Review, PID Controller

Md Hazrat Ali
Department of Mechanical Engineering
School of Engineering,
Nazarbayev University

Today's Quote:

"Live as if you were to die tomorrow. Learn as if you were to live forever."

— **Mahatma Gandhi**

Steady State Error (ess)

Steady-state error is defined as the difference between the input (command) and the output of a system in the limit as time goes to infinity (i.e. when the response has reached steady state). The steady-state error will depend on the type of input (step, ramp, etc.) as well as the system type (0, I, or II).

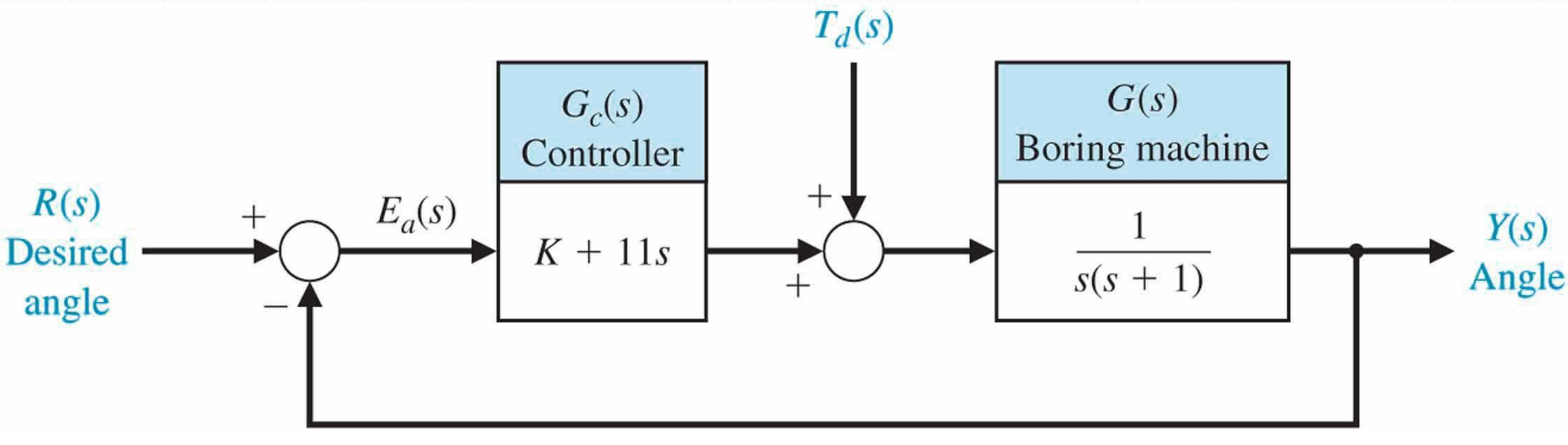
Note: Steady-state error analysis is only useful for stable systems. You should always check the system for stability before performing a steady-state error analysis.

Steady State Error (ess)

Table 5.5 Summary of Steady-State Errors

Number of Integrations in $G_c(s)G(s)$, Type Number	Input: Step, $r(t) = A$, $R(s) = A/s$	Input: Ramp, At, A/s^2	Input: Parabola, $At^2/2$, A/s^3
0	$e_{ss} = \frac{A}{1 + K_p}$	Infinite	Infinite
1	$e_{ss} = 0$	$\frac{A}{K_v}$	Infinite
2	$e_{ss} = 0$	0	$\frac{A}{K_a}$

Steady State Error (ess)- Multiple inputs



output due to the two inputs is

$$Y(s) = \frac{K + 11s}{s^2 + 12s + K} R(s) + \frac{1}{s^2 + 12s + K} T_d(s).$$

Classical Controller-
PID Controller

Introduction

- More than half of the industrial controllers in use today utilize PID or modified PID control schemes.
- When the mathematical model of the plant is not known and therefore analytical design methods cannot be used, PID controls prove to be most useful.

Design PID control

- Know mathematical model 👉 various design techniques
- Plant is complicated, can't obtain mathematical model 👉 experimental approaches to the tuning of PID controllers

PID Control

- A closed loop (feedback) control system, generally with Single Input-Single Output (SISO)
- A portion of the signal being fed back is:
  - Proportional to the signal (**P**)
  - Proportional to integral of the signal (**I**)
  - Proportional to the derivative of the signal (**D**)

