Panel Methods

What are panel methods?

- Panel methods are techniques for solving incompressible potential flow over thick 2-D and 3-D geometries.
- In 2-D, the airfoil surface is divided into piecewise straight line segments or panels or "boundary elements" and vortex sheets of strength γ are placed on each panel.
  - We use vortex sheets (miniature vortices of strength γds, where ds is the length of a panel) since vortices give rise to circulation, and hence lift.
  - Vortex sheets mimic the boundary layer around airfoils.

Analogy between boundary layer and vortices

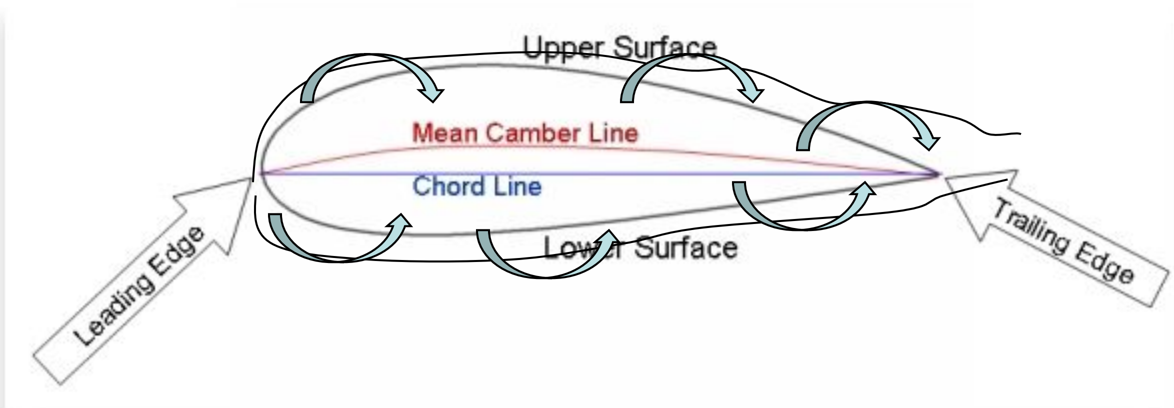


FIGURE 3

Upper surface boundary layer contains, in general, clockwise rotating vorticity

Lower surface boundary layer contains, in general, counter clockwise vorticity.

Because there is more clockwise vorticity than counter clockwise Vorticity, there is net clockwise circulation around the airfoil.

In panel methods, we replace this boundary layer, which has a small but finite thickness with a thin sheet of vorticity placed just outside the airfoil.

Panel method treats the airfoil as a series of line segments

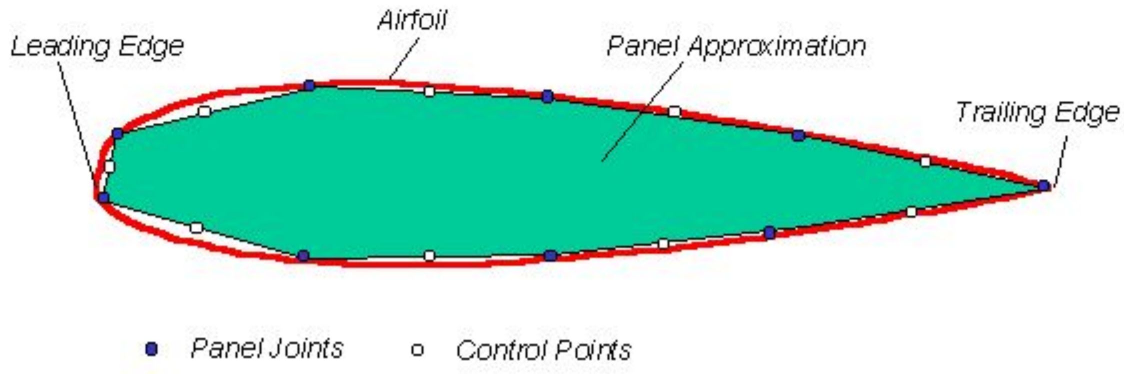


Figure 1. Vortex panel approximation to an airfoil.

On each panel, there is vortex sheet of strength $\Delta\Gamma = \gamma_0\, ds_0$
Where ds_0 is the panel length.

Each panel is defined by its two end points (panel joints)
and by the control point, located at the panel center, where we will
Apply the boundary condition ψ= Constant=C.

The more the number of panels, the more accurate the solution,
since we are representing a continuous curve by a series
of broken straight lines

Boundary Condition

- We treat the airfoil surface as a streamline.
  - This ensures that the velocity is tangential to the airfoil surface, and no fluid can penetrate the surface.
- We require that at all control points (middle points of each panel) $\psi = C$
- The stream function is due to superposition of the effects of the free stream and the effects of the vortices $\gamma_0\ ds_0$ on each of the panel.

