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Physics 1

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Lecture 9

- Insulators and Conductors in electric field.
- Capacitance, Dielectrics.
- Current, resistance.
- Electromotive Force.

Conductors and Insulators

- Electrical **conductors** are materials in which some of the electrons are free, that are not bound to atoms and can move relatively freely through thematerial.
- Electrical **insulators** are materials in which all electrons are bound to atoms and can not move freely through the material.

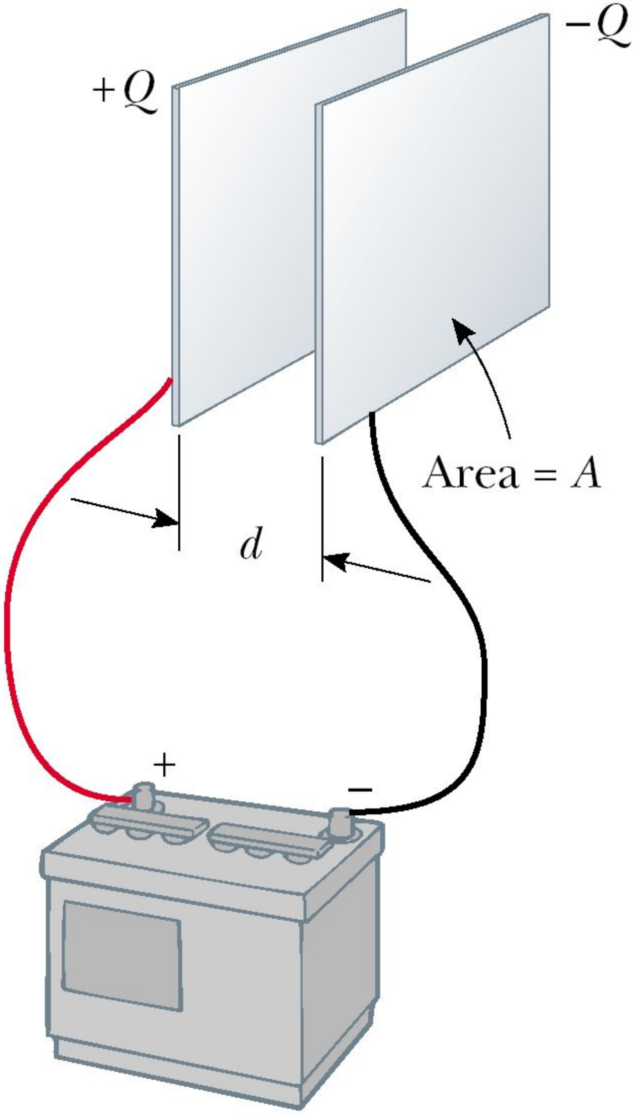
Capacitance

- The capacitance C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors:

$$C \equiv \frac{Q}{\Delta V}$$

- Note: net charge of a capacitor is zero. A capacitor consists of 2 conductors, and Q is the charge on one of each, and correspondingly –Q is the charge on the other.
- Do not confuse C for capacitance with C for the unit coulomb.
- Usually V is taken instead of ΔV for simplicity.

Parallel – Plate Capacitor



A parallel-plate capacitor consists of two parallel conducting plates, each of area *A*, separated by a distance *d*. When the capacitor is charged the plates carry equal amounts of charge. One plate carries positive charge, and the other carries negative charge.

- Using the Gauss theorem we can find that the value of the electric field between plates is

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

- The magnitude of the potential difference between the plates equals:

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

- And finally: $$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}$$
- So the capacitance of a parallel-plate capacitor is

