Constant Jerk Trajectory Generator (TG)

Purpose:

This chapter introduces the ideal constant jerk S-curve (jerk is the derivative of acceleration), represented by a 2nd order polynomial in velocity. Its shape is governed by the motion conditions at the start and end of the transition.

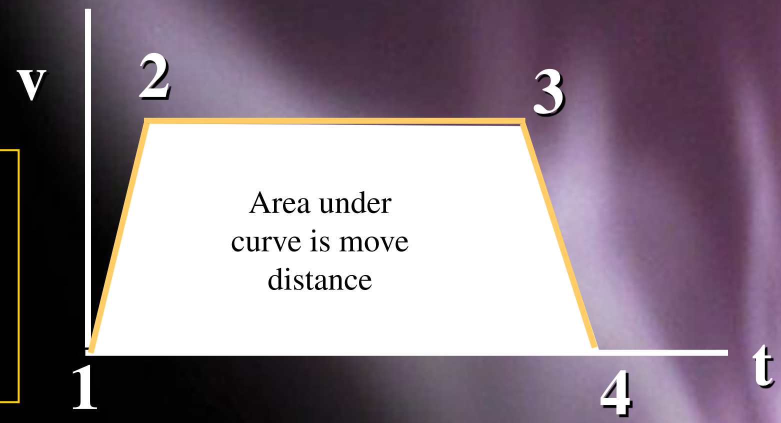
An S-curve with an intermediate constant acceleration (linear portion) is often used to reduce the time to make large speed changes. The jerk can be used to determine how much of the rise or fall period can be made under constant acceleration.

In particular, you will

1. **Determine why S-curves are necessary**
2. **Review the ideal S-curve.**
3. **Consider constant acceleration jerk transitions.**
4. **Consider the speed transition when the velocity change is too small to reach the desired accel (or decel) value.**
5. **Consider the trajectory generator in the context of joint moves or curvilinear moves.**

Why S-curves?

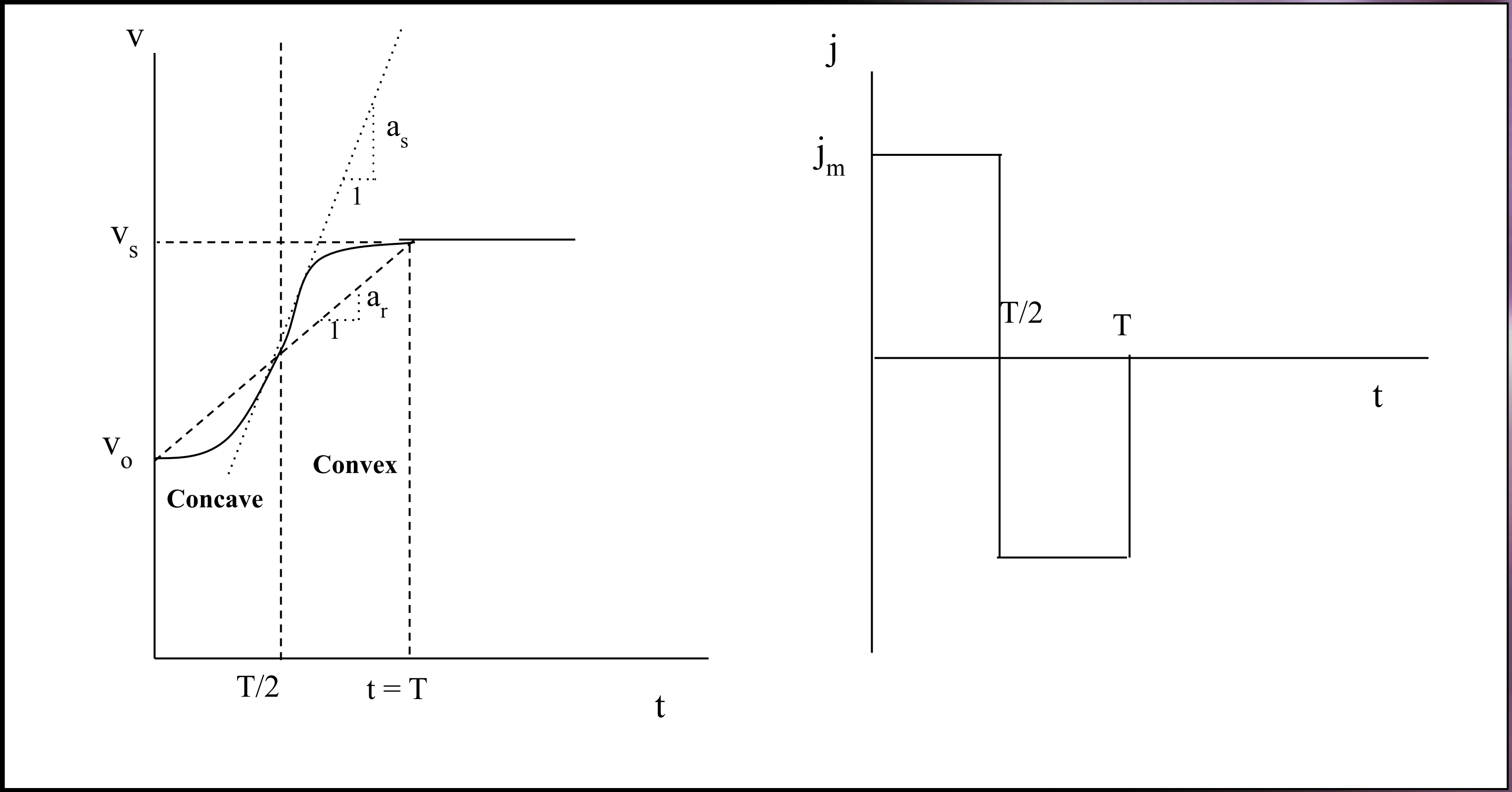
The trapezoidal profile to the right was the trajectory generator of choice for many years, but is now being replaced by S-curve profiles. Why?



