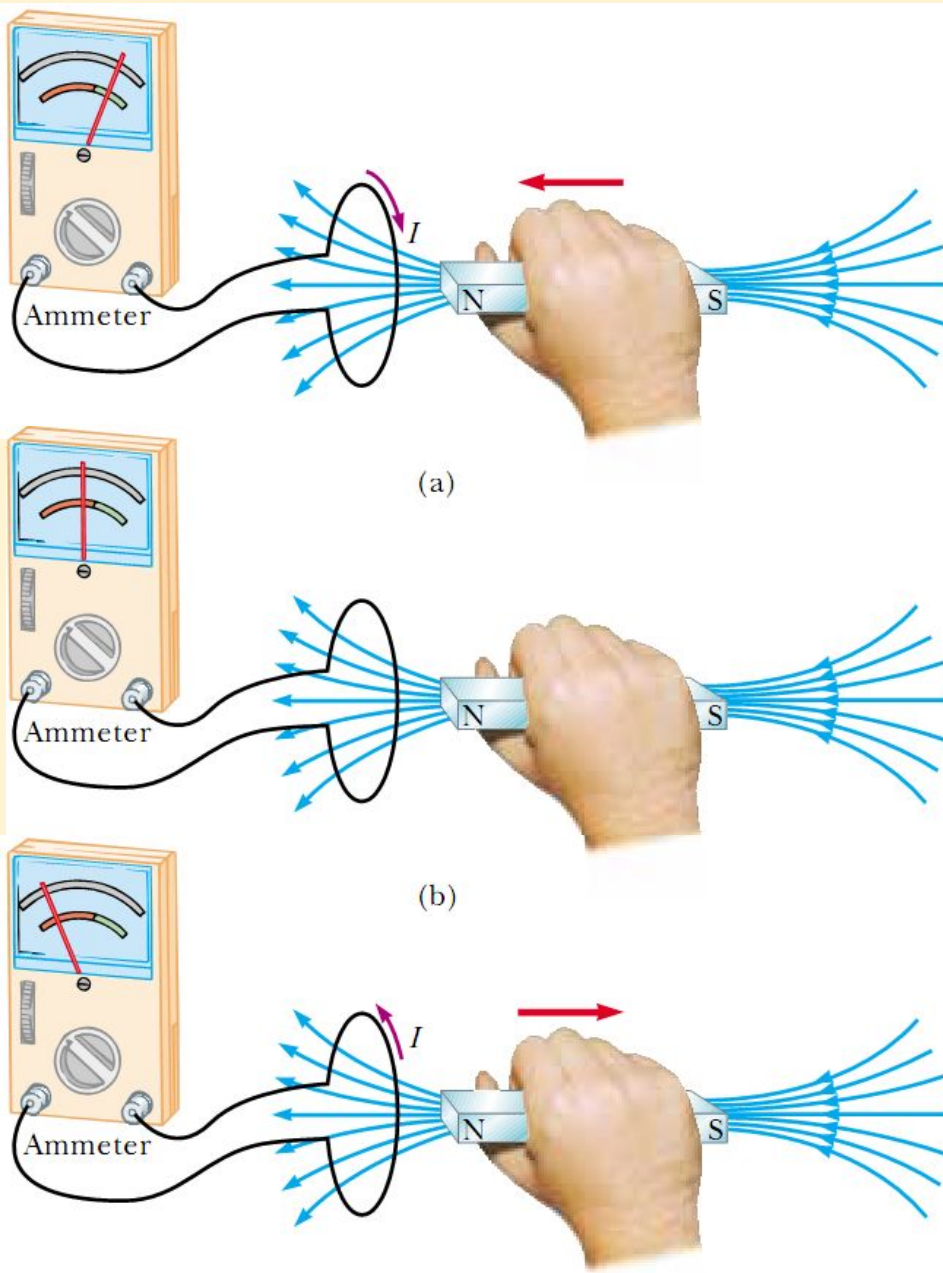


Physics 1

Voronkov Vladimir Vasilyevich

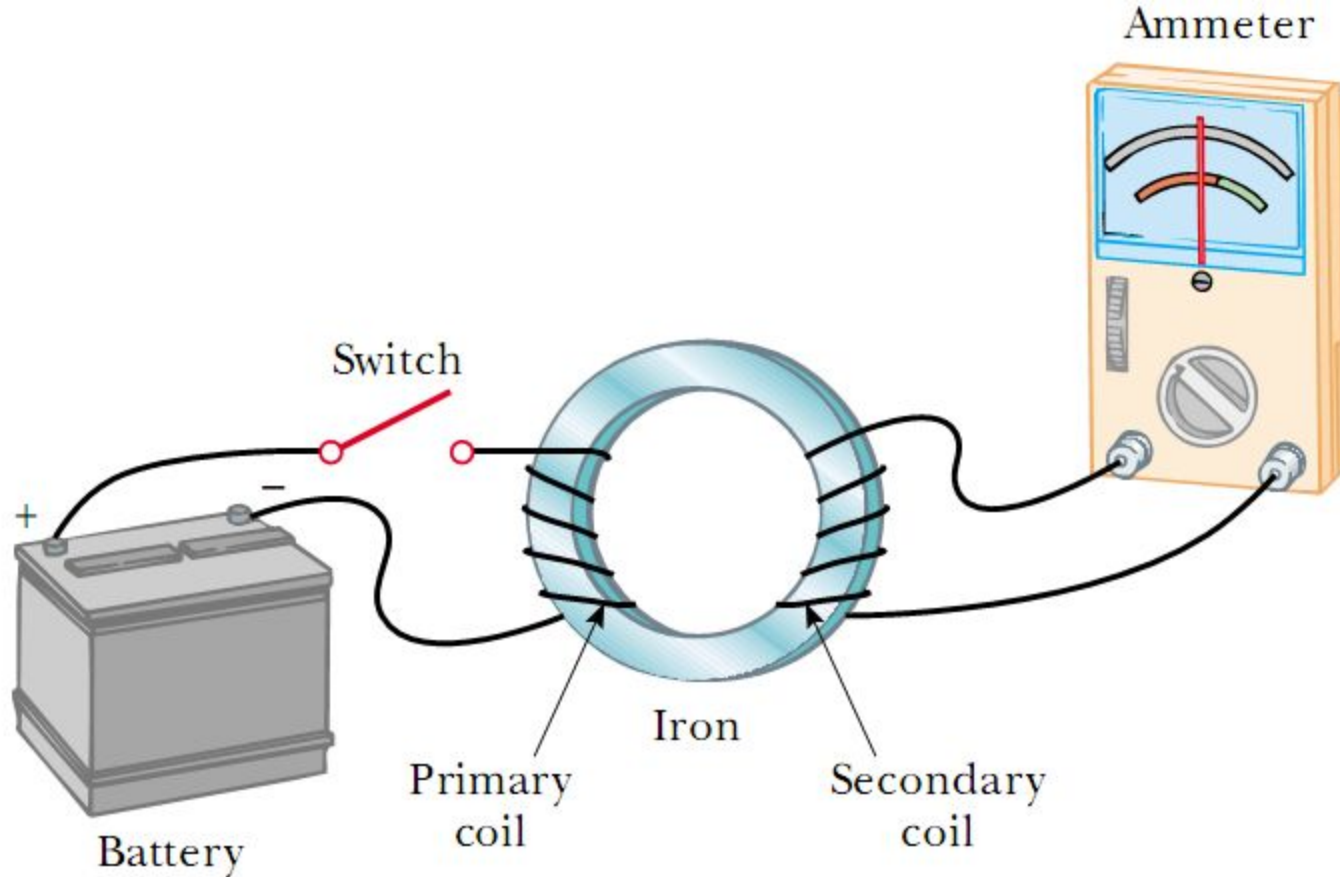
Lecture 13

- Faraday's law of induction
- Lenz' law
- Eddy currents
- Maxwell's equations



- When a magnet is moved toward a loop of wire connected to a sensitive ammeter, the ammeter shows that a current is induced in the loop.
- When the magnet is held stationary, there is no induced current in the loop, even when the magnet is inside the loop.
- When the magnet is moved away from the loop, the ammeter deflects in the opposite direction, indicating that the induced current is opposite that shown in part (a).

- From these observations, we conclude that the loop detects that the magnet is moving relative to it and we relate this detection to a change in magnetic field.
- Thus, it shows that a relationship exists between **current** and **changing magnetic field**.
- So a current is set up even though no batteries are present in the circuit! We call such a current an **induced current** and say that it is produced by an **induced emf**.