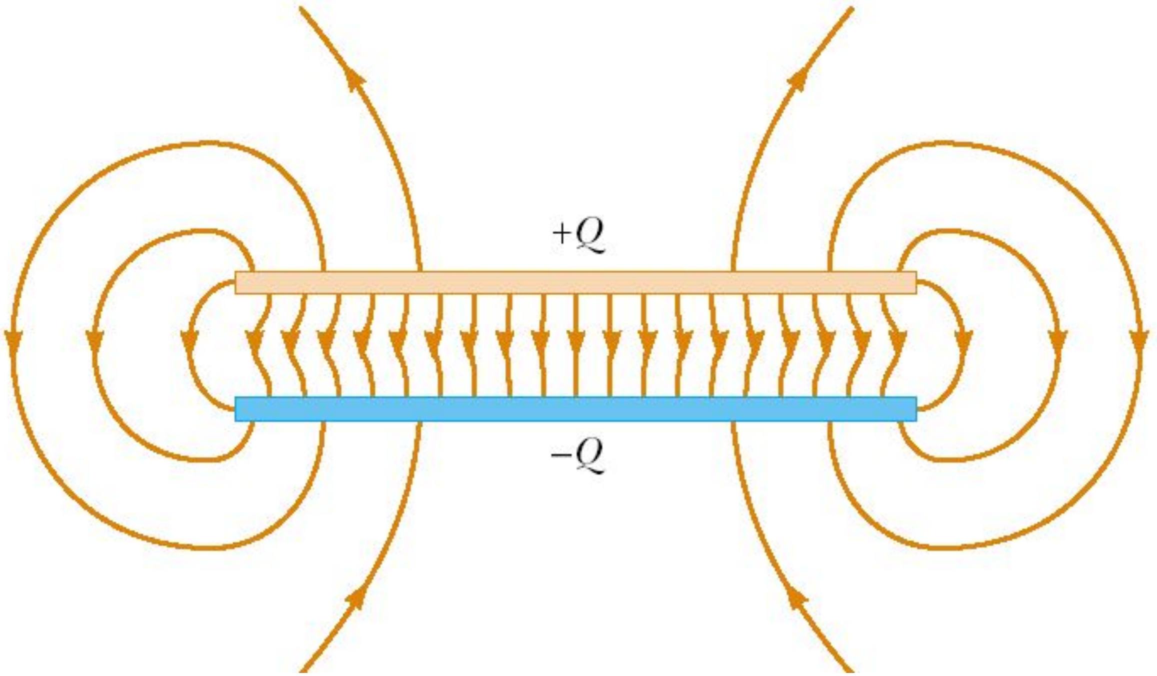
$$C = \frac{\epsilon_0 A}{d}$$

- Here A is the area of each plate, d is the distance between plates.

Capacitance of various Capacitors

Capacitance and Geometry

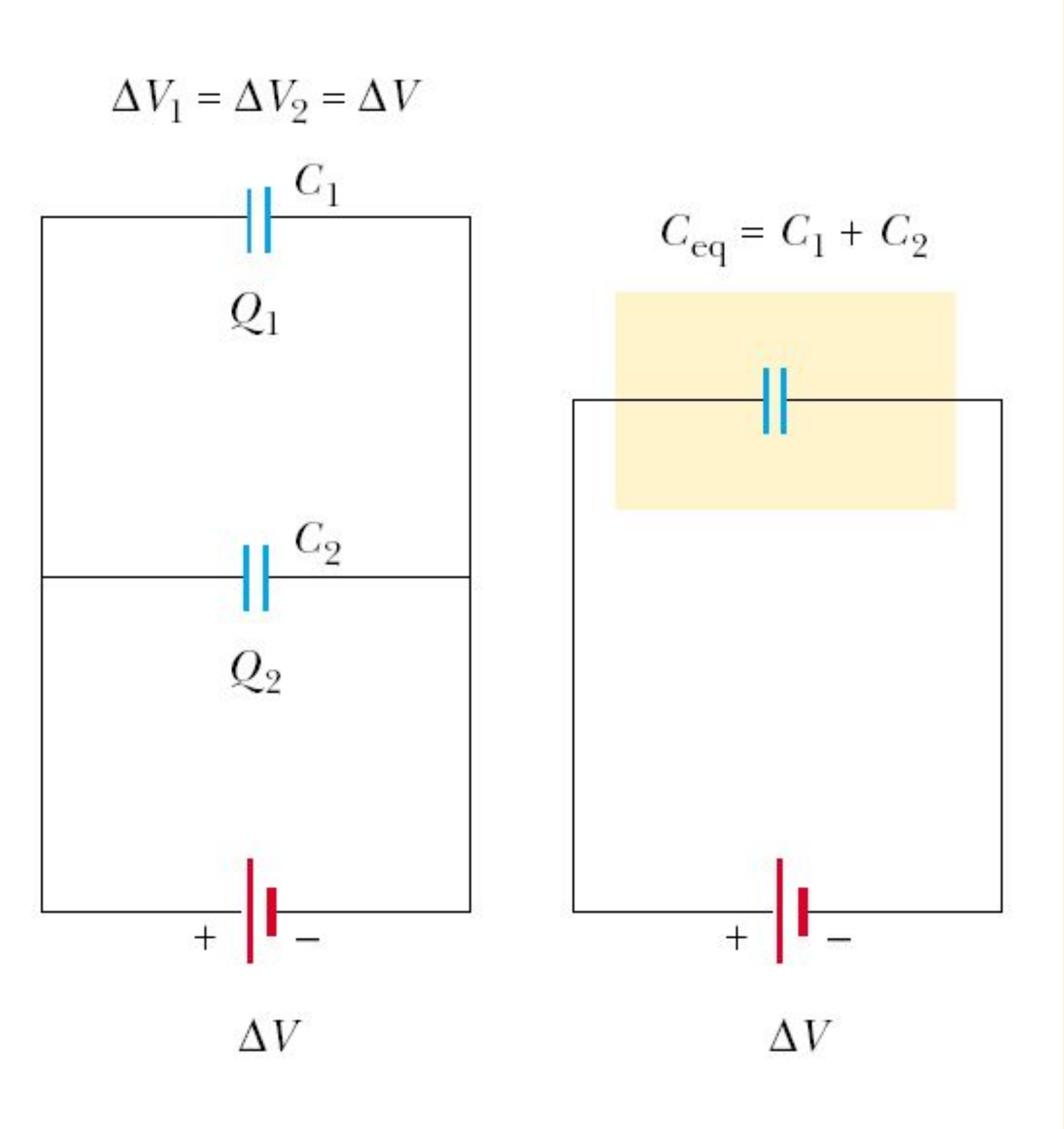
Geometry	Capacitance	Equation
Isolated sphere of radius R (second spherical conductor assumed to have infinite radius)	$C = 4\pi\epsilon_0 R$	26.2
Parallel-plate capacitor of plate area A and plate separation d	$C = \epsilon_0 \dfrac{A}{d}$	26.3
Cylindrical capacitor of length ℓ and inner and outer radii a and b, respectively	$C = \dfrac{\ell}{2k_e \ln(b/a)}$	26.4
Spherical capacitor with inner and outer radii a and b, respectively	$C = \dfrac{ab}{k_e(b - a)}$	26.6