Stream Function due to freestream

- The free stream is given by $u_\infty y - v_\infty x$

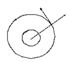
Recall $$\frac{\partial \psi}{\partial y} = u; \qquad -\frac{\partial \psi}{\partial y} = v$$

This solution satisfies conservation of mass $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

And irrotationality $$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0$$

It also satisfies the Laplace's equation. Check!

Stream function due to a Counterclockwise Vortex of Strengh Γ



$$\psi = -\frac{\Gamma}{2\pi}\ln(r)$$

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = 0$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = \frac{\Gamma}{2\pi r}$$

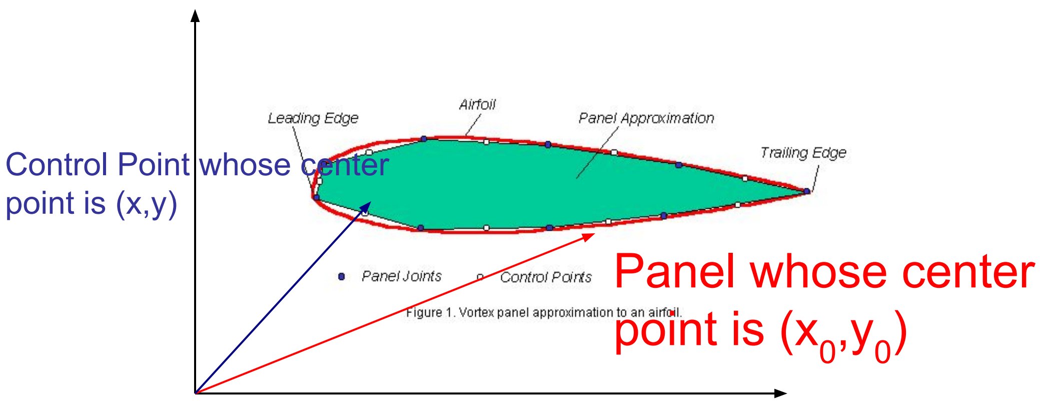
Stream function Vortex, continued..

- Pay attention to the signs.
- A counter-clock wise vortex is considered "positive"
- In our case, the vortex of strength $\gamma_0 ds_0$ had been placed on a panel with location (x_0 and y_0).
- Then the stream function at a point (x,y) will be

$$\psi = -\frac{\gamma_0 ds_0}{2\pi}\ln\left(\left|\bar{r}-\bar{r}_o\right|\right)$$

where

$$\left(\left|\bar{r}-\bar{r}_o\right|\right)=\sqrt{\left(x-x_0\right)^2+\left(y-y_0\right)^2}$$



Superposition of All Vortices on all Panels

- In the panel method we use here, ds_0 is the length of a small segment of the airfoil, and γ_0 is the vortex strength per unit length.
- Then, the stream function due to all such infinitesimal vortices at the control point (located in the middle of each panel) may be written as the interval below, where the integral is done over all the vortex elements on the airfoil surface.

$$-\oint \frac{\gamma_0}{2\pi} \ln\left(\bar{r} - r_0\right) ds_0$$

Adding the freestream and vortex effects..

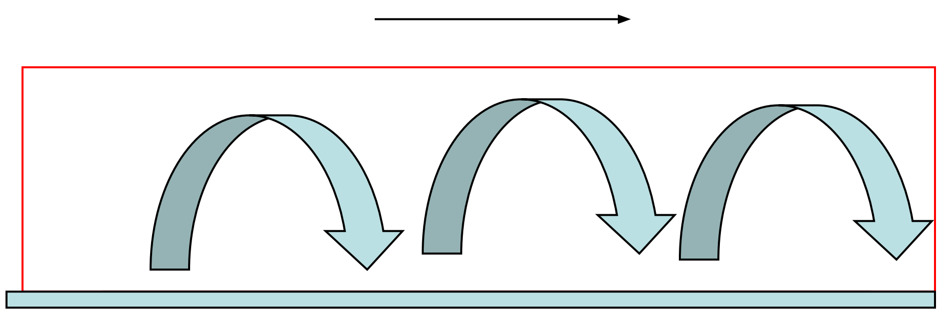
$$u_\infty y - v_\infty x - \frac{1}{2\pi}\oint \gamma_0 \ln\left(\left|\bar{r} - \bar{r}_0\right|\right) ds_o = C$$

The unknowns are the vortex strength γ_0 on each panel, and the value of the
Stream function C.

Before we go to the trouble of solving for γ_0, we ask what is the purpose..