- Initially, when the switch is open, no current is detected by the ammeter. The same is when the switch is closed ammeter detects no current.
- When the switch in the primary circuit is either opened or thrown closed. At the instant the switch is closed, the galvanometer needle deflects in one direction and then returns to zero. At the instant the switch is opened, the needle deflects in the opposite direction and again returns to zero.

- So when the switch was open, and is closed, the **changing current** in the primary circuit creates **changing magnetic field** in the iron ring, which creates current in the secondary coil.
- When the switch was closed, and is opened, the current in the primary circuit also is changing – from some constant value to zero, it creates changing magnetic field in the iron ring, which creates current in the secondary coil.

Summary of the two experiments

An electric current can be induced in a circuit by a changing magnetic field. The induced current exists only while the magnetic field through the secondary coil is changing. Once the magnetic field reaches a steady value, the current in the coil disappears.

Faraday's Law of Induction

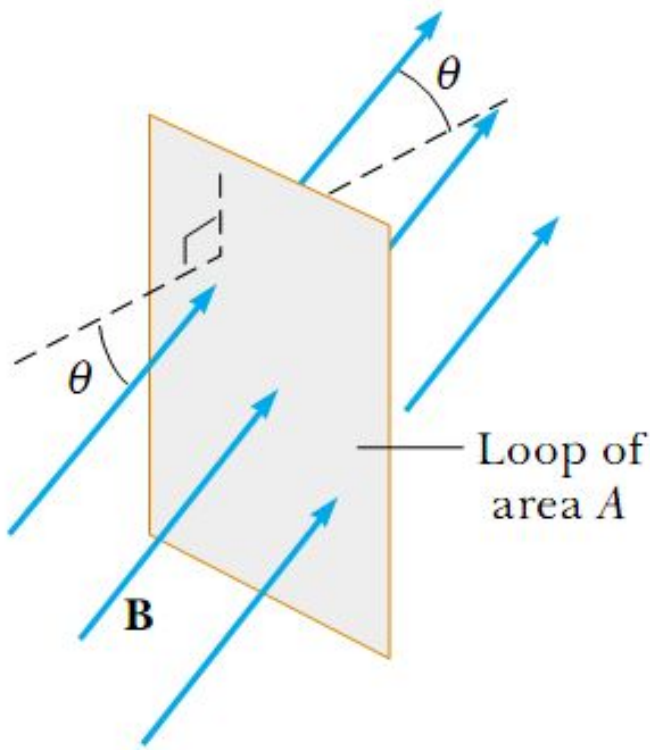
- The emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit.

$$\mathcal{E} = - \frac{d\Phi_B}{dt}$$

- Here Φ_B is the magnetic flux through the circuit:

$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$$

Loop in a uniform magnetic field



A conducting loop that encloses an area A in the presence of a uniform magnetic field B . The angle between B and the normal to the loop is Θ .