- The electric field between the plates of a parallel-plate capacitor is uniform near the center but nonuniform near the edges.
- That's why we applied formula for electric field between two infinite uniformly charged planes:

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

Parallel Combination of Capacitors



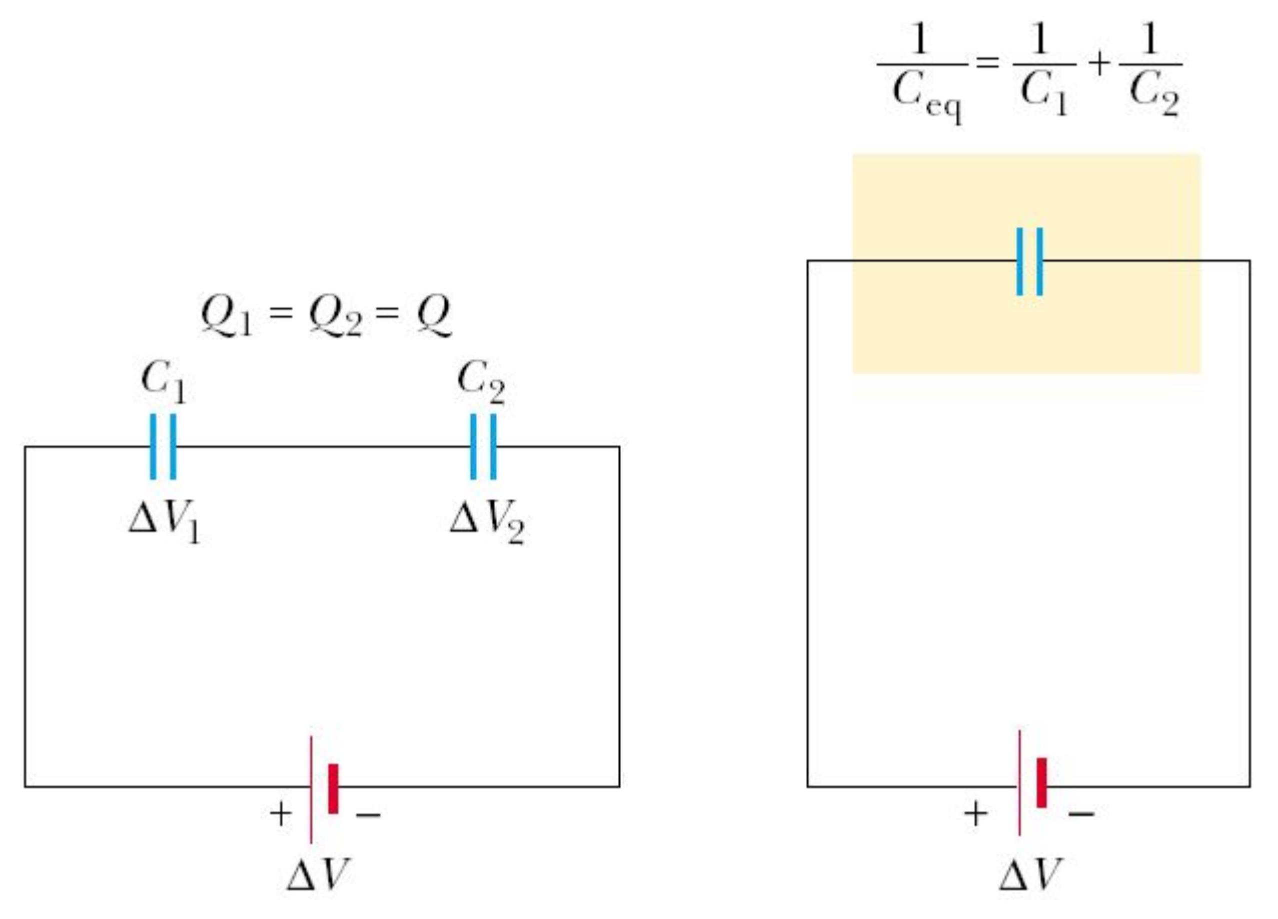
- $C_{eq}=C_1+C_2$
- $Q_{net}=Q_1+Q_2$
- $V=V_1=V_2$

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

Parallel Combination of Capacitors

- The equivalent capacitance of a parallel combination of capacitors is the algebraic sum of the individual capacitances and is greater than any of the individual capacitances.
- $C_{eq}=C_1+C_2+C_3+\ldots$
- The total charge on capacitors connected in parallel is the sum of the charges on the individual capacitors:
- $Q_{net}=Q_1+Q_2+Q_3+\ldots$
- The individual potential differences across capacitors connected in parallel are the same and are equal to the potential difference applied across the combination:
- $V=V_1=V_2=V_3=\ldots$