Reviewing the trapezoidal trajectory profile in speed v, we examine points 1, 2, 3, and 4. Each of these points has a discontinuity in acceleration. This discontinuity causes a very large jerk, which impacts the machine dynamics, also stressing the machine's mechanical components.

An S-curve is a way to impose a limited jerk on the speed transitions, thus smoothing out the robot's (or machine tool) motion.

Ideal S-curve



Ideal S-curve equations

The form assumed for the S-curve velocity profile is

$$v(t) = c_o + c_1 t + c_2 t^2 \qquad (5.1)$$

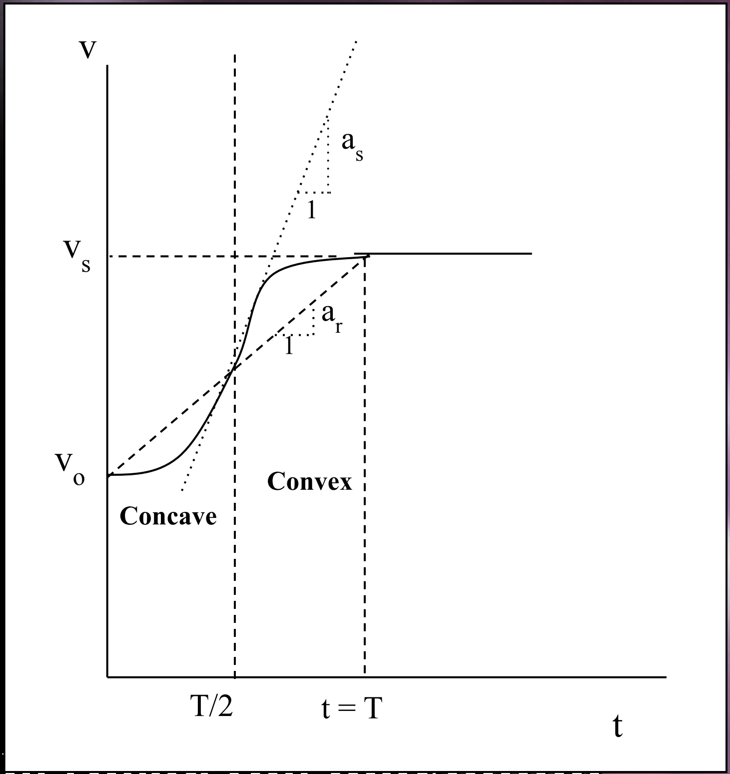
giving the acceleration and constant jerk equations:

$$a(t) = c_1 + 2 c_2 t \qquad (5.2)$$

$$j(t) = 2 c_2 \qquad (5.3)$$

The rise motion can be divided into 2 periods - a concave period followed by a convex period.

Concave period



The concave conditions are

$$v(0) = v_o$$

$$a(0) = 0$$

$$a(T/2) = a_s$$

$$j(0) = j_m$$

where j_m is the jerk set for the profile (near the maximum allowed for the robot), and a_s is the maximum acceleration encountered at the S-curve inflection point.

