Математический анализ 2 семестр Занятие №1

Неопределенный интеграл

Занятие 1. Интеграл от степенной функции

$$\int \sqrt[3]{x^2} \, dx = \int x^{2/3} dx = \frac{3}{5} x^{5/3} + C$$

1. $\int \sqrt{x} \, dx = \frac{2}{3} x^{3/2} + C$

2. $\int \sqrt[3]{x} \, dx = \frac{3}{4} x^{4/3} + C$

3. $\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + C$

4. $\int \sqrt[5]{x^4} \, dx = \frac{5}{9} x^{9/5} + C$

5. $\int \frac{dx}{\sqrt[3]{x}} = \frac{3}{2} x^{2/3} + C$

6. $\int \frac{dx}{\sqrt[5]{x^3}} = \frac{5}{3} x^{2/5} + C$

7. $\int \frac{dx}{\sqrt[10]{x^7}} = \frac{10}{3} x^{3/10} + C$

Занятие 1. Метод разложения

$$\int \frac{(1-x)^2}{x\sqrt{x}} dx = \int \frac{x^2 - 2x + 1}{x^{3/2}} dx =$$

$$= \int \sqrt{x}\, dx - 2\int \frac{dx}{\sqrt{x}} + \int \frac{dx}{x^{3/2}} = \frac{2}{3} x^{3/2} - 4\sqrt{x} - \frac{2}{\sqrt{x}} + C$$

1. $\int \left(3 - x^2\right)^3 dx = -\frac{1}{7} x^7 + \frac{9}{5} x^5 - 9x^3 + 27x + C$

2. $\int x^2 (5 - x^2) dx = -\frac{1}{5} x^5 + \frac{5}{3} x^3 + C$

3. $\int \left(\frac{1-x}{x}\right)^2 dx = x - 2\ln|x| - \frac{1}{x} + C$

4. $\int \frac{x+1}{\sqrt{x}} dx = 2\sqrt{x} + \frac{2}{3} x^{3/2} + C$

5. $\int \frac{x^2}{1 + x^2} dx = x - \operatorname{arctg} x + C$

Занятие 1. Метод разложения

$$\int \frac{(1-x)^2}{x\sqrt{x}} dx = \int \frac{x^2 - 2x + 1}{x^{3/2}} dx =$$

$$= \int \sqrt{x} dx - 2\int \frac{dx}{\sqrt{x}} + \int \frac{dx}{x^{3/2}} = \frac{2}{3}x^{3/2} - 4\sqrt{x} - \frac{2}{\sqrt{x}} + C$$

6. $\int (1 + \sin x + \cos x) dx = x - \cos x + \sin x + C$

7. $\int \frac{\sqrt[3]{x} - xe^x + \sqrt{x}}{x} dx = 3\sqrt[3]{x} + e^x + 2\sqrt{x} + C$

8. $\int \left(\frac{1}{x} + \cos x - \frac{1}{\sqrt{1-x^2}} \right) dx = \ln|x| + \sin x - \arcsin x + C$

9. $\int \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1-x^4}} dx = \arcsin x + \operatorname{arcsh} x + C$

Занятие 1. Замена переменных

$$\int \sin x \cdot \cos x \, dx = \left\langle \begin{array}{c} \cos x = t \\ dt = -\sin x \, dx \end{array} \right\rangle =$$

$$= -\int t \, dt = -\frac{1}{2} t^2 + C = -\frac{1}{2} \cos^2 x + C$$

1. $\int \frac{dx}{(2x-3)^5} = \langle 2x - 3 = t \rangle = -\frac{1}{8(2x-3)^4} + C$

2. $\int \sqrt[5]{(8-3x)^6} \, dx = \langle 8 - 3x = t \rangle = -\frac{5}{33}(8-3x)^{11/5} + C$

3. $\int x \sqrt{1-x^2} \, dx = \langle 1 - x^2 = t \rangle = -\frac{1}{3}\left(1-x^2\right)^{3/2} + C$

Занятие 1. Замена переменных

$$\int \frac{xdx}{\sqrt{x^2+1}} = \left\langle \begin{array}{c} x^2 + 1 = t \\ dt = 2xdx \end{array} \right\rangle =$$

$$= \frac{1}{2} \int \frac{dt}{\sqrt{t}} = t^{1/2} + C = \sqrt{x^2+1} + C$$

4. $\int \sin^3 x \cdot \cos x dx = \langle \sin x = t \rangle = \frac{1}{4} \sin^4(x) + C$