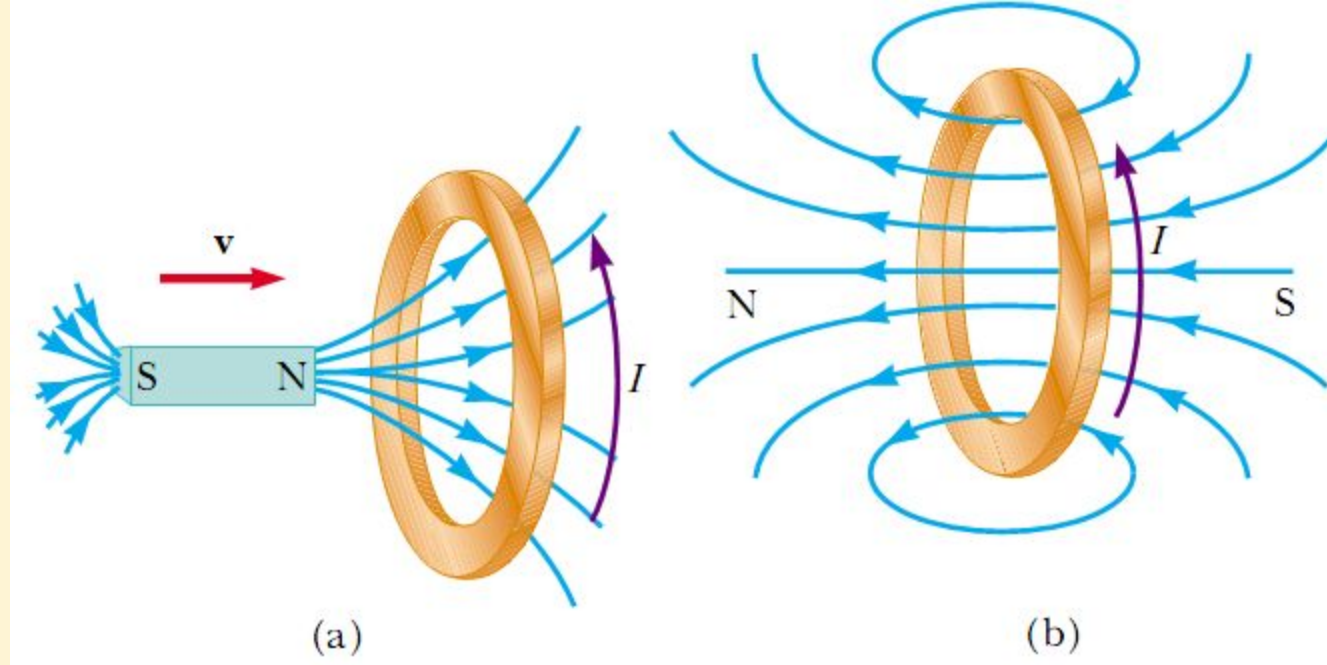
$$\Phi_B = BA \cos \Theta$$

An emf can be induced in the circuit in several ways:

- The magnitude of B can change with time.
- The area enclosed by the loop can change with time.
- The angle Θ between B and the normal to the loop can change with time.
- Any combination of the above can occur.

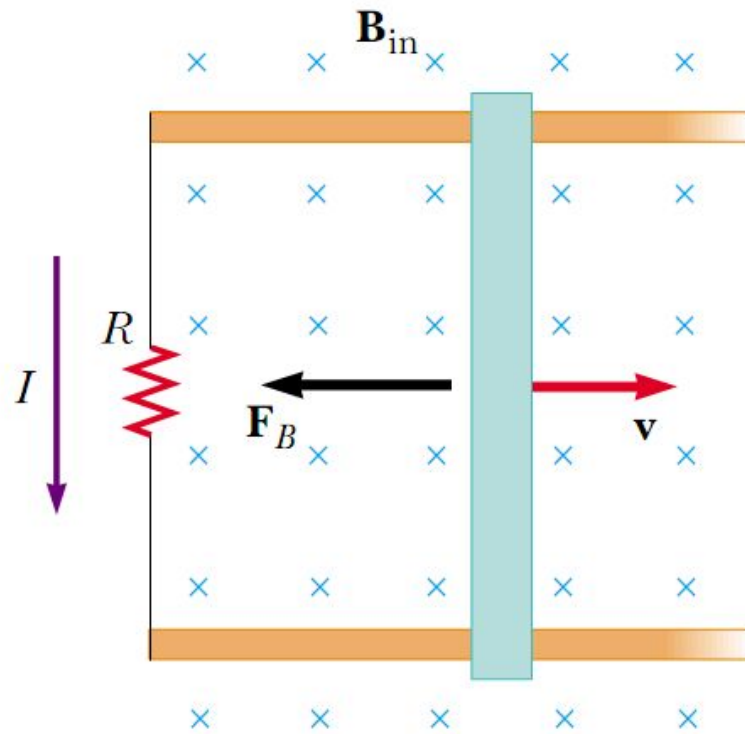
Lenz's Law

- Faraday's law ($\varepsilon = -d\Phi_B/dt$) indicates that the induced emf and the change in flux have opposite algebraic signs. This has a very real physical interpretation that has come to be known as Lenz's law (*2 equivalent variants*):
 - 1. The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop.**
 - 2. Induced currents produce magnetic fields which tend to cancel the flux changes that induce them.**