When PID Control is Used

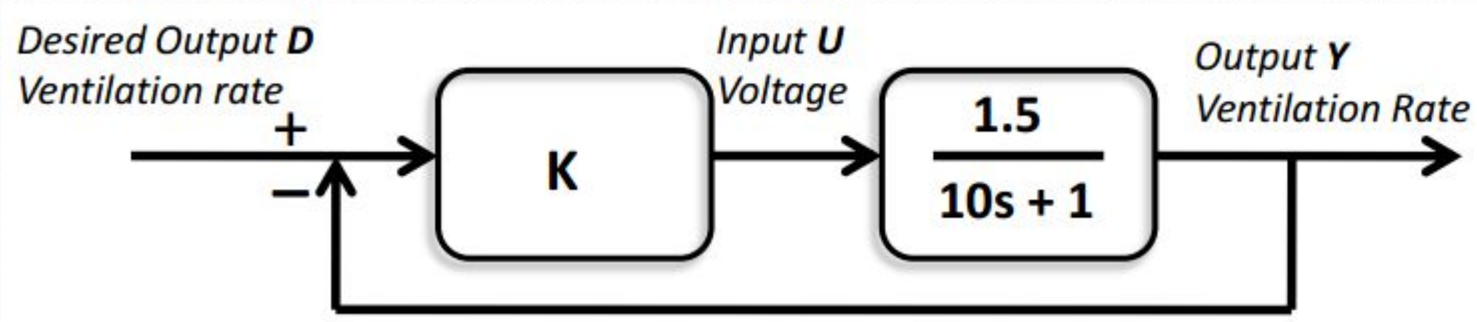
- PID control works well on SISO systems of 2nd Order, where a desired Set Point can be supplied to the system control input
- PID control handles step changes well to the Set Point especially when :
  - Fast Rise Times
  - Little or No Overshoot
  - Fast settling Times
  - Zero Steady State Error
- PID controllers are often fine tuned on-site, using established guidelines

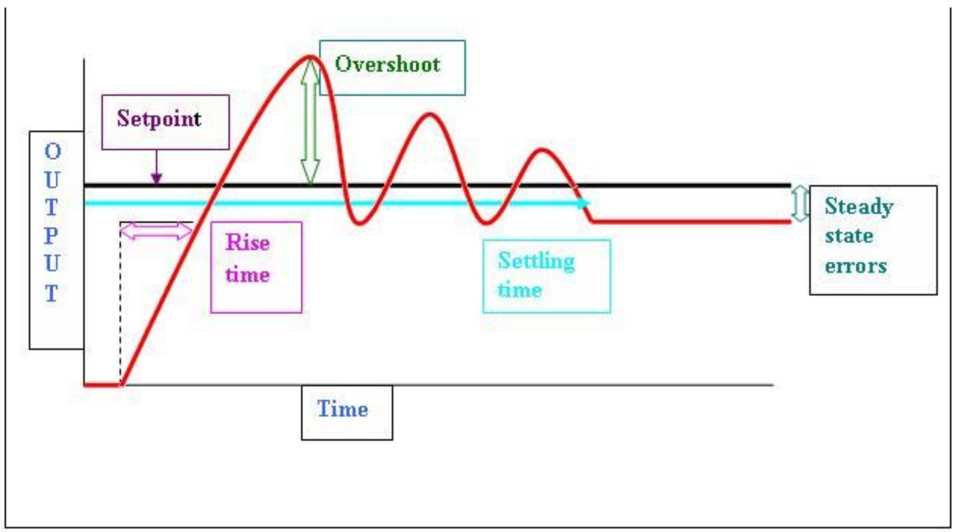
Output equation of PID controller in time domain

$$u(t) = K_p e(t) + K_I \int e(t)\, dt + K_D \frac{de(t)}{dt}$$

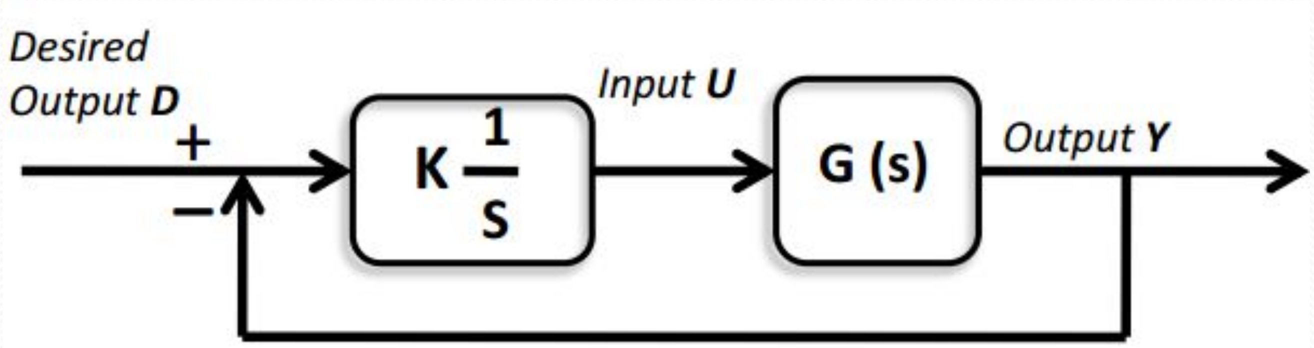
Proportional Control

- A proportional controller attempts to perform better than the On-off type by applying power in proportion to the difference in temperature between the measured and the set-point.
- The P-controller usually has steady-state errors (the difference in set point and actual outcome) unless the control gain is large.
- As the control gain becomes larger, issues arise with the stability of the feedback loop.





