KAZAKH KBTU BRITISH
TECHNICAL
UNIVERSITY

Physics 1

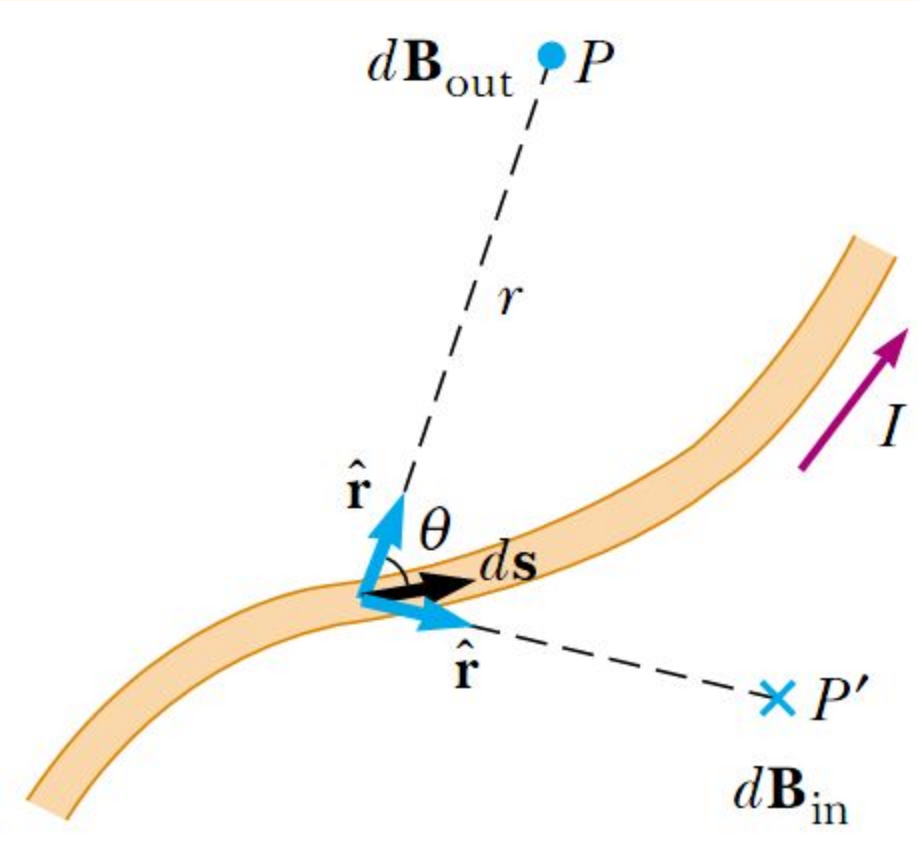
Voronkov Vladimir Vasilyevich

Lecture 12

- Sources of the Magnetic Field
  - The Biot-Savart Law
  - Ampere's Law

- The effects of magnetic fields.
- The production and properties of magnetic fields.

Current Produces Magnetic Field



The magnetic field **dB** at a point P due to the current I through a length element **ds** is given by the Biot–Savart law. The direction of the field is out of the page at P and into the page at P´.