Physical meaning of γ_0

V = Velocity of the flow just outside the boundary layer



Sides of our contour have zero height
Bottom side has zero
Tangential velocity
Because of viscosity

Panel of length ds_0 on the airfoil

Its circulation = $\Delta\Gamma = \gamma_0\, ds_0$

$$Circulation = \oint_{Contour} \vec{V} \bullet \vec{ds} = \gamma_0 ds_0 = -V ds_0$$

$$Or, \quad V = -\gamma_0$$

If we know γ_0 on each panel, then we know the velocity of the flow outside the boundary layer for that panel, and hence pressure over that panel.

Pressure distribution and Loads

$$\text{Bernoulli says: } p + \frac{1}{2}\rho\left(u^2 + v^2\right) = p_\infty + \frac{1}{2}\rho_\infty V_\infty^2$$

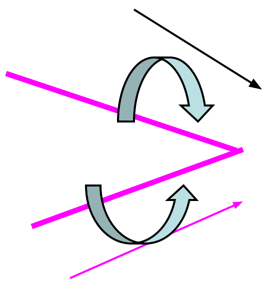
$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} = 1 - \frac{u^2 + v^2}{V_\infty^2} = 1 - \frac{V^2}{V_\infty^2}$$

Since $V = -\gamma_0$

$$C_p = 1 - \frac{\gamma_0^2}{V_\infty^2}$$

Kutta Condition

- Kutta condition states that the pressure above and below the airfoil trailing edge must be equal, and that the flow must smoothly leave the trailing edge in the same direction at the upper and lower edge.



$$\gamma^2_{upper} = V^2_{upper}$$
$$\gamma^2_{lower} = V^2_{lower}$$

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From this sketch above, we see that pressure will be equal, and the flow will leave the trailing edge smoothly, only if the voritcity on each panel is equal in magnitude above and below, but spinning in opposite Directions relative to each other.

$$\gamma_{Upper} = -\gamma_{lower}$$

Summing up..

- We need to solve the integral equation derived earlier
- And, satisfy Kutta condition.

$$u_\infty y - v_\infty x - \frac{1}{2\pi}\oint \gamma_0 \ln\left(\left|\bar{r} - \bar{r}_0\right|\right) ds_o = C$$

$$\gamma_{Upper} = -\gamma_{lower}$$

Numerical Procedure

- We divide the airfoil into N panels. A typical panel is given the number j, where J varies from 1 to N.
- On each panel, we assume that γ_0 is a piecewise constant. Thus, on a panel numbered j, the unknown strength is $\gamma_{0,j}$
- We number the control points at the centers of each panel as well. Each control point is given the symbol "i", where i varies from 1 to N.
- The integral equation becomes

$$u_\infty y_i - v_\infty x_i - \sum_{j=1..N} \frac{\gamma_{0,j}}{2\pi} \int_j \ln\left(\left|\bar{r}_i - \bar{r}_0\right|\right) ds_o - C = 0$$

Numerical procedure, continued

- Notice that we use two indices ‘i’ and ‘j’. The index ‘I’ refers to the control point where equation is applied.
- The index ‘j’ refers to the panel over which the line integral is evaluated.
- The integrals over the individual panels depends only on the panel shape (straight line segment), its end points and the control point í’.
- Therefore this integral may be computed analytically.
- We refer to the resulting quantity as

$$u_\infty y_i - v_\infty x_i - \sum_{j=1}^{N} A_{i,j}\gamma_{0j} - C = 0$$

$where,$

$$A_{i,j} = \text{Influence of Panel j on index i} = \frac{1}{2\pi}\int \ln\left(\left|\bar{r}_i - \bar{r}_0\right|\right)ds_0$$

Numerical procedure, continued..

- We thus have N+1 equations for the unknowns $\gamma_{0,j}$ (j=1…N) and C.
- We assume that the first panel (j=1) and last panel (j=N) are on the lower and upper surface trailing edges.

$$u_\infty y_i - v_\infty x_i - \sum_{j=1}^{N} A_{i,j}\gamma_{0j} - C = 0$$

$$\gamma_{0,1} = -\gamma_{0,N}$$

This linear set of equations may be solved easily, and γ_0 found.
Once go is known, we can find pressure, and pressure coefficient Cp.

Panel code

- Our web site contains a Matlab code I have written, if you wish to see how to program this approach in Matlab.
- See http://www.ae.gatech.edu/people/lsankar/AE3903/Panel.m
- And, sample input file http://www.ae.gatech.edu/people/lsankar/AE3903/panel.data.txt

- An annotated file telling you what the avrious numbers in the input means is found at
  - http://www.ae.gatech.edu/people/lsankar/AE3903/Panel.Code.Input.txt

PABLO

- A more powerful panel code is found on the web.
- It is called PABLO: Potential flow around Airfoils with Boundary Layer coupled One-way
- See http://www.nada.kth.se/~chris/pablo/pablo.html
- It also computes the boundary layer growth on the airfoil, and skin friction drag.
- Learn to use it!
- We will later on show how to compute the boundary layer characteristics and drag.