Concave period

Applying the initial and final conditions, we get the equations for s (position), v, and a along the concave portion of the S-curve:

$$s(t) = v_o t + j_m t^3/6 \qquad (5.7)$$

$$v(t) = v_o + j_m t^2/2 \qquad (5.8)$$

$$a(t) = j_m t \qquad (5.9)$$

Note: It is assumed that s is 0 at the beginning of the S-move. Thus, s represents a position delta.

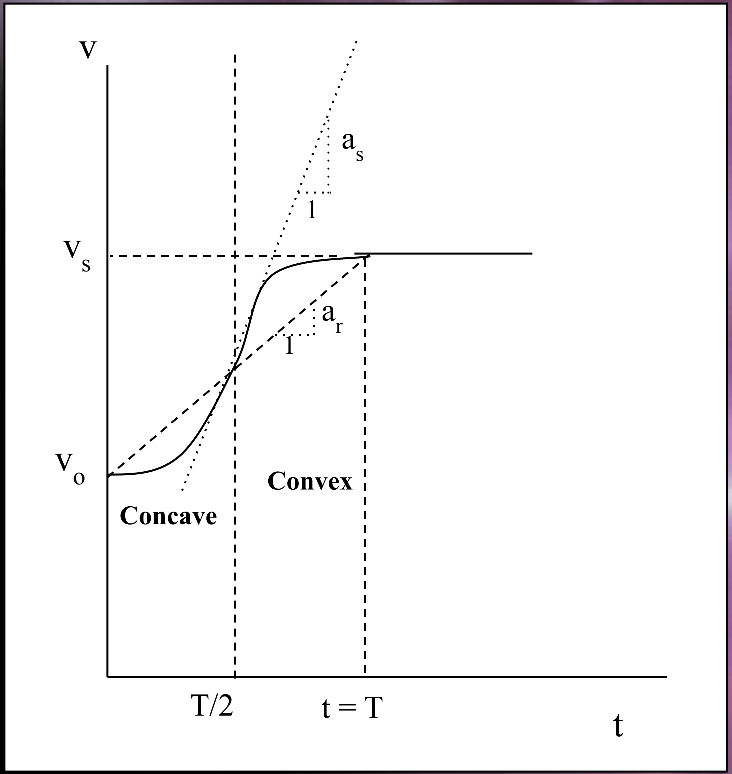
Ideal S-curve observations

1. If we let $\Delta v = v_s - v_o$ and define $a_r = \Delta v/T$ to be the acceleration of a constant acceleration ramp from v_o to v_s, then we note that a_s is twice a_r. It is also true that $T = 2\Delta v/a_s$.

2. The trapezoidal profile can be used to predict the time and distance required to transition the accel and decel periods of the ideal S-curve. This exercise is commonly called motion or path planning.

Convex period

This period applies for $T/2 \le t \le T$. Letting time be zero measured from the beginning of the convex period ($0 \le t \le T/2$), the pertinent motion conditions are:



$$v(0) = v_h = (v_s + v_o)/2$$

$$a(0) = a_s$$

$$a(T/2) = 0$$

$$j(0) = -j_m$$

where $-j_m$ is the jerk set for the profile, and a_s is the maximum acceleration encountered at the S-curve inflection point.

Convex period

Applying the initial and final conditions, we get the equations for s (position), v, and a along the convex portion of the S-curve:

$$s(t) = v_h\, t + a_s\, t^2/2 - j_m\, t^3/6 \qquad (5.11)$$

$$v(t) = v_h + a_s\, t - j_m\, t^2/2 \qquad (5.12)$$

$$a(t) = a_s - j_m\, t \qquad (5.13)$$

Note: It is assumed that s is 0 at the beginning of the S-move. Thus, s represents a position delta.

Distance traversed

Adding in the distance at the halfway point gives the total distance traversed in the S-curve, including both concave and convex sections:

$$S = (v_s^2 - v_o^2)/a_s$$

Max jerk transitions

An ideal S-curve cannot transition smoothly between any speed change using a specified max jerk value!

Why?