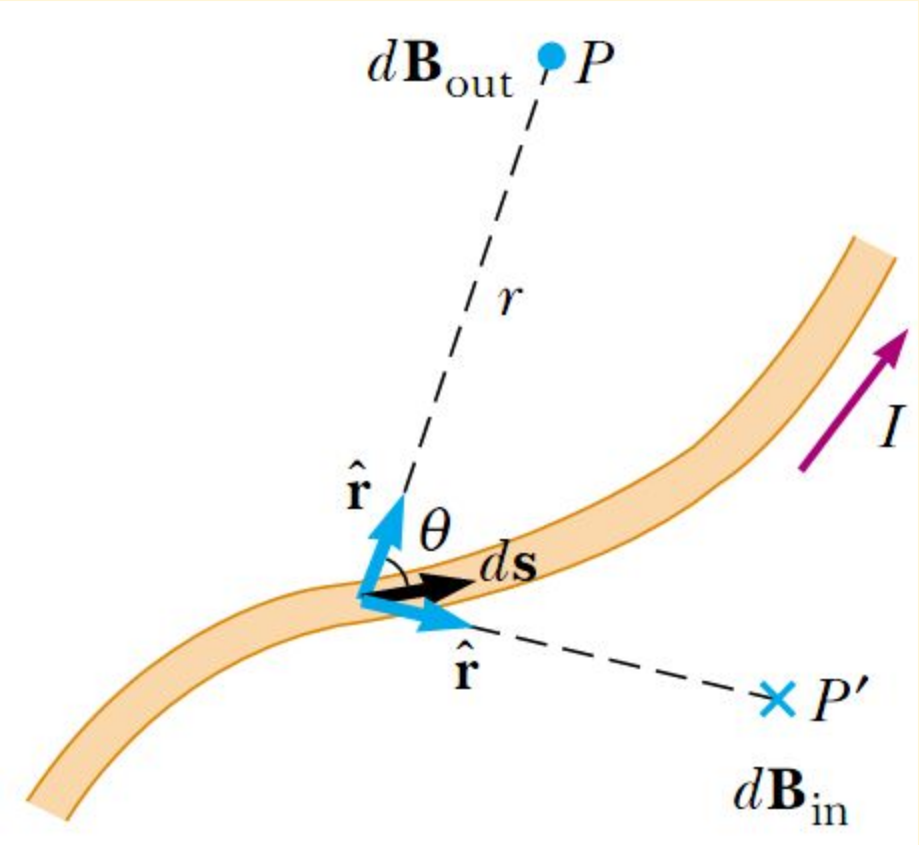
The Biot-Savart Law

The experimental observations for the magnetic field ***dB*** at a point *P* associated with a length element ***ds*** of a wire carrying a steady current ***I*:**

- The vector **dB** is perpendicular both to **ds** (which points in the direction of the current) and to the unit vector $\hat{r}$ directed from **ds** toward P.
- The magnitude of **dB** is inversely proportional to r^2, where r is the distance from **ds** to P.
- The magnitude of **dB** is proportional to the current and to the magnitude **ds** of the length element **ds**.
- The magnitude of **dB** is proportional to $\sin\Theta$, where Θ is the angle between the vectors **ds** and $\hat{r}$.

- The foregoing experimental observations can be expressed in one formula:

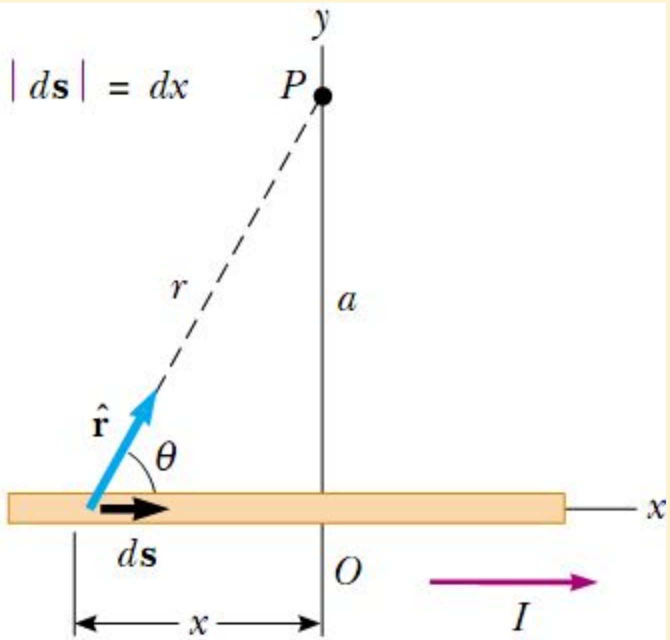
$$\boxed{d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\, d\mathbf{s} \times \hat{\mathbf{r}}}{r^2}}$$

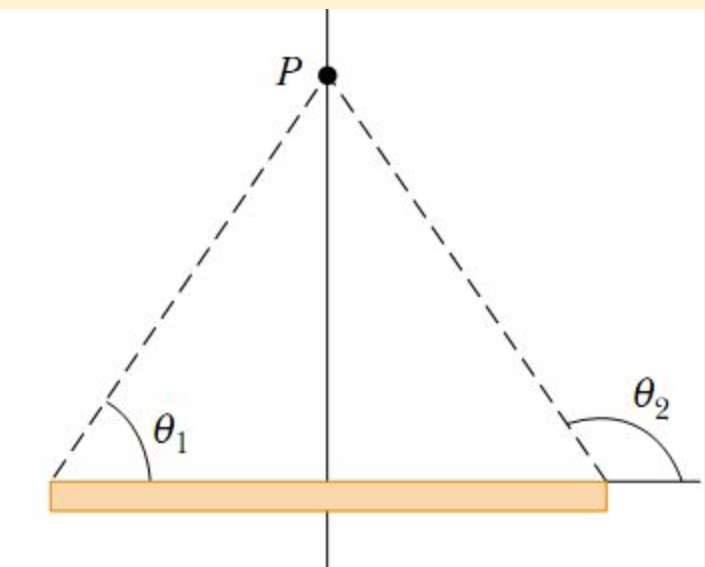


- Here ***dB*** is a magnetic force at a point P associated with a length element ***ds*** of a wire carrying a steady current *I*.
- Unit vector $\hat{r}$ is directed from **ds** toward P.
- r is the distance from **ds** to P.
- μ_0 is the permeability of free space: $\mu_0 = 4\pi \times 10^{-7}\, \mathrm{T \cdot m/A}$