Integral Control



•Gain is applied to *integral* of error

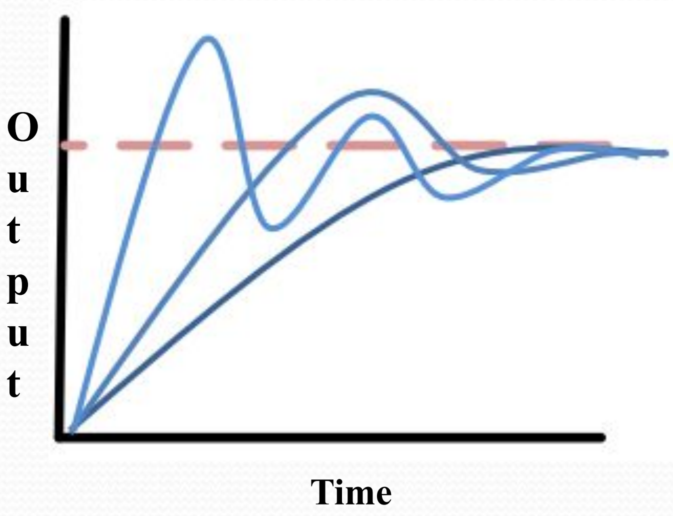
Proportional to both magnitude & duration

•Summing error over time gives an accumulated offset previously uncorrected

•Results in Zero Steady State Error

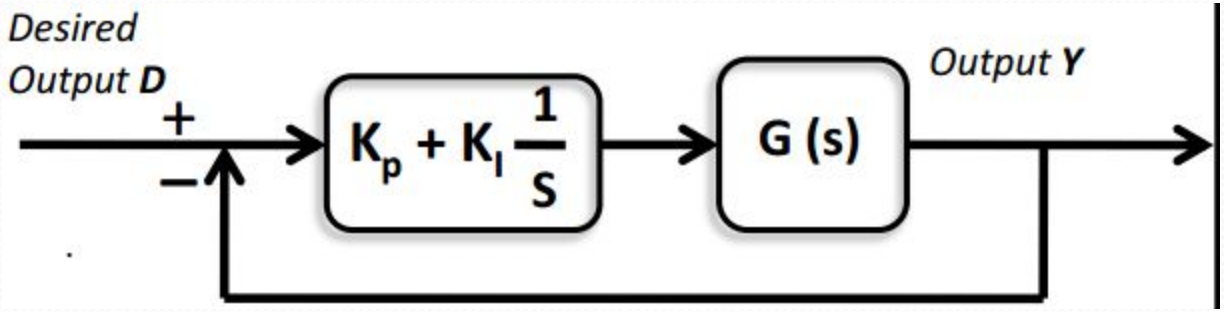
•Can cause overshoot of setpoint

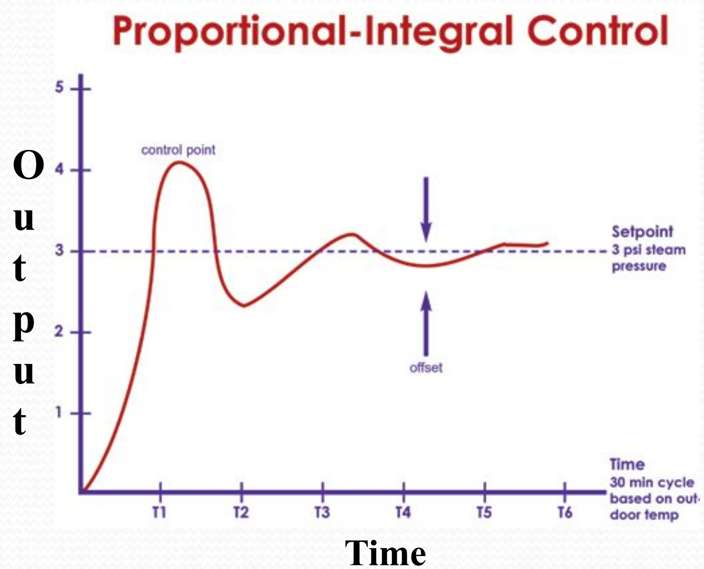
•Greater complexity in closed loop Transfer Function – may become unstable



Proportional-Integral Control

- The combination of proportional and integral terms is important to increase the speed of the response.
- Eliminate the steady state error.