Series Combination of Capacitors



- $Q=Q_1=Q_2$
- $V=V_1+V_2$
- $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

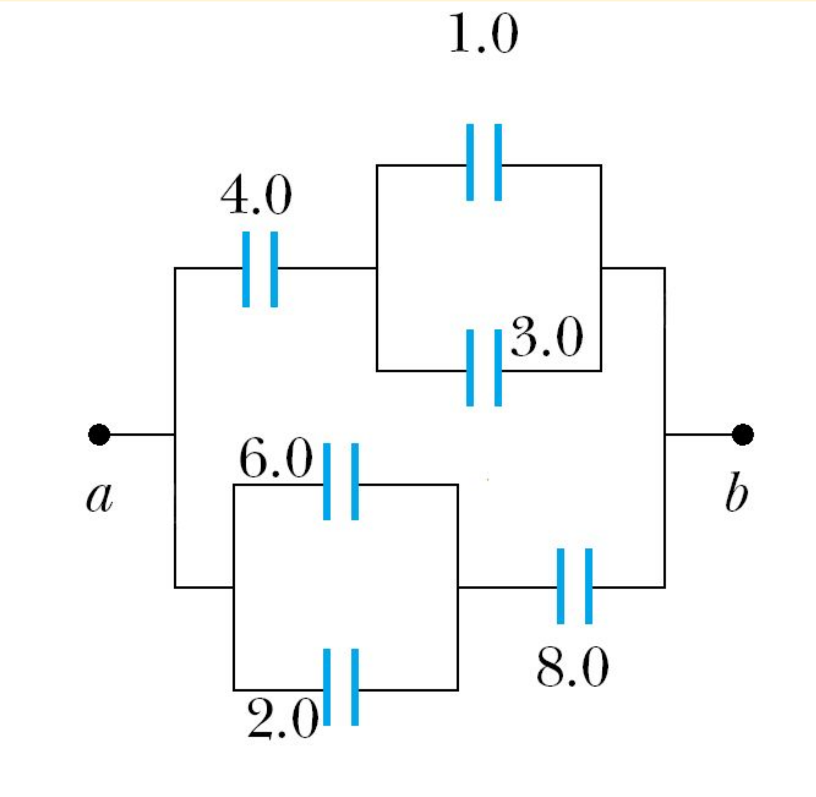
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

Series Combination of Capacitors

- The charges on capacitors connected in series are the same:
- $Q=Q_1=Q_2=Q_3=\dots$
- The total potential difference across any number of capacitors connected in series is the sum of the potential differences across the individual capacitors:
- $V_{net}=V_1+V_2+V_3+\dots$
- The inverse of the equivalent capacitance is the algebraic sum of the inverses of the individual capacitances and the equivalent capacitance of a series combination is always less than any individual capacitance in the combination:

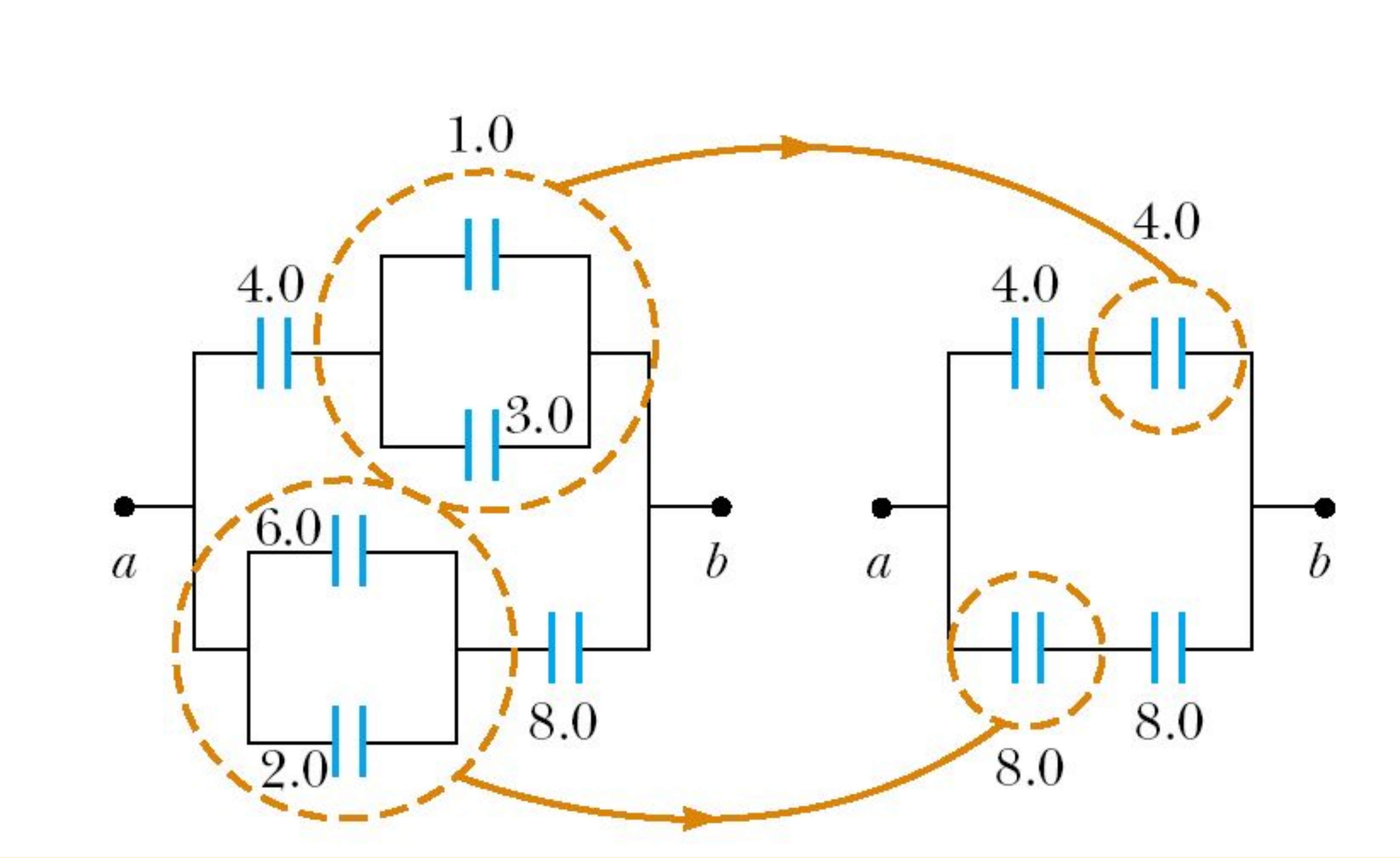
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