$$\mathbf{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$$

Magnetic Field of a Thin Straight Wire





- Using the Biot-Savart law we can find the magnetic field at point P, created by a thin straight wire with current in it:

$$B = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2)$$

- a is the distance from the wire to P
- Θ_1, Θ_2 are the angles shown in the picture.

Magnetic Field of an Infinitely Long Wire

- For a very long thin straight wire we can consider $\Theta_1=0$, $\Theta_2=\pi$, then:

$$B = \frac{\mu_0 I}{2\pi a}$$

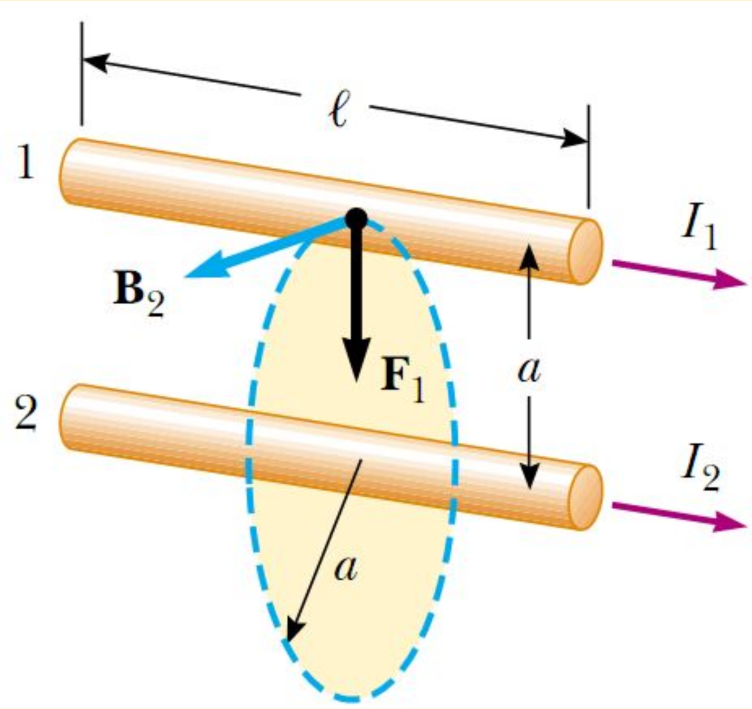
- a is the distance from the wire to P
- I is the current in the wire
- This expression shows that the magnitude of the magnetic field is proportional to the current and decreases with increasing distance from the wire.

Magnetic Field around a Wire

Because of the symmetry of the wire, the magnetic field lines are circles concentric with the wire and lie in planes perpendicular to the wire. The magnitude of ***B*** is constant on any circle of radius ***a*** and is given by the expression on the previous slide:

$$B = \frac{\mu_0 I}{2\pi a}$$

Magnetic Force Between Two Parallel Conductors



$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

Parallel conductors carrying currents

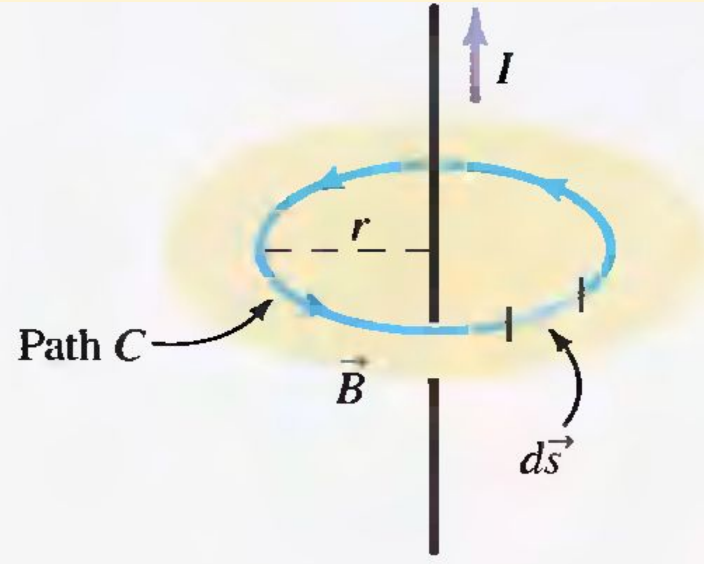
- in the same direction attract each other.
- in opposite directions repel each other.