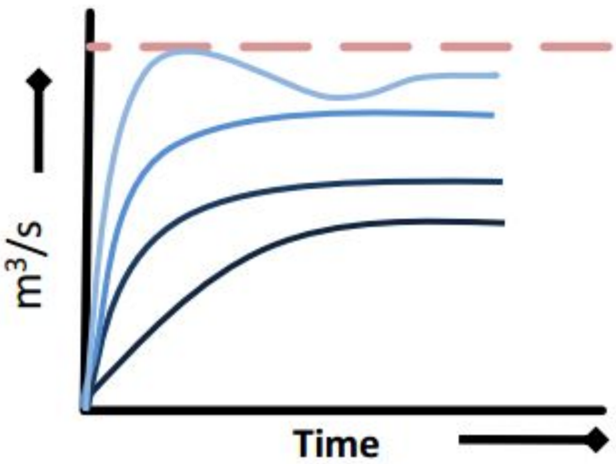
Proportional Derivative Control

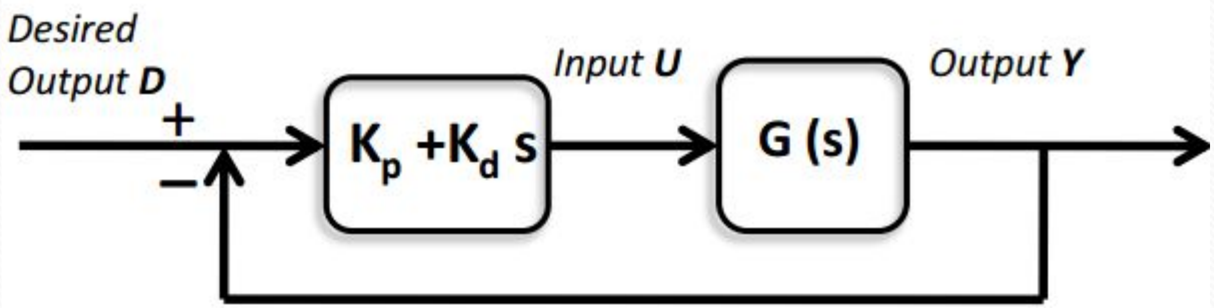
Used; Where a steady state error can be tolerated

Avoids; Destabilising nature of Integral action

Reduces overshoot effects

However; Susceptible to noise, acts to amplify it





Tips for Designing a PID Controller

1. Obtain an open-loop response and determine what needs to be improved
2. Add a proportional control to improve the rise time
3. Add a derivative control to improve the overshoot
4. Add an integral control to eliminate the steady-state error
5. Adjust each of Kp, Ki, and Kd until you obtain a desired overall response.

Lastly, please keep in mind that you do not need to implement all three controllers (proportional, derivative, and integral) into a single system, if not necessary. For example, if a PI controller gives a good enough response (like the above example), then you don't need to implement derivative controller to the system. Keep the controller as simple as possible.

The Characteristics of P, I, and D controllers

A proportional controller (Kp) will have the effect of reducing the rise time and will reduce, but never eliminate, the steady-state error.

An integral control (Ki) will have the effect of eliminating the steady-state error, but it may make the transient response worse.

A derivative control (Kd) will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response.

PID Controller (Conti…)

Proportional Control

By only employing proportional control, a steady state error occurs.

Proportional and Integral Control

The response becomes more oscillatory and needs longer to settle, the error disappears.

Proportional, Integral and Derivative Control

All design specifications can be reached.

Tips for Designing a PID Controller

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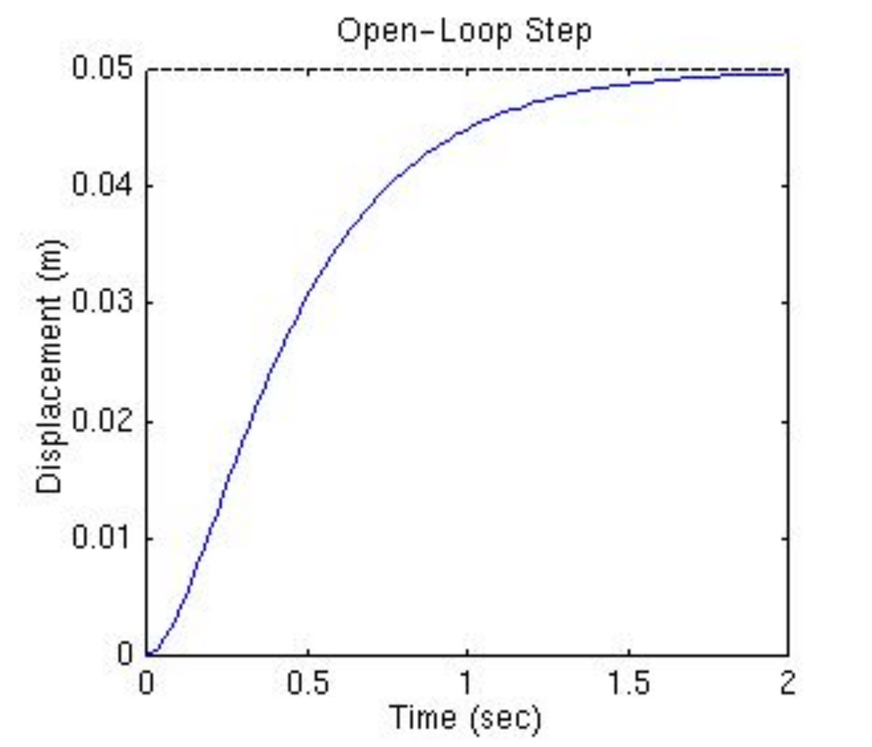
Open-Loop Control - Example