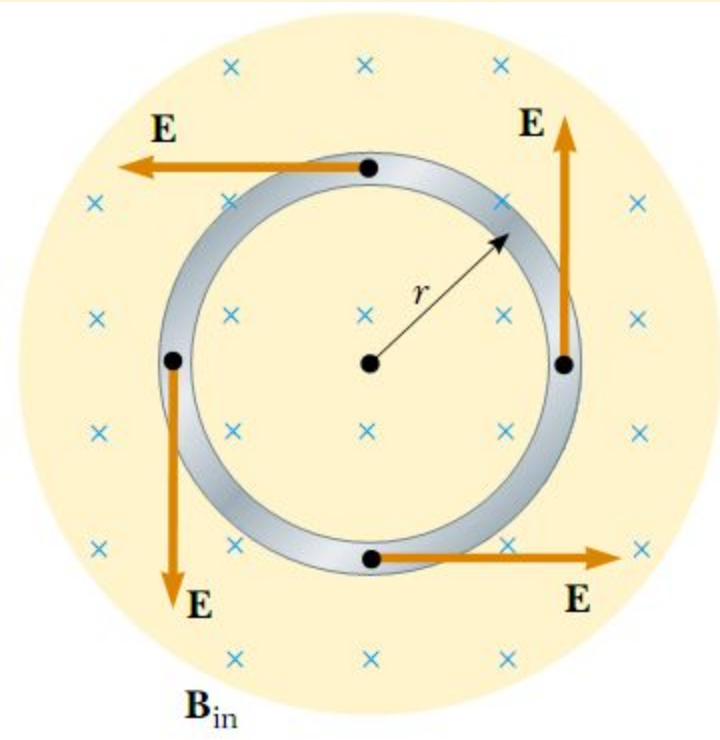
- Picture (a): When the magnet is moved toward the stationary conducting loop, a current is induced in the direction shown. The magnetic field lines shown belong to the bar magnet.

- Picture (b): The induced current produces its own magnetic field directed to the left that counteracts the increasing external flux. The magnetic field lines shown are those due to the induced current in the ring.



As the conducting bar slides on the two fixed conducting rails, the magnetic flux due to the external magnetic field, through the area enclosed by the loop, increases in time. By Lenz's law, the induced current must be counterclockwise so as to produce a counteracting magnetic field directed out of the page.

Induced emf and Electric Fields



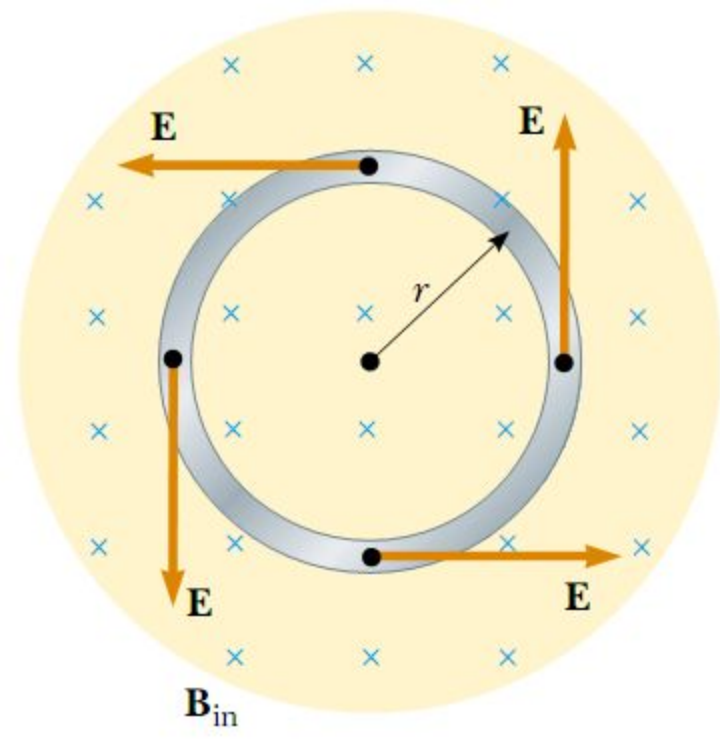
A conducting loop of radius r in a uniform magnetic field perpendicular to the plane of the loop. If B changes in time, an electric field is induced in a direction tangent to the circumference of the loop.