5. $\int \frac{\sqrt{\ln x}}{x} dx = \langle \ln x = t \rangle = \frac{2}{3} (\ln(x))^{3/2} + C$

6. $\int \frac{dx}{(\arcsin x)^3 \sqrt{1-x^2}} = \langle \arcsin x = t \rangle = -\frac{1}{2} \arcsin^{-2}(x) + C$

7. $\int \sin(2x-3) dx = \langle 2x-3 = t \rangle = -\frac{1}{2} \cos(2x-3) + C$

Занятие 1. Замена переменных

8. $\displaystyle\int \mathrm{tg}\, x\, dx = \langle \cos x = t \rangle = -\ln\left|\cos x\right| + C$

9. $\displaystyle\int \frac{e^x dx}{e^x + 1} = \langle e^x + 1 = t \rangle = \ln(e^x + 1) + C$

10. $\displaystyle\int \frac{dx}{x \ln x} = \langle \ln x = t \rangle = \ln\left|\ln x\right| + C$

11. $\displaystyle\int e^{\sin x} \cos x\, dx = \langle \sin x = t \rangle = e^{\sin x} + C$

12. $\displaystyle\int \frac{dx}{1 + 9x^2} = \langle 3x = t \rangle = \frac{1}{3} \mathrm{arctg}\, 3x + C$

13. $\displaystyle\int \frac{dx}{(x-1)^2 + 4} = \langle x - 1 = 2t \rangle = \frac{1}{2} \mathrm{arctg}\left(\frac{1}{2} x - \frac{1}{2}\right) + C$
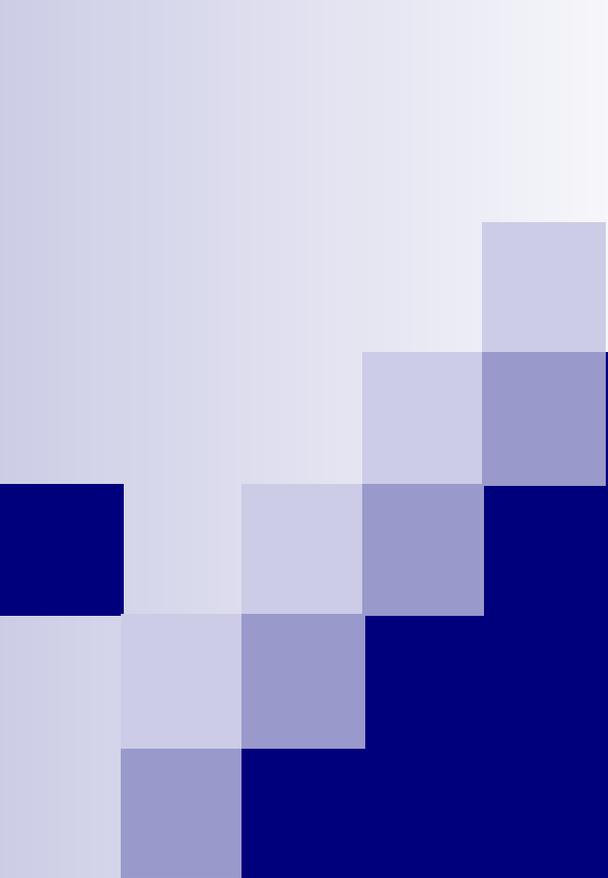
Занятие 1. Замена переменных

14. $\int \cos^3 x\, dx = \langle \sin x = t \rangle = \dfrac{1}{3}\cos^2 x \cdot \sin x + \dfrac{2}{3}\sin x + C$

15. $\int \dfrac{\sin^3 x}{\cos x}\, dx = \langle \cos x = t \rangle = -\dfrac{1}{2}\sin^2 x - \ln|\cos(x)| + C$

16. $\int \dfrac{dx}{1 + \sqrt{x+1}} = \langle x = t^2 - 1 \rangle$

17. $\int \dfrac{\sqrt{x}\, dx}{\sqrt{x} - \sqrt[3]{x}} = \langle x = t^6 \rangle$

18. $\int \dfrac{dx}{4x^2 + 4x + 5} = \left\langle x + \dfrac{1}{2} = t \right\rangle$

Спасибо за внимание

Занятие окончено