$$G(s) = \frac{1}{s^2 + 10s + 20}$$

```
num=1;
den=[1 10 20];
step(num,den)
```



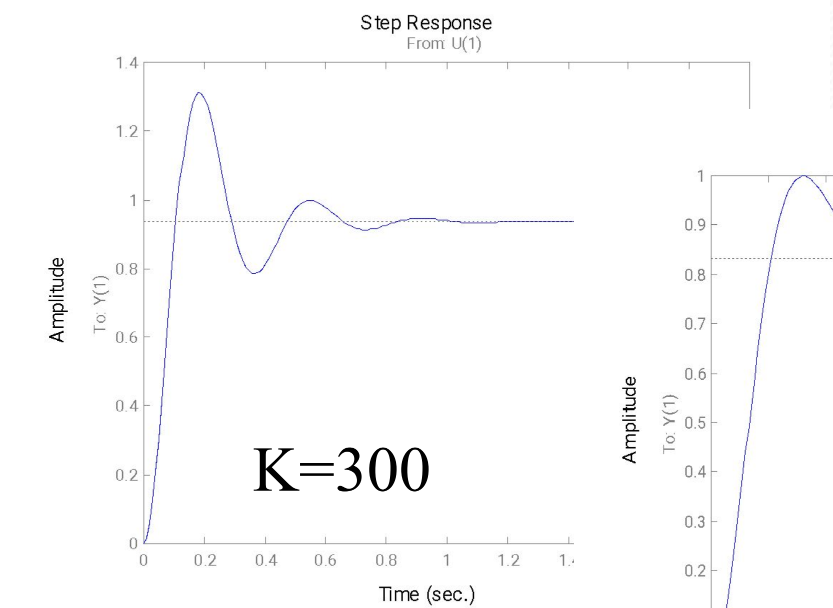
Proportional Control - Example

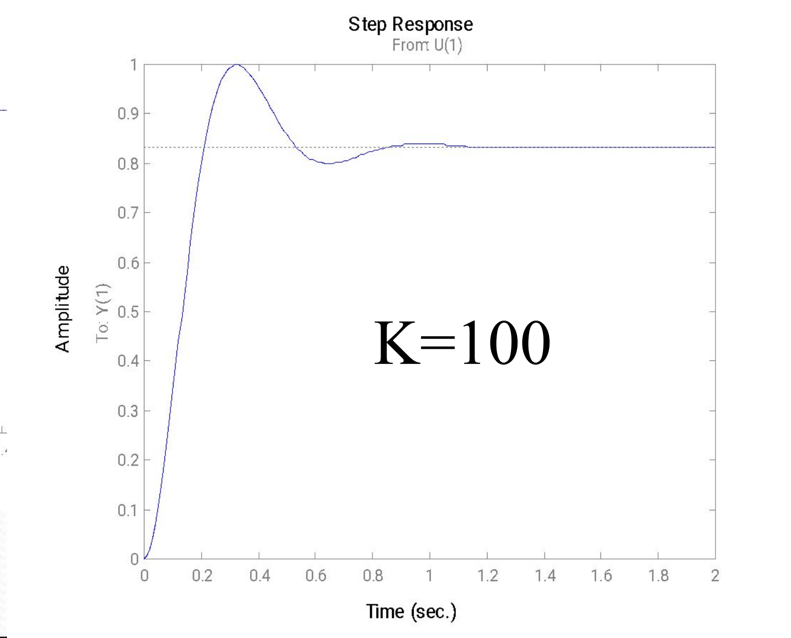
The proportional controller (Kp) reduces the rise time, increases the overshoot, and reduces the steady-state error.

MATLAB Example

$$T(s) = \frac{Kp}{s^2 + 10 \cdot s + (20 + Kp)}$$

```
Kp=300;
num=[Kp];
den=[1 10 20+Kp];
t=0:0.01:2;
step(num,den,t)
```





PID Controller (Conti…)