Capacitors Parallel-Series Combinations:



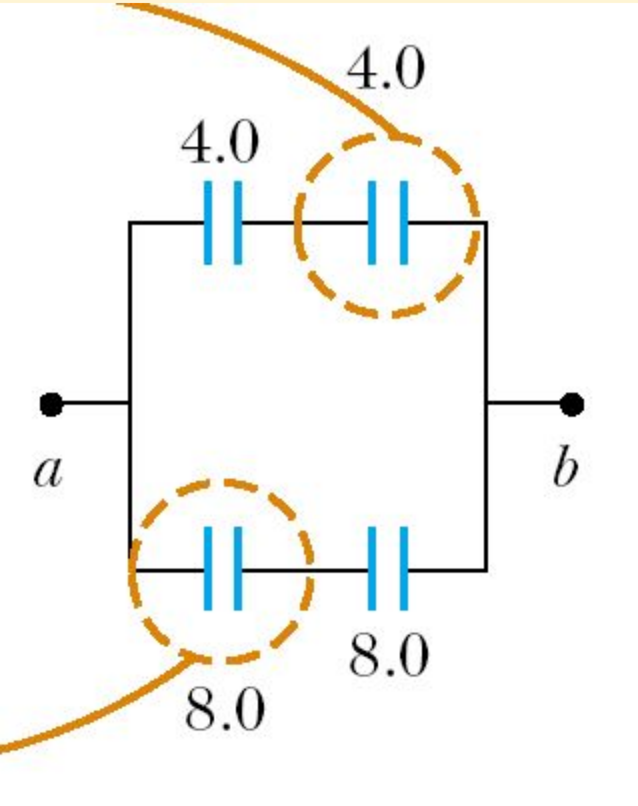
- Let's calculate the equivalent capacitance step by step, using the mentioned above properties of capacitors:
- First we merge parallel capacitors into one: using that $C_{parallel}=C_1+C_2+\ldots$