Ampere's Law

- The line integral of B*ds around any closed path equals $\mu_0 I$, where I is the total steady current passing through any surface bounded by the closed path.

$$\boxed{\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I}$$

Example for the Ampere's Law



- We choose integration along the path C:

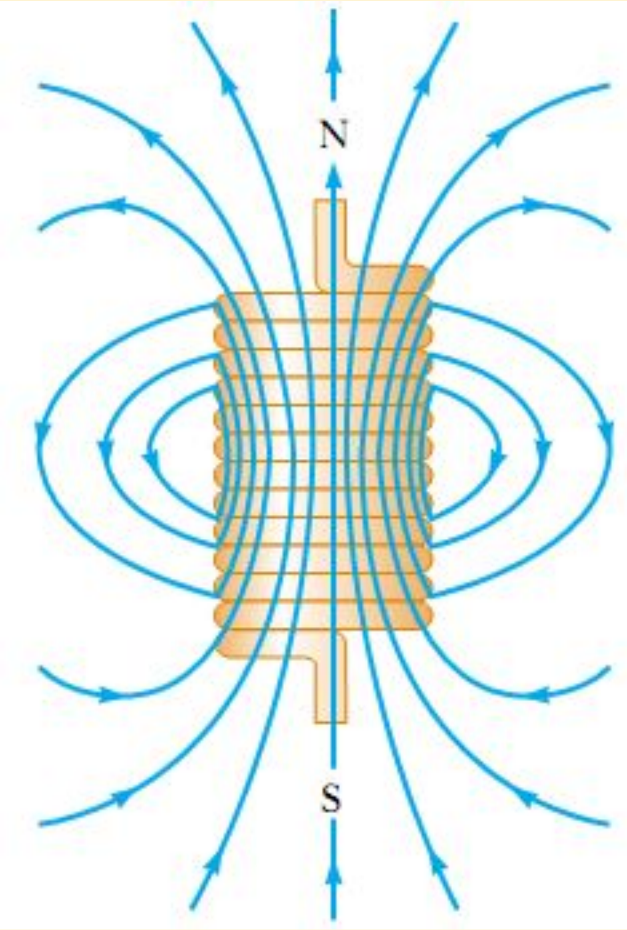
$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds =$$

$$= B(2\pi r) = \mu_0 I$$

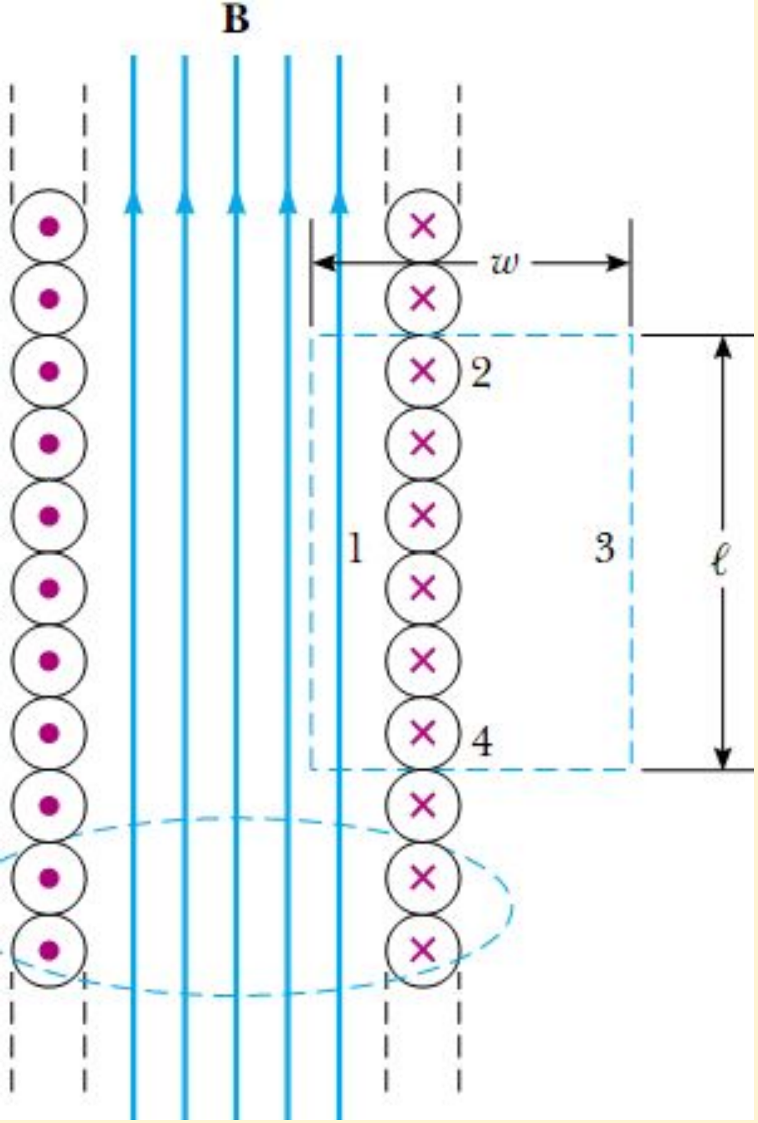
- And finally we have the result (cf. slide 7)