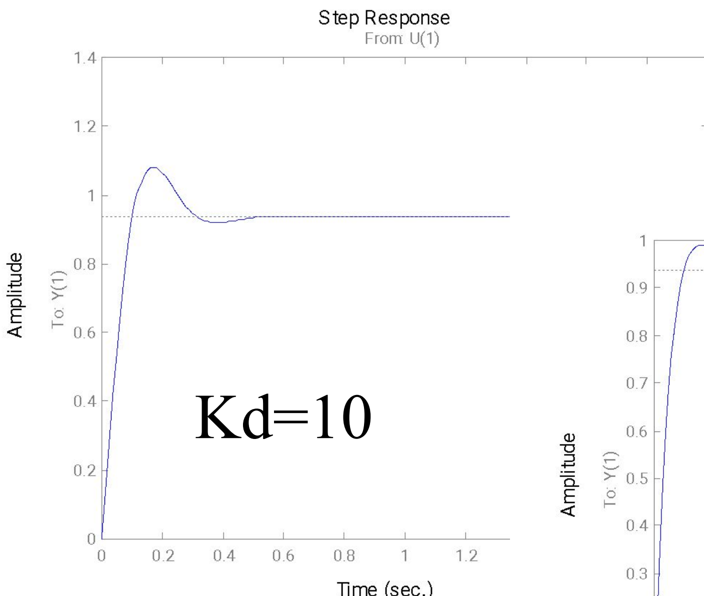
Proportional - Derivative - Example

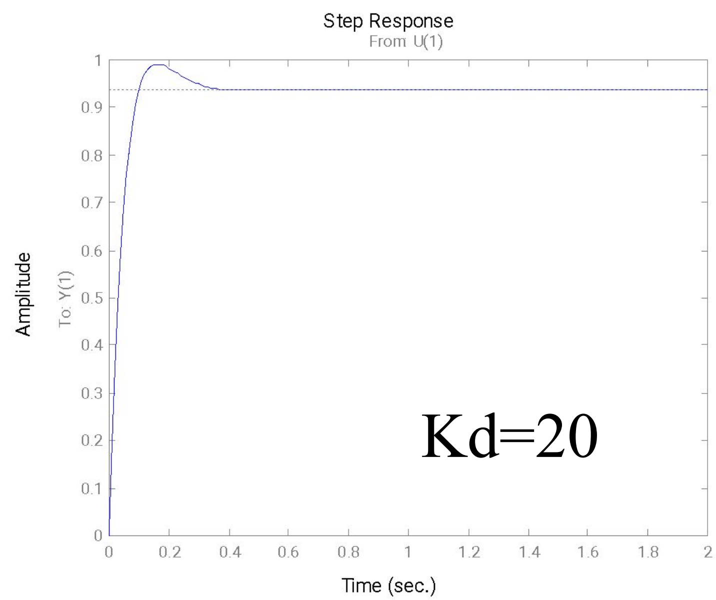
The derivative controller (Kd) reduces both the overshoot and the settling time.

MATLAB Example

$$T(s) = \frac{Kd \cdot s + Kp}{s^2 + (10 + Kd) \cdot s + (20 + Kp)}$$

```
Kp=300;
Kd=10;
num=[Kd Kp];
den=[1 10+Kd 20+Kp];
t=0:0.01:2;
step(num,den,t)
```





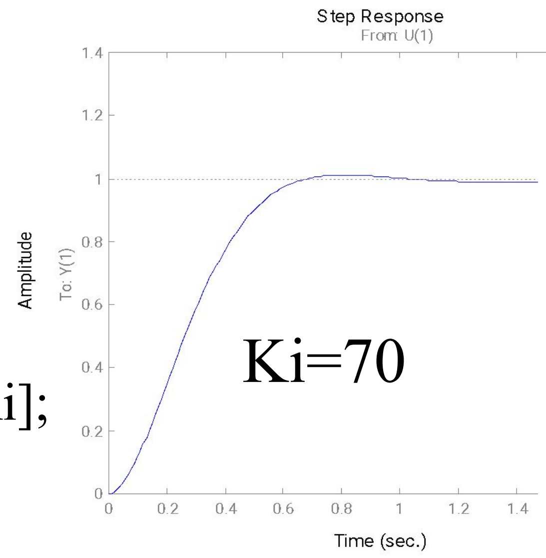
Proportional - Integral - Example

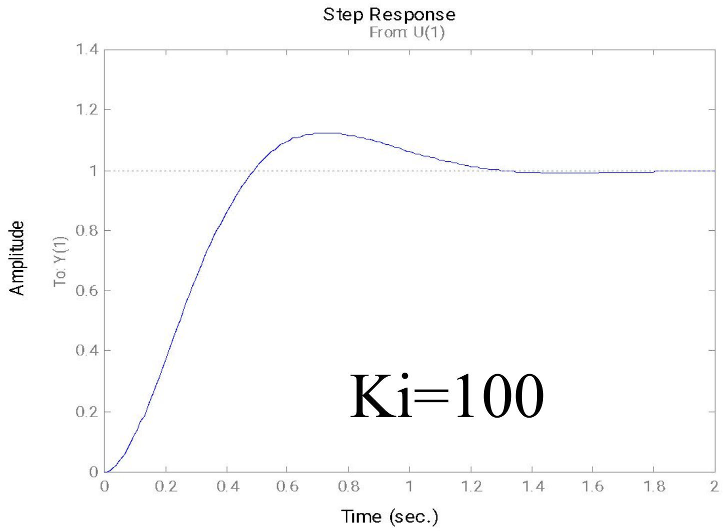
The integral controller (Ki) decreases the rise time, increases both the overshoot and the settling time, and eliminates the steady-state error

$$T(s) = \frac{Kp \cdot s + Ki}{s^3 + 10 \cdot s^2 + (20 + Kp) \cdot s + Ki}$$

MATLAB Example

```
Kp=30;
Ki=70;
num=[Kp Ki];
den=[1 10 20+Kp Ki];
t=0:0.01:2;
step(num,den,t)
```





PID Controller (Conti…)

The Characteristics of P, I, and D controllers

CL RESPONSE	RISE TIME	OVERSHOOT	SETTLING TIME	S-S ERROR
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	Small Change

PID Controller (Conti…)