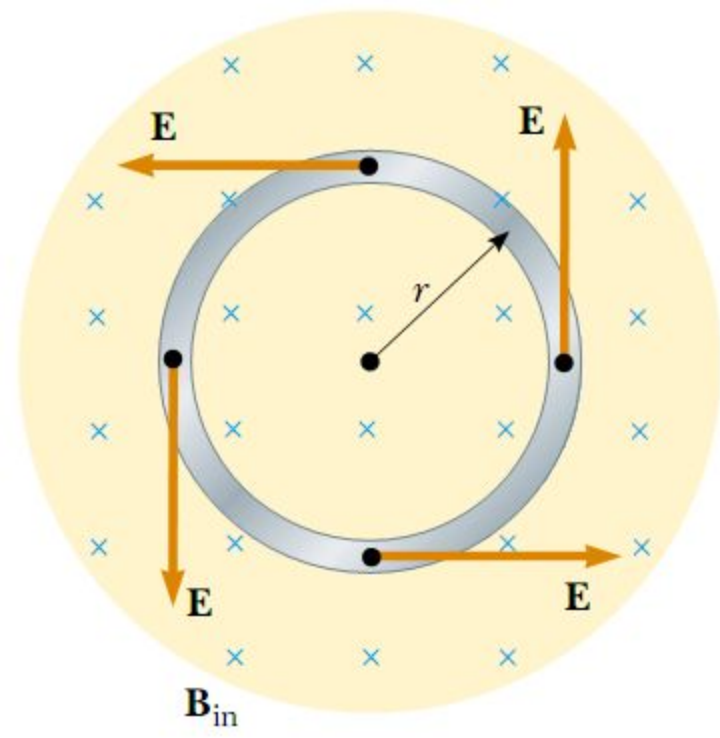
$$\mathcal{E} = -d\Phi_B/dt$$

The work done by the electric field in moving a test charge q once around the loop is equal to $q\mathcal{E}$. Because the electric force acting on the charge is qE , the work done by the electric field in moving the charge once around the loop is $qE(2\pi r)$:

$$q\mathcal{E} = qE(2\pi r)$$

$$E = \frac{\mathcal{E}}{2\pi r}$$





$$\Phi_B = BA = \pi r^2 B$$

$$E = -\frac{1}{2\pi r} \frac{d\Phi_B}{dt} = -\frac{r}{2} \frac{dB}{dt}$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

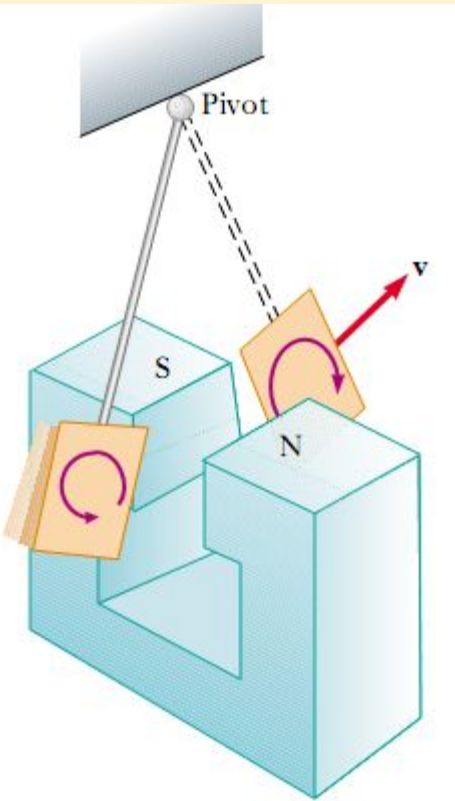
General form of Faraday's Law of Induction

$$\oint \mathbf{E} \cdot d\mathbf{s} = - \frac{d\Phi_B}{dt}$$

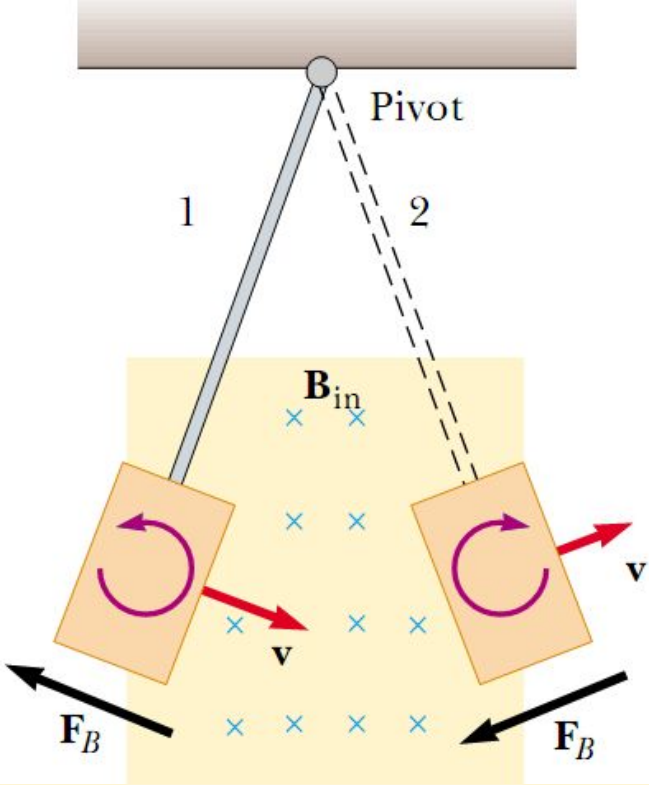
- Here $\oint \mathbf{E} \cdot d\mathbf{s}$ is integration along an enclosed path.
- Φ_B is a magnetic flux through that enclosed path (along which the integration $\oint \mathbf{E} \cdot d\mathbf{s}$ goes).
- The induced electric field \mathbf{E} in this expression is a nonconservative field that is generated by a changing magnetic field.