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

Magnetic Field of a Solenoid



- A solenoid is a long wire wound in the form of a helix.
- Magnetic field lines for a tightly wound solenoid of finite length, carrying a steady current. The field in the interior space is strong and nearly uniform.



Cross-sectional view of an ideal solenoid, where the interior magnetic field is uniform and the exterior field is close to zero.

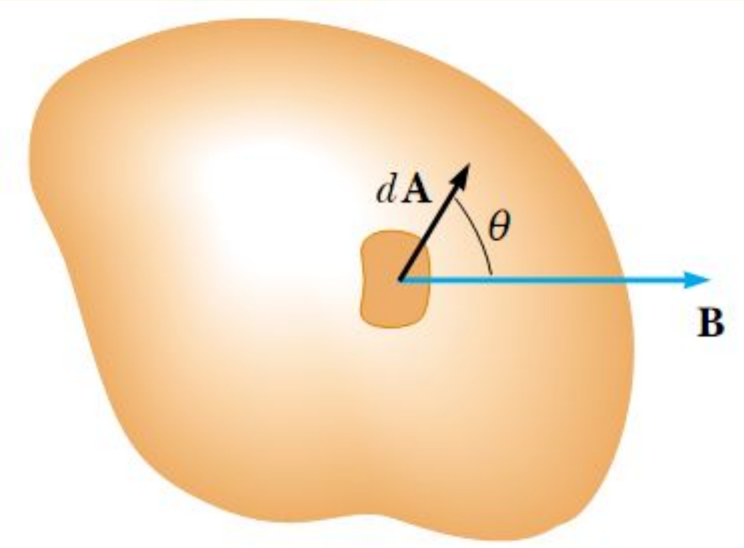
$$\oint \mathbf{B} \cdot d\mathbf{s} = \int_{\text{path 1}} \mathbf{B} \cdot d\mathbf{s} = B \int_{\text{path 1}} ds = B\ell$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = B\ell = \mu_0 N I$$

$$B = \mu_0 \frac{N}{\ell} I = \mu_0 n I$$

Where $n = N/\ell$ umber of turns per unit length.

Magnetic Flux



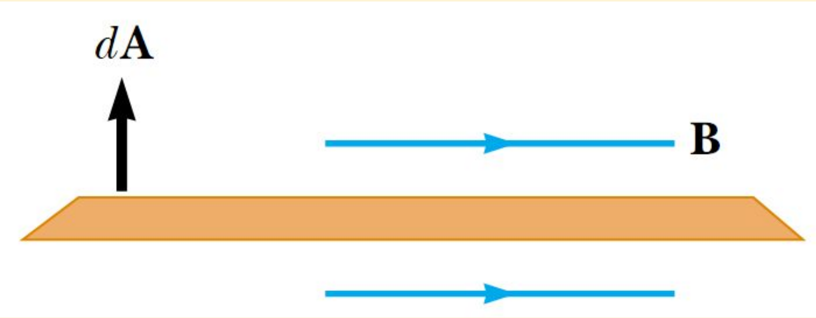
- The magnetic flux through an area element dA is

