Special Theory of Relativity





Postulates

• The laws of physics are the same in all inertial reference frames. No experiment can be performed to decide who in a set of inertial frames is moving and who is at rest.

 The speed of light in empty space is the same in all inertial frames

The Lorentz transformations

Inertial frame at rest:

O (x,y,z,t)

Inertial frame moving with velocity v: O' (x',y',z',t')

$$x' = y (x-vt)$$
 $x = y (x'+vt')$
 $t' = y (t - vx/c^2)$ $t = y (t' + vx'/c^2)$
 $y = 1/\sqrt{(1-v^2/c^2)}$ $y = 1/\sqrt{(1-v^2/c^2)}$
 $y' = y$ $z' = z$ $y = y'$ $z = z'$
 $V_x' = (V_x - V)/(1 - V_x V/c^2)$ $V_x = (V_x' + V)/(1 + V_x' V/c^2)$

Length constriction

$$t_1 = t_1' = 0$$

We measure
$$L = X_2 - X_1$$

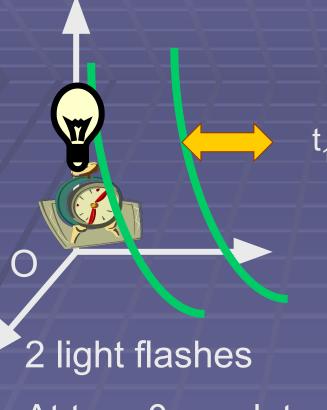
at $t_1 = t_2 = 0$

We have to calculate L' at $t_1' = t_2' = 0$ $t_1' = \gamma (t_1 - vx_1/c^2) = 0$ $t_2' = \gamma (t_2 - vx_2/c^2) = \gamma (t_2 - v L/c^2) = 0$ $t_1 = 0 \quad and \quad t_2 = v L/c^2$

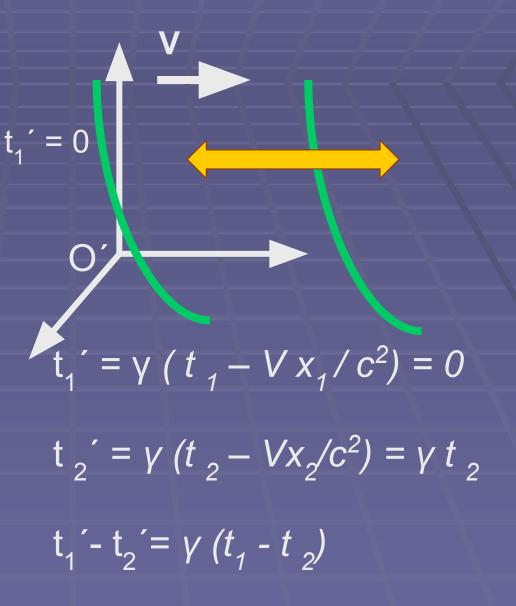
$$x_{1}' = \gamma (x_{1} - vt_{1}) = 0$$

 $x_{2}' = \gamma (x_{2} - vt_{2}) = \gamma (L - v^{2}L/c^{2}) = \gamma L(1-v^{2}/c^{2}) = L/\gamma$
 $L' = x_{2}' - x_{1}' = L/\gamma$

Time dilation



At
$$t_1 = 0$$
, and t_2
 $X_1 = X_2 = 0$



Momentum and energy

The relativistic momentum:

P = mV
$$/\sqrt{(1-v^2/c^2)} = \gamma mv$$

 $\gamma = 1/\sqrt{(1-v^2/c^2)}$

The relativistic energy:

$$E = mc^2 / \sqrt{(1-v^2/c^2)} = \gamma mc^2$$

$$K = mc^2 (\gamma - 1)$$

The energy and momentum are related by:

$$E = \sqrt{p^2 c^2 - m^2 c^4}$$