1. Merging parallel capacitors:

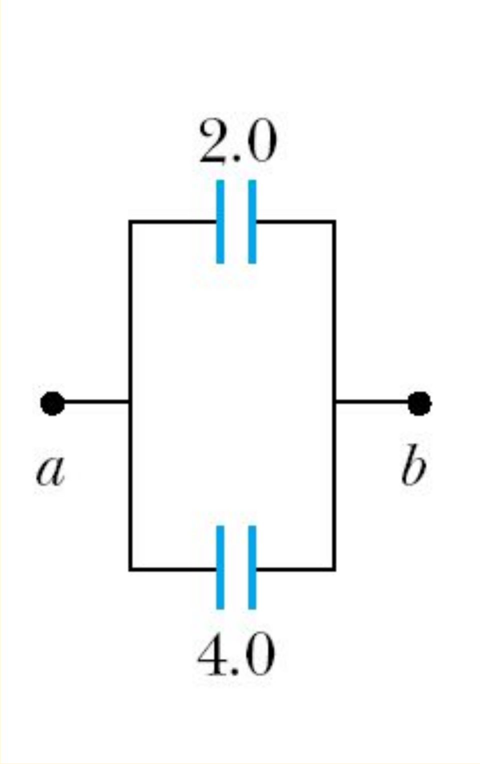


2. Joining serial capacitors:

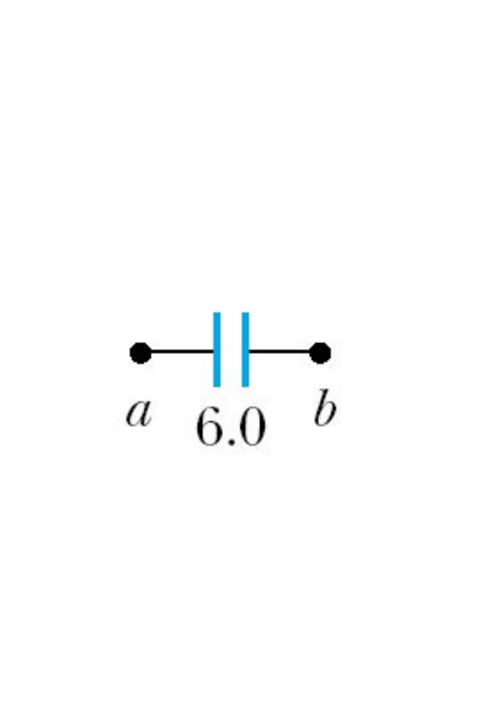
- In circles we have merged capacitors:



- Then we join series conductors:



- And finally we join the two parallel conductors into one:



Energy Stored in a Charged Capacitor

If a capacitor has charge Q then it's difference of potentials V is $V=Q/C$, then the work dW, necessary to transfer small charge dq from one capacitor's conductor to another is:

$$dW = \Delta V \, dq = \frac{q}{C} \, dq$$

Then the total work required to charge the capacitor from $q = 0$ to final charge $q = Q$ is

$$W = \int_0^Q \frac{q}{C} \, dq = \frac{1}{C} \int_0^Q q \, dq = \frac{Q^2}{2C}$$

Energy Stored in a Charged Capacitor

- The work done in charging the capacitor appears as electric potential energy *U* stored in the capacitor

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}Q\,\Delta V = \tfrac{1}{2}C(\Delta V)^2$$

- Here U is the energy, stored in the capacitor,
  ΔV – difference of potentials on the capacitor
- This result applies to any capacitor, regardless of its geometry.

Energy in a Capacitor

Usually **V** is used instead of **ΔV** for the difference of potentials, then the expressions for energy, stored in a capacitor is:

$$U = \frac{Q^2}{2C}.$$

$$U = \frac{QV}{2}.$$

$$U = \frac{CV^2}{2}.$$

Energy in Electric Fields

- Let's take a parallel-plate capacitor:
- *V* - the potential difference between the plates of a capacitor,
- *d* - distance between the plates,
- *A* – the area of each plate,
- *E* - the electric field between the plates of a capacitor. Then *V=Ed*.
- Then the energy of the electric field in the capacitor is:

$$U = \frac{1}{2}\frac{\epsilon_0 A}{d}(E^2 d^2) = \frac{1}{2}(\epsilon_0 A d)E^2$$

Energy density of Electric Field

- The volume, occupied by the electric field is *Ad*, then the energy density of the electric field is:

$$u_E = \frac{1}{2}\epsilon_0 E^2$$

- The energy density in any electric field is proportional to the square of the magnitude of the electric field at a given point.

Dielectrics

- Many materials (like paper, rubber, plastics, glass …) do not conduct electricity easily – we call them insulators.
- But they modify the electric field they are placed in, that's why they are called **dielectrics**.

$$\mathbf{E} = \frac{\mathbf{E}_0}{\kappa}$$

- E_0 – the electric field without the dielectric
- E – the electric field in the presence of the dielectric
- k – the dielectric constant

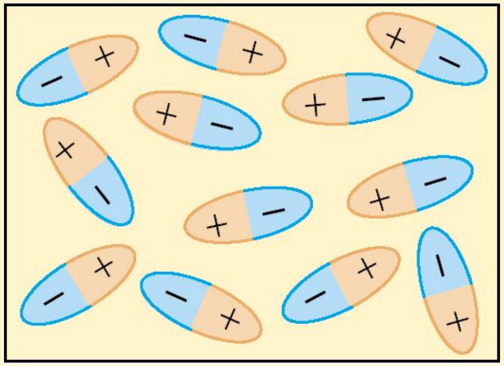
Dielectric strength

- The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. Note that these values depend strongly on the presence of impurities and flaws in the materials.

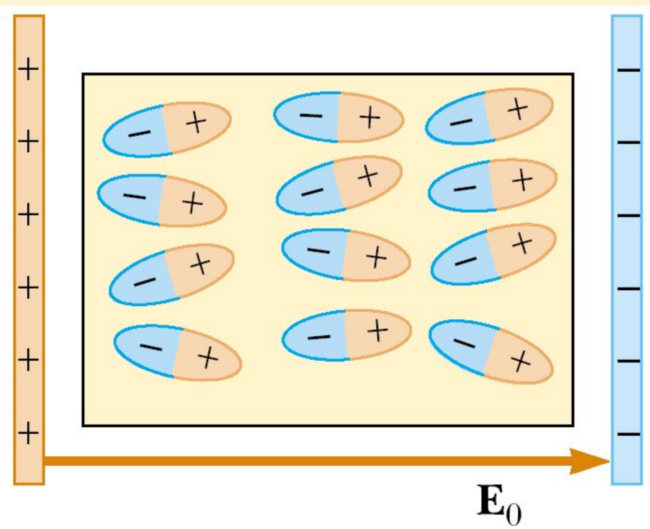
Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

Atomic Description of Dielectrics



- Dielectric can be made up of polar molecules. The dipoles are randomly oriented in the absence of an electric field.