  B·dA = BdA cosΘ

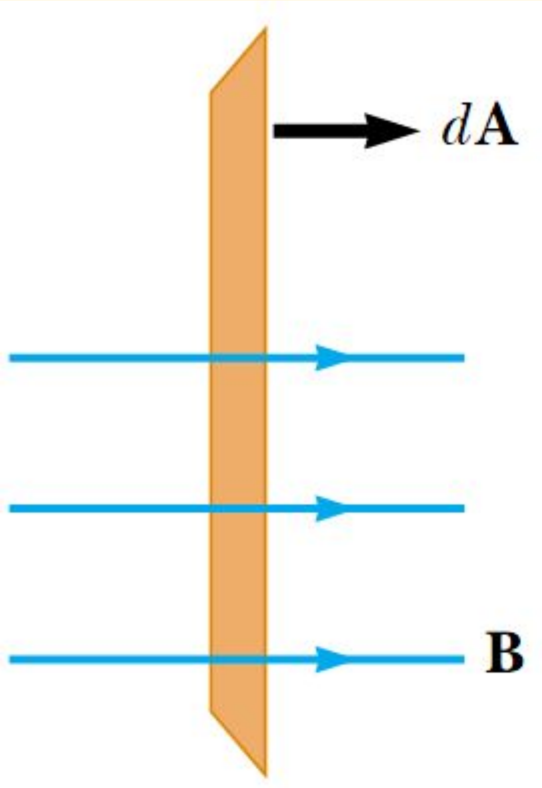
  where dA is a vector perpendicular to the surface and has a magnitude equal to the area dA.
- Therefore, the total magnetic flux Φ_B through the surface is

$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$$

Magnetic flux through a plane lying in a magnetic field



The flux through the plane is zero when the magnetic field is parallel to the plane surface.



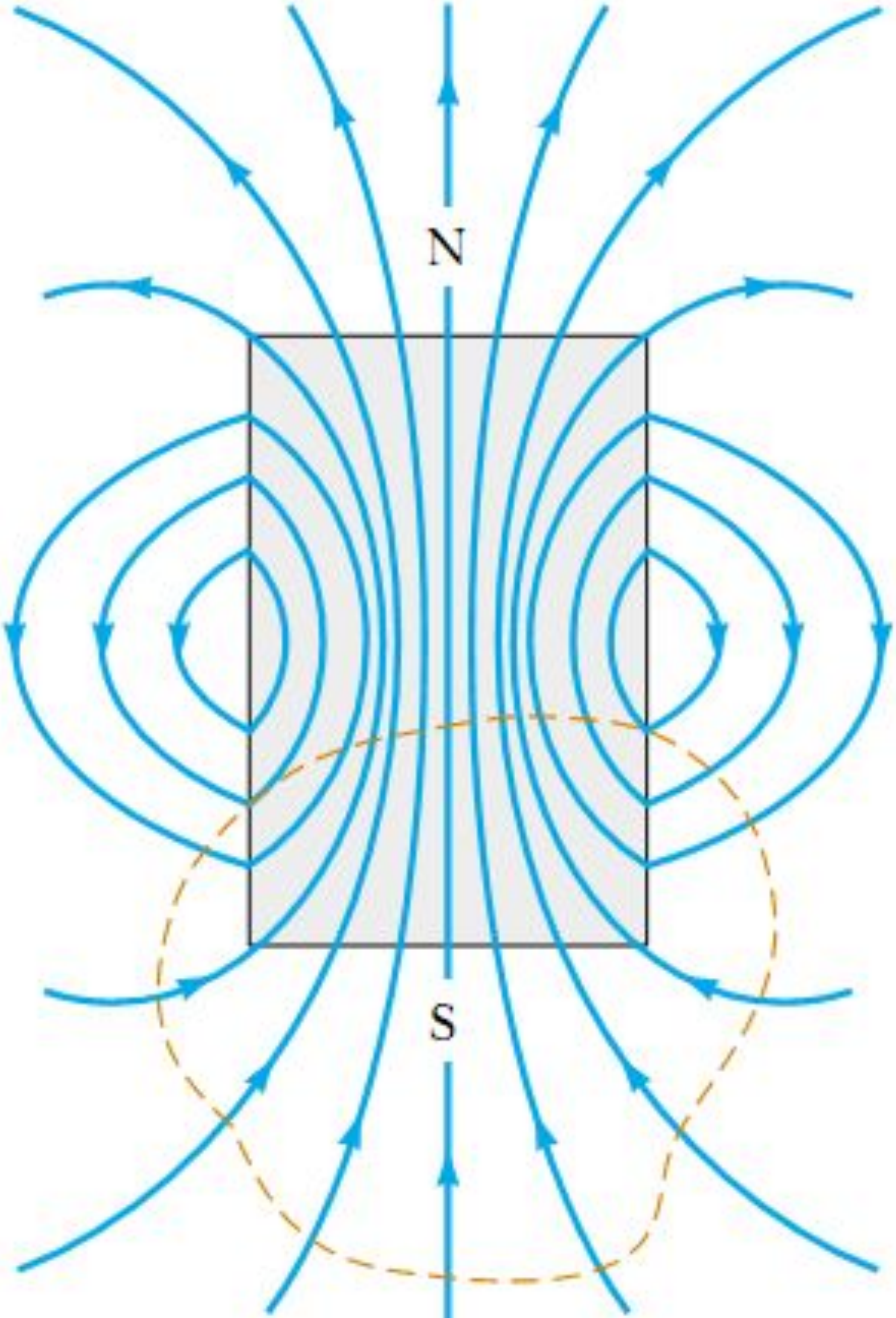
The flux through the plane is a maximum when the magnetic field is perpendicular to the plane.