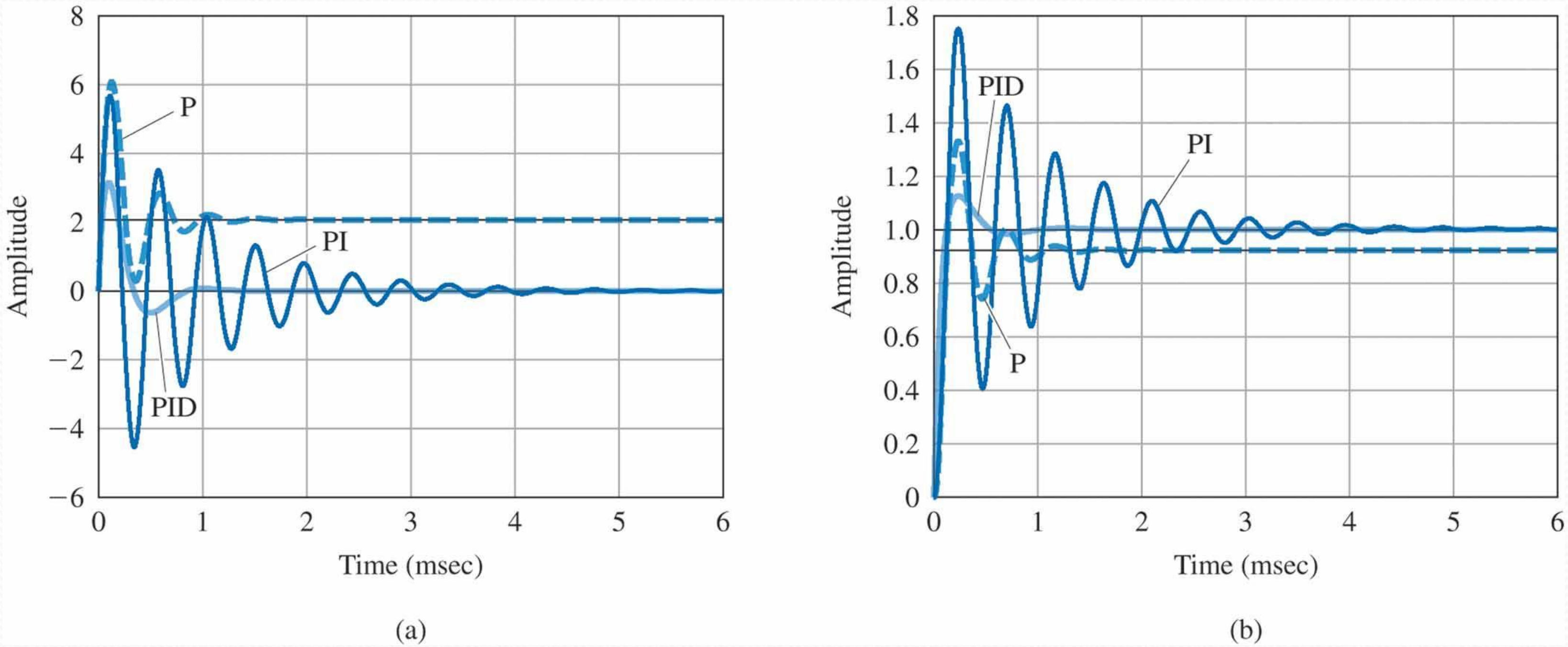


Figure 4.9 Responses of P, PI, and PID control to (a) step disturbance input (b) step reference input

PID Controller (Conti…)