Max jerk transitions

Given a jerk j_m, a starting speed v_o, and the ending speed v_s, we can determine v_1 and v_2, where these are the velocities that end the concave transition and begin the convex transition at max accel a_s for the ideal S-curve transition:

$$v_1 = v_o + a_s^2/(2j_m)$$

$$v_2 = v_s - a_s^2/(2\,j_m)$$

By setting $v_1 = v_2$, we can also determine the max jerk for a given a_s and $\Delta v = v_s - v_o$:

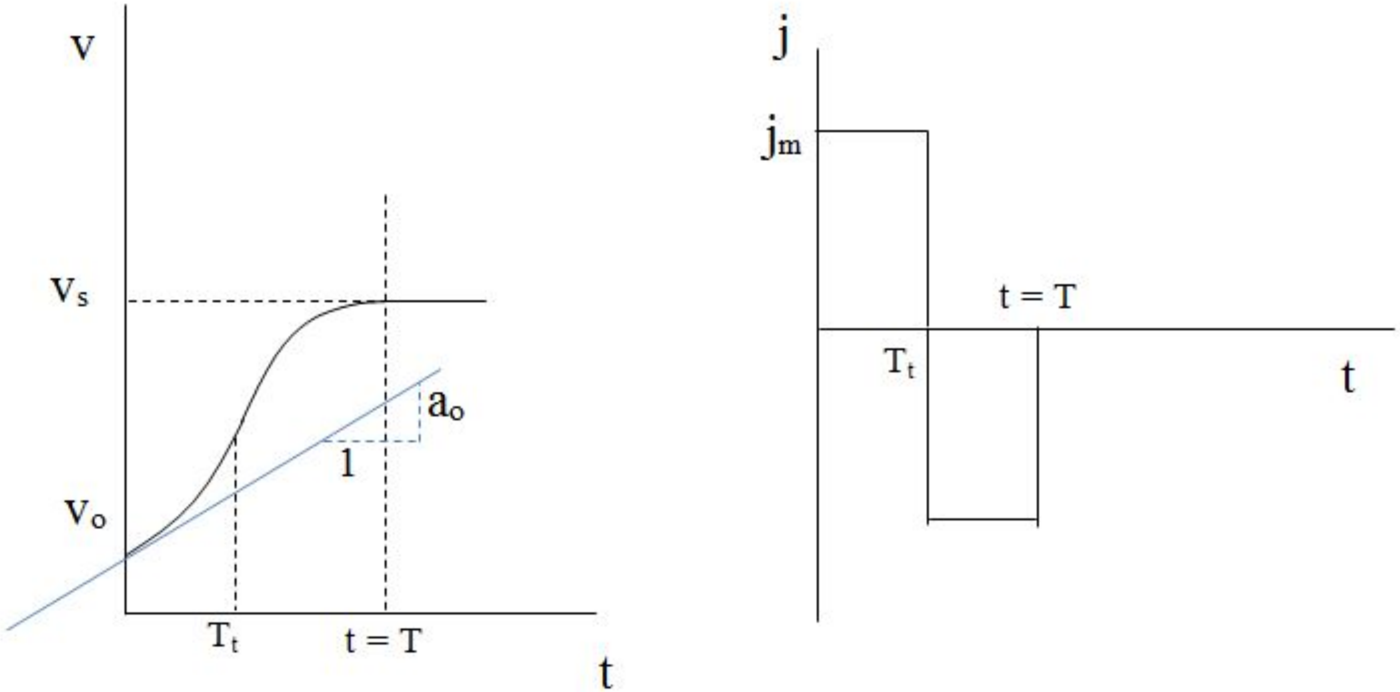
$$j_m = a_s^2 / \Delta v$$

Speed transitions

If $v_1 > v_2$ (overlap), we can determine an intermediate transition point using speed and acceleration continuity.

Note that the velocity and acceleration for the previous concave curve and the new convex curve must be equal at T_t where the velocity is v_t. We cannot reach the maximum acceleration a_s by applying maximum jerk transitions. Nevertheless, there exists a point where the concave profile will be tangent to the convex profile. This point will lie between v_o and v_s. At this point the acceleration and speed of both profiles are the same, although there will be a sign change in jerk.

Speed transitions



S-Curve profile with speed transition

Speed transitions

The pertinent equations are:

$$v_o + a_o T_t + j_m T_t^2/2 = v_s - j_m (T - T_t)^2/2 \quad (5.20)$$

$$a_o + j_m T_t = j_m (T - T_t) \quad (5.21)$$

Solving these we get:

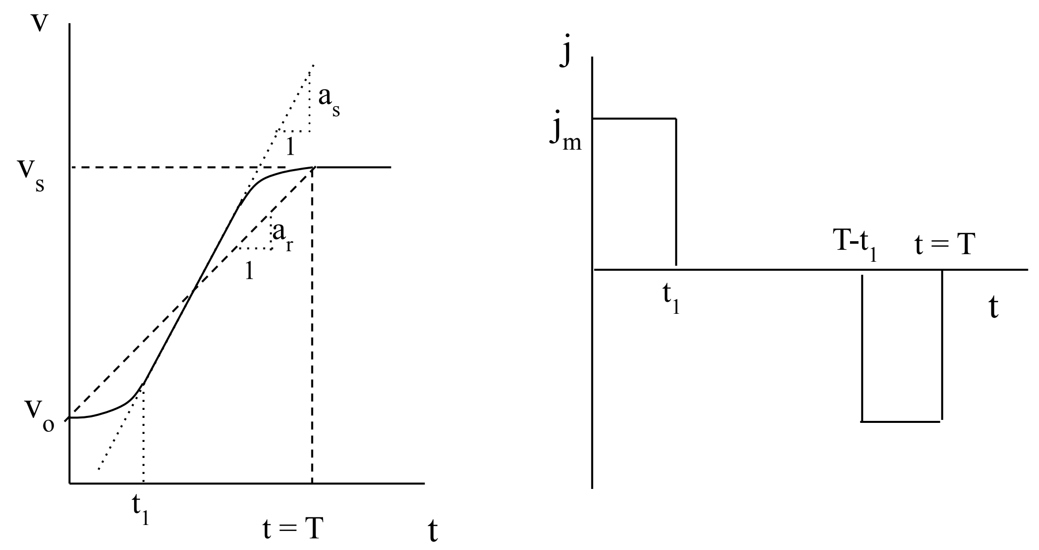
$$T = [-a_o + \mathrm{sqrt}(\, 2\, a_o^2 + 4\, \Delta v\, j_m)\,]/\, j_m \quad (5.22)$$

$$T_t = (j_m T - a_o)/(2\, j_m) \quad (5.23)$$

where $\Delta v = (v_s - v_o)$.

S-curve with linear period

If $v_1 < v_2$, then we must insert a linear (constant acceleration) period. The desired maximum S accel (a_s) is known, as is the maximum jerk (j_m).



S-Curve profile with linear period

S-curve with linear period

Motion conditions:

Phase 1 - Concave	Phase 2 – Linear	Phase 3 - Convex
$0 \le t \le t_1$	$0 \le t \le T - 2t_1$	$0 \le t \le t_1$
$v(0) = v_o$	$v(0) = v_1$	$v(0) = v_2$
$a(0) = 0$	$a(0) = a_s$	$a(0) = a_s$
$a(t_1) = a_s$	$a(T-2t_1) = a_s$	$v(t_1) = v_s$
$j(0) = j_m$	$v(T-2t_1) = v_2$	$a(t_1) = 0$
$v(t_1) = v_1$		$j(0) = - j_m$

S-curve with linear period

Phase 1 – Concave motion conditions:

$$s(t) = v_o\, t + j_m\, t^3/6 \qquad (5.25)$$

$$v(t) = v_o + j_m\, t^2/2 \qquad (5.26)$$

$$a(t) = j_m\, t \qquad (5.27)$$

S-curve with linear period

Phase 2 – Linear motion conditions:

$$s(t) = v_1\, t + a_s\, t^2/2 \qquad (5.30)$$

$$v(t) = v_1 + a_s\, t \qquad (5.31)$$

S-curve with linear period

Phase 3 – Convex motion conditions:

$$s(t) = v_2\, t + a_s\, t^2/2 - j_m\, t^3/6 \qquad (5.35)$$

$$v(t) = v_2 + a_s\, t - j_m\, t^2/2 \qquad (5.36)$$

$$a(t) = a_s - j_m\, t \qquad (5.37)$$

S-curve context

How is the S-curve applied in the real world?

Robots and machine tools are commanded to move in either joint space or Cartesian space.

In joint space the slowest joint becomes the controlling move. Its set speed and joint distance is used for the trajectory motion planning. Desired acceleration and jerk values are applied for this joint to specify the S-curve profiles.

In Cartesian space either the path length or tool orientation change dominates the motion. The associated speeds , accelerations, and jerk values specify the S-curve profiles. The trajectory generator processes length or orientation change, whichever is dominant. The other change is proportioned.

TG summary

- S-curve is used to smooth speed transitions by eliminating points of extremely high jerk.
- S-curve is limited by jerk and acceleration settings, and also by desired speed change.
- The equations that govern the decel period of the TG are similar to the accel period, but use a negative acceleration setting.
- The S-curve profiles can be applied to joint moves or to Cartesian moves.