Gauss's Law in Magnetism

- The net magnetic flux through any closed surface is always zero:

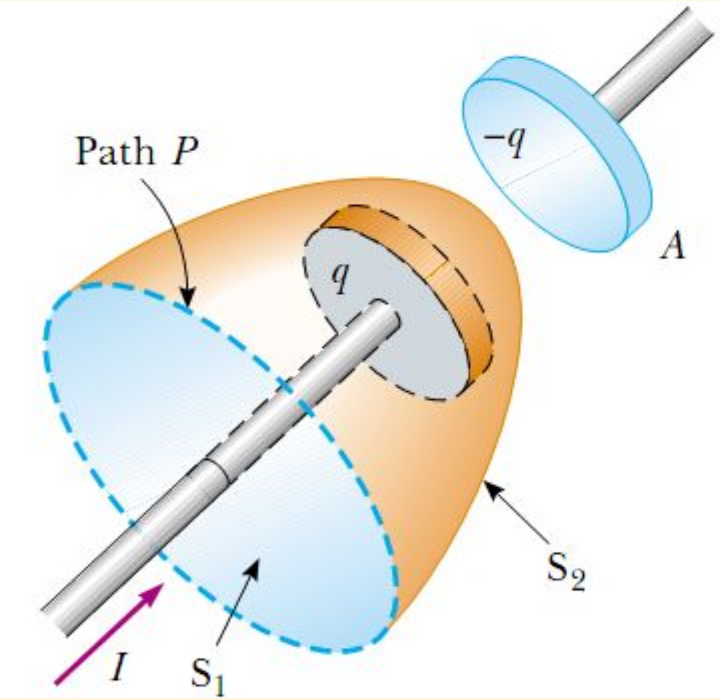
$$\boxed{\oint \mathbf{B} \cdot d\mathbf{A} = 0}$$

- Here $\mathbf{B} \cdot d\mathbf{A}$ scalar multiplication of two vectors.
- Zero net magnetic flux through any closed surface means that magnetic field lines has no source. It is based on the fact that there exist no magnetic monopoles.



- The magnetic field lines of a bar magnet form closed loops. Note that the net magnetic flux through a closed surface surrounding one of the poles (or any other closed surface) is zero. (The dashed line represents the intersection of the surface with the page.)
- The number of lines entering the surface equals the number of lines leaving it.

Displacement Current



- There is a charging capacitor, with current I two imaginary surfaces S_1 and S_2, and path P, bounding to S_1 and S_2.
- When the path P is considered as bounding S_1, then

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I$$

because the conduction current passes through S_1.
- When the path is considered as bounding S_2, then

$$\oint \mathbf{B} \cdot d\mathbf{s} = 0$$

because no conduction current passes through S_2. Thus, we have a <u>contradiction</u>.

- This contradiction is resolved by introducing a new quantity – the displacement current:

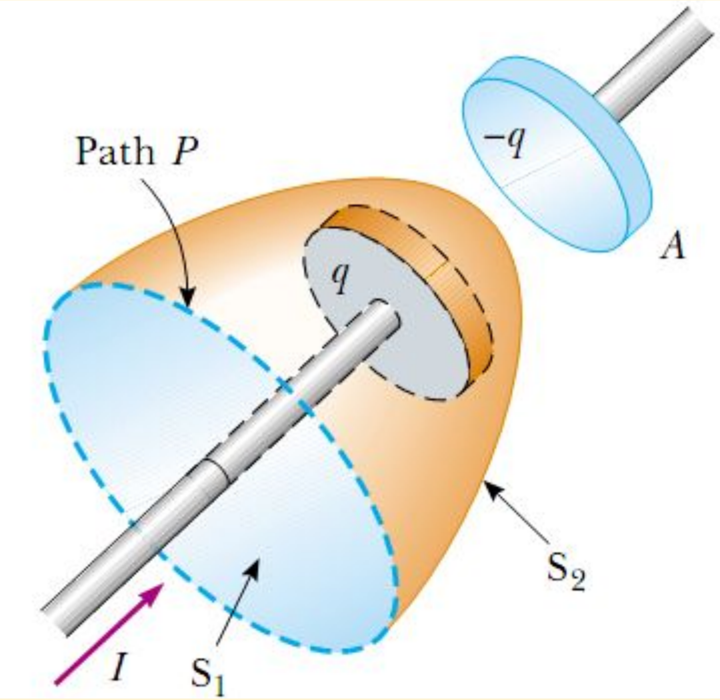
$$I_d \equiv \epsilon_0 \frac{d\Phi_E}{dt}$$