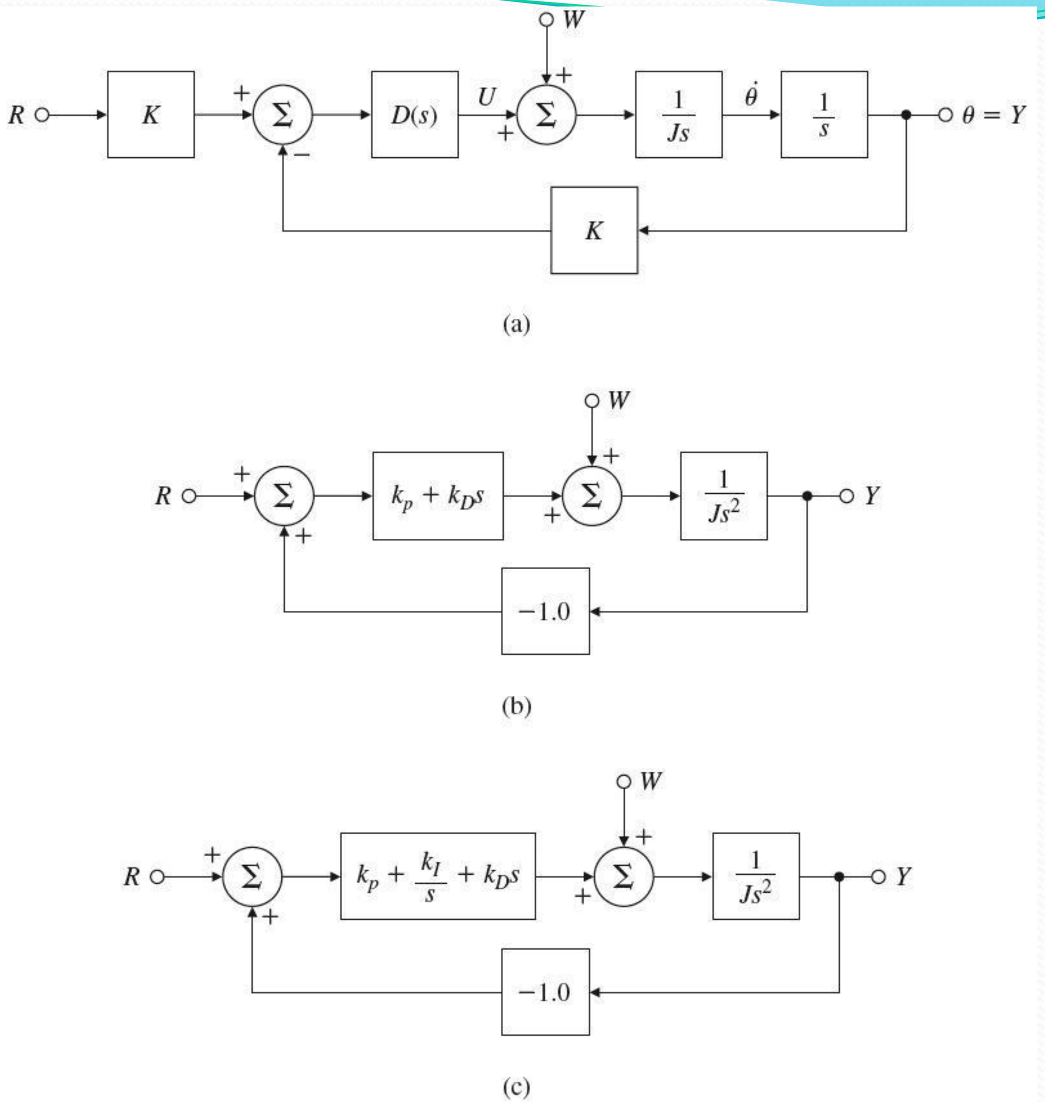


Figure 4.10 Model of a satellite attitude control: (a) basic system; (b) PD control; (c) PID control

PID Controller

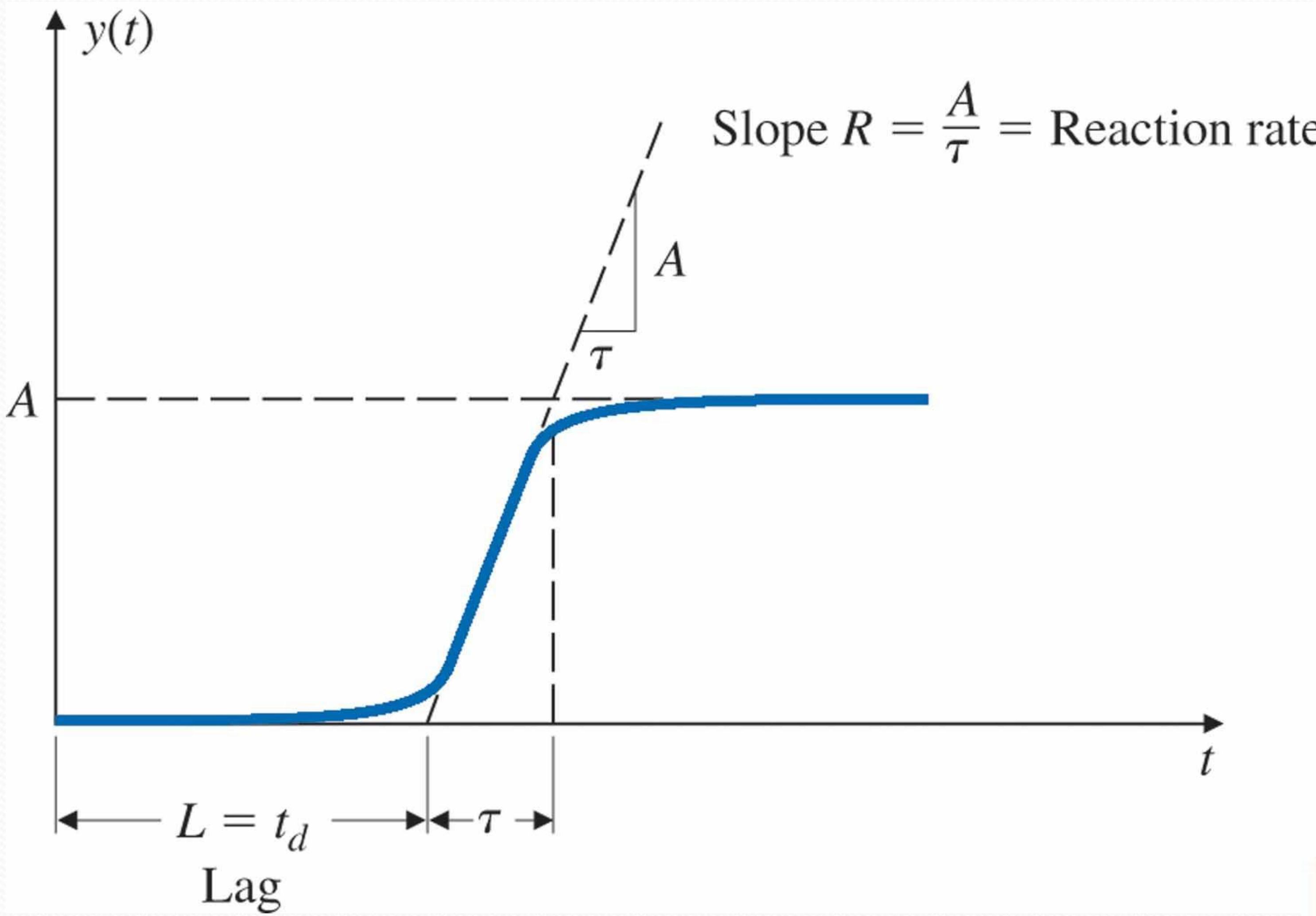
