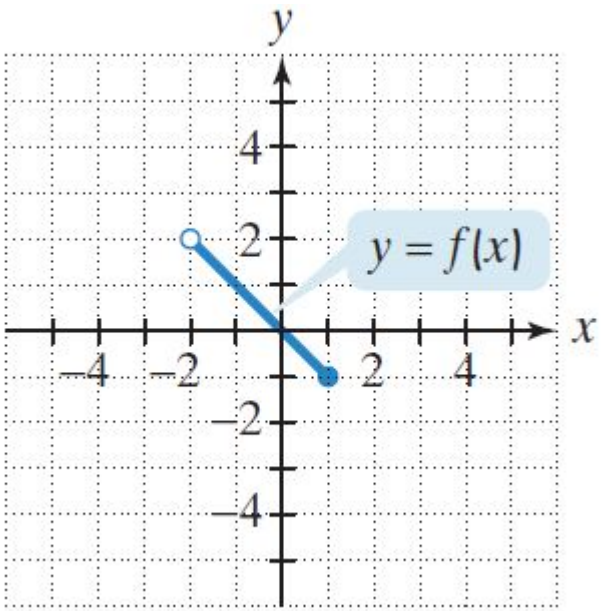
Use the graph of the function to identify its domain and its range.

Domain $\{x|-2 < x \leq 1\}$

$(-2,1]$

Range $\{y|-1 \leq y < 2\}$

$[-1,2)$



Identifying Intercepts from a Function's Graph

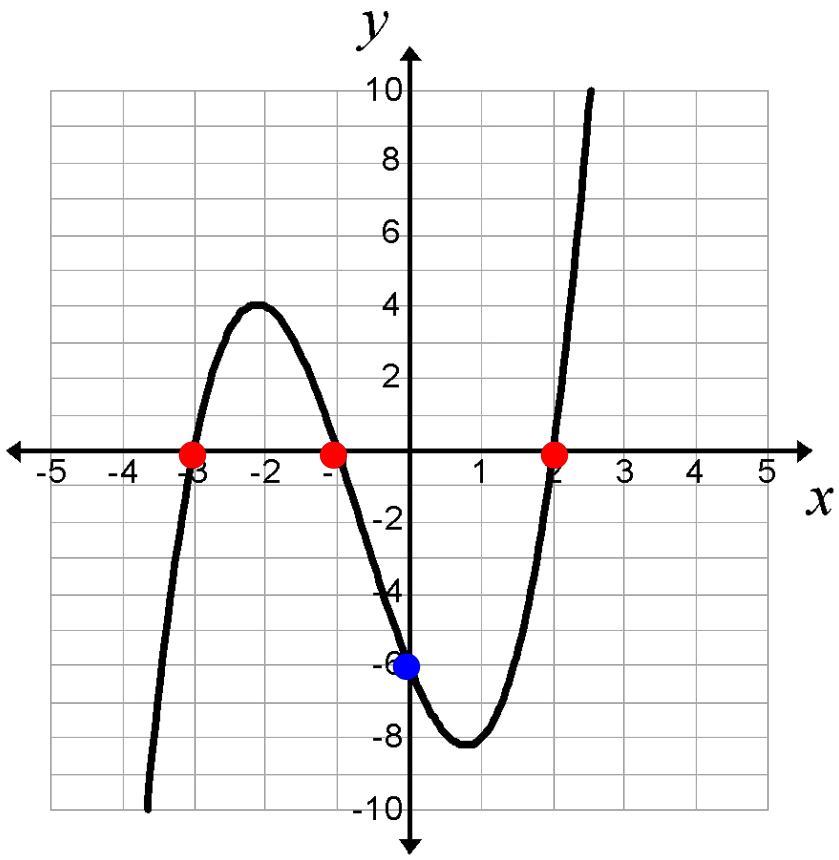
To find the *x*-intercepts, look for the points at which the graph crosses the *x*-axis.

To find the *y*-intercept, look for the point at which the graph crosses the *y*-axis.

A function can have more than one *x*-intercept but at most one *y*-intercept.

Example: Identifying Intercepts from a Function's Graph

Identify the x- and y-intercepts for the graph of $f(x)$.



The x-intercepts are
$(-3, 0)$
$(-1, 0)$
and $(2, 0)$

The y-intercept is $(0, -6)$