Eddy Currents

- Circulating currents called **eddy currents** are induced in bulk pieces of metal moving through a magnetic field.
- Eddy currents are used in many braking systems of subway and rapid-transit cars.
- Eddy currents are often undesirable because they represent a transformation of mechanical energy to internal energy. To reduce this energy loss, conducting parts are often laminated—that is, they are built up in thin layers separated by a nonconducting material. This layered structure increases the resistance of eddy current paths and effectively confines the currents to individual layers.



Eddy currents appears in a **solid conducting plate** moving through a magnetic field. As the plate enters or leaves the field, the **changing magnetic flux** induces an emf, which causes eddy currents in the plate.



As the conducting plate enters the field (position 1), the eddy currents are counterclockwise. As the plate leaves the field (position 2), the currents are clockwise. In either case, the force on the plate is opposite the velocity, and eventually the plate comes to rest.



When slots are cut in the conducting plate, the eddy currents and retarding force are reduced and the plate swings more freely through the magnetic field.

Maxwell's Equations

Maxwell's equations applied to free space, that is, in the absence of any dielectric or magnetic material:

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0 \quad (2)$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

- Equation (1):

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$$

- Here integration goes across an enclosed surface, q is the charge inside it.
- This is the Gauss's law: the total electric flux through any closed surface equals the net charge inside that surface divided by ϵ_0 .
- This law relates an electric field to the charge distribution that creates it.

- Equation (2):

$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0$$

- Here integration goes across an enclosed surface. It can be considered as Gauss's law in magnetism, states that **the net magnetic flux through a closed surface is zero.**
- That is, the number of magnetic field lines that enter a closed volume must equal the number that leave that volume. This implies that magnetic field lines cannot begin or end at any point. It means that there is no isolated magnetic monopoles exist in nature.

- Equation (3):

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

- Here integration goes along an enclosed path, Φ_B is a magnetic flux through that enclosed path.
- This equation is Faraday's law of induction, which describes the creation of an electric field by a changing magnetic flux.
- This law states that the emf, which is the line integral of the electric field around any closed path, equals the rate of change of magnetic flux through any surface area bounded by that path.

- Equation (4):

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

- This is Ampère–Maxwell law, or the generalized form of Ampère’s law. It describes the creation of a magnetic field by an electric field and electric currents: **the line integral of the magnetic field around any closed path is the sum of μ_0 times the net current through that path and $\epsilon_0 \mu_0$ times the rate of change of electric flux through any surface bounded by that path.**

Maxwell's Equations Symmetry

- Equations (1) and (2) are symmetric, apart from the absence of the term for magnetic monopoles in Equation (2).
- Equations (3) and (4) are symmetric in that the line integrals of \mathbf{E} and \mathbf{B} around a closed path are related to the rate of change of magnetic flux and electric flux, respectively.
- Maxwell's equations are of fundamental importance not only to electromagnetism but to all of science.

- Maxwell's equations predict the existence of electromagnetic waves (traveling patterns of electric and magnetic fields), which travel with a speed

$$c = 1/\sqrt{\mu_0\epsilon_0} = 3.00 \times 10^8 \text{ m/s}$$

the speed of light. Furthermore, the theory shows that such waves are radiated by accelerating charges.

Units in Si

- Emf ε V
- Electric Flux Φ_E V*m
- Magnetic Flux Φ_B T*m²
- Electric permeability
of free space: $\epsilon_0 = 8.8542 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$
- Magnetic permeability
of free space: $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A}$