- When an external Electric field is applied, its molecules partially align with the field. Now the dielectric is polarized.

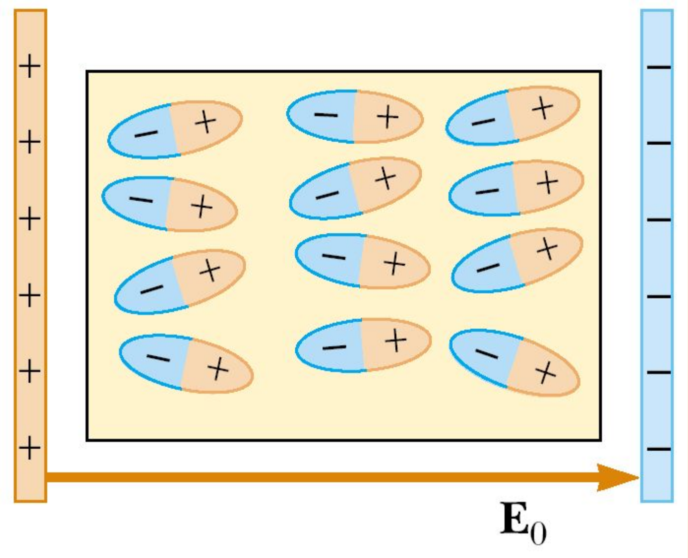
Polar and Nonpolar molecules of Dielectric

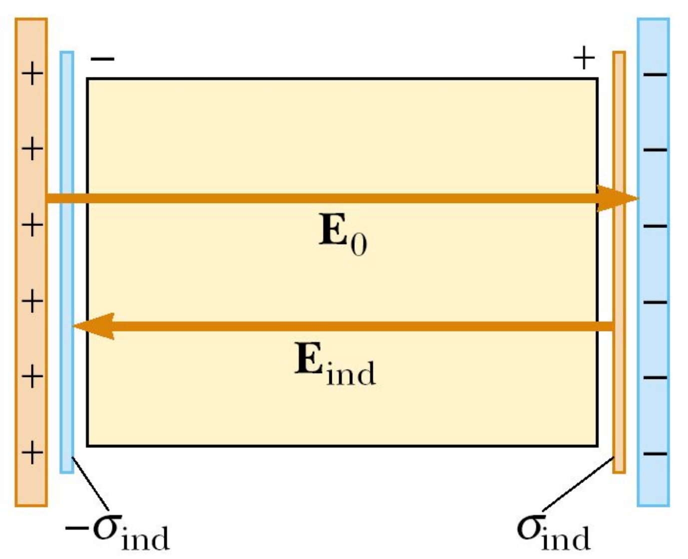
- The molecules of the dielectric can be polar or nonpolar.
- The case of polar molecules are considered in the previous slide.
- If the molecules of the dielectric are nonpolar then the electric field produces some charge separation in every molecule of the dielectric, and an *induced dipole moment* is created. These induced dipole moments tend to align with the external field, and the dielectric is polarized.
- Thus, we can polarize a dielectric with an external field regardless of whether the molecules are polar or nonpolar.

Dielectric polarization

- The degree of alignment of the molecules with the electric field depends on temperature and on the magnitude of the electric field.
- In general, the alignment increases with decreasing temperature and with increasing electric field.

Induced Electric field in Dielectric





- When an external field E_0 is applied, a torque is exerted on the dipoles, causing them to partially align with the field.

- That's why dielectric's molecules produces induced electric field E_{ind}, opposite to the external E_0.

Capacitor with Dielectric

- So the electric field is *k* times less in a capacitor with a dielectric, its dielectric constant is *k*:

$$\mathbf{E} = \frac{\mathbf{E}_0}{\kappa}$$

- Then, the potential difference is *k* times less:

without dielectric: $\mathbf{V_0 = E_0 d}$

with dielectric: $\mathbf{V = Ed = E_0 d/k.}$

$\mathbf{V = V_0/k.}$

- As the charge *Q* on the capacitor is not changed:
- $C_0 = Q/V_0$, $V = V_0/k$
- $C = Q/V = kC_0V_0/V_0 = kC_0$

$$C = kC_0$$

- So the capacitance increases in *k* if a dielectric completely fills the distance between the plates of a capacitor.

Usage of Dielectrics in Capacitors

- Insulating materials have $k>1$ and dielectric strength greater than that of air, so usage of dielectrics has following advantages:
  - Increase in capacitance.
  - Increase in maximum operating voltage.
  - Possible mechanical support between the plates, which allows the plates to be close together without touching, thereby decreasing d and increasing C.