- ϵ_0 is a free space permittivity, a constant
- Φ_E is the electric flux: $\Phi_E = \int \mathbf{E} \cdot d\mathbf{A}$
- As the capacitor is being charged (or discharged), the changing electric field between the plates may be considered equivalent to a current that acts as a continuation of the conduction current in the wire.

General form of Ampere’s Law

- So considering the displacement current, we can write the General form of Ampere’s Law (or Ampere-Maxwell law):

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 (I + I_d) = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$



- So the electric flux through S_2 is

$$\Phi_E = \int \mathbf{E} \cdot d\mathbf{A} = EA$$

- Where E is the electric field between the plates, A is the area of the plates, then

$$E = q/(\epsilon_0 A)$$

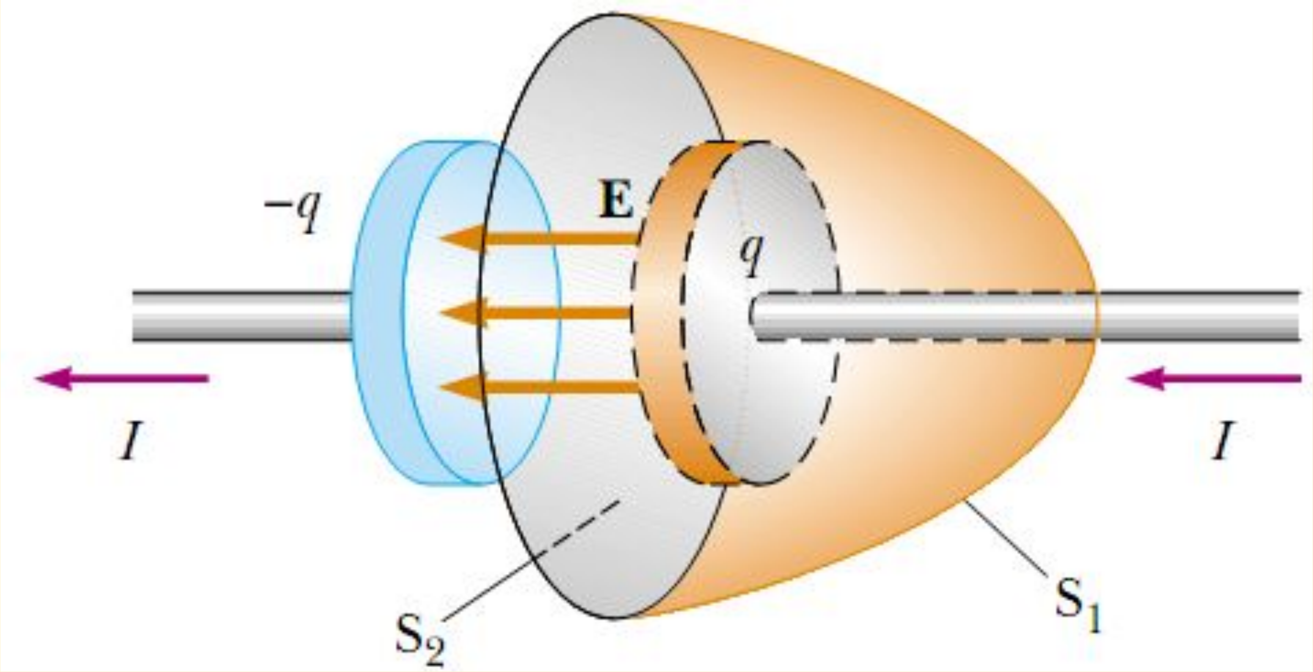
- So the electric flux through S_2 is

$$\Phi_E = EA = \frac{q}{\epsilon_0}$$

- Then the displacement current through S_2 is

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dq}{dt}$$

- That is, the displacement current I_d through S_2 is precisely equal to the conduction current I through S_1!



Units in Si

- Magnetic field B T= N*s/(C*m)
  T= N/(A*m)
- Electric Field E V/m=N/C

- Magnetic permeability of free space: $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}$