Electric Current

- Electric current (or just current) is defined as the total charge that passes through a given cross-sectional area per unit time.
- Current can be composed of
  - moving negative charges such as electrons or negatively charged ions;
  - moving positive charges such as protons or positively charged ions.
- *ΔQ* - the amount of charge *passing through the cross sectional area of a wire*
- *ΔT* - a time interval of the passing.
- The average current is:

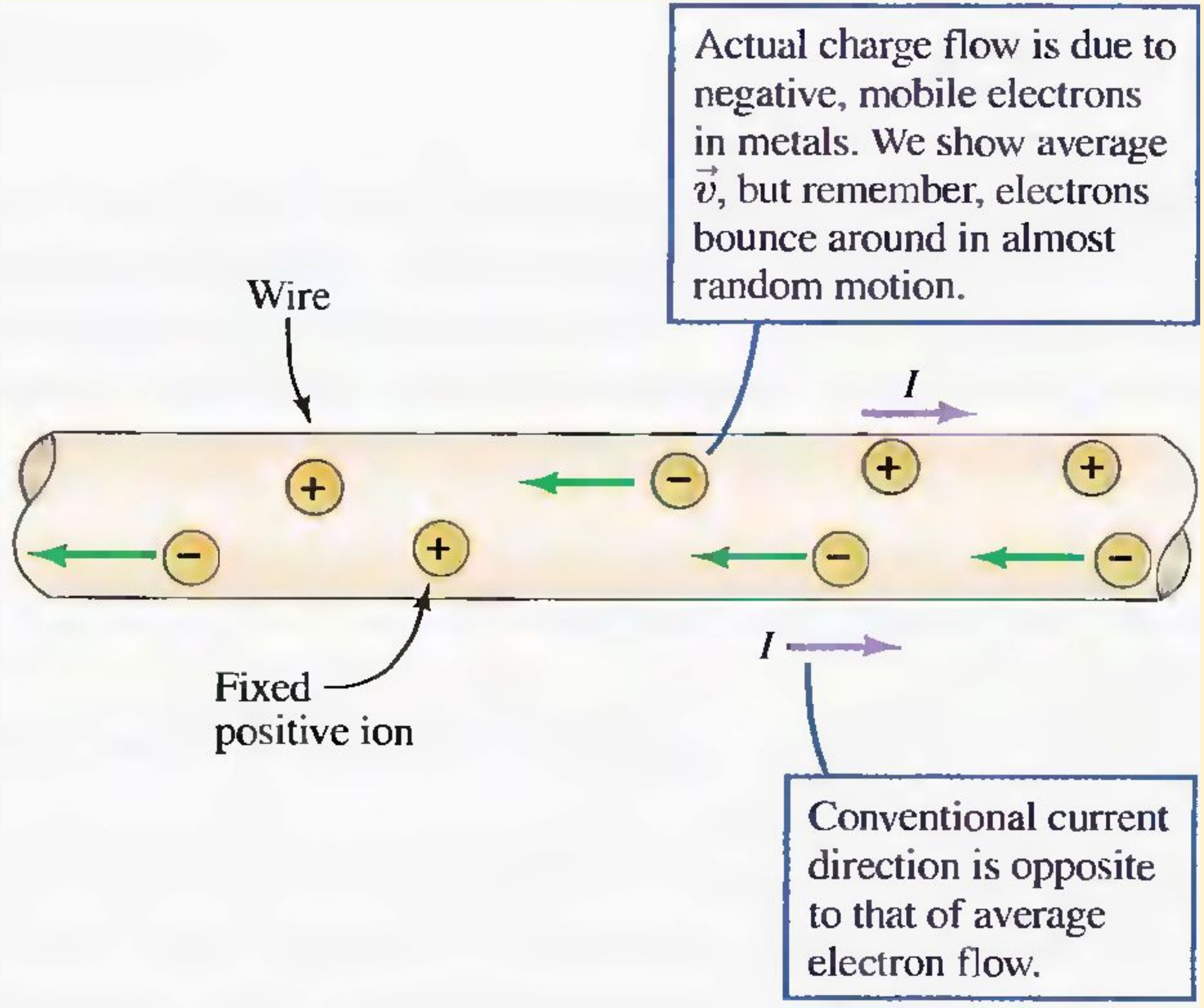
$$I_{av} = \frac{\Delta Q}{\Delta t}$$

- The instantaneous current is:

$$I \equiv \frac{dQ}{dt}$$

Current direction

- By convention the direction of the current is the direction of positive charges would move.



Ohm's Law

- Ohm's law states that
- For many materials the resistance is constant over a wide range of potential differences:

$$V = IR.$$

- Resistance is defined as the opposition to the flow of electric charge.

Electromotive Force

- A device with the ability to maintain potential difference between two points is called a **source of electromotive force** (emf). The most familiar sources of emf are *batteries* and *generators*.
- Batteries convert chemical energy into electric energy.
- Generators transforms mechanical energy into electric energy.
- Since emf is work per unit charge, it is expressed in the same unit as potential difference: the *joule per coulomb*, or volt.

Units in Si

- Capacitance C F=C/V
- Current I A=C/s
- Resistance R Ohm=V/A
- Electro motive force (emf) ε V
- Energy density u_E $J/m^3 = kg/(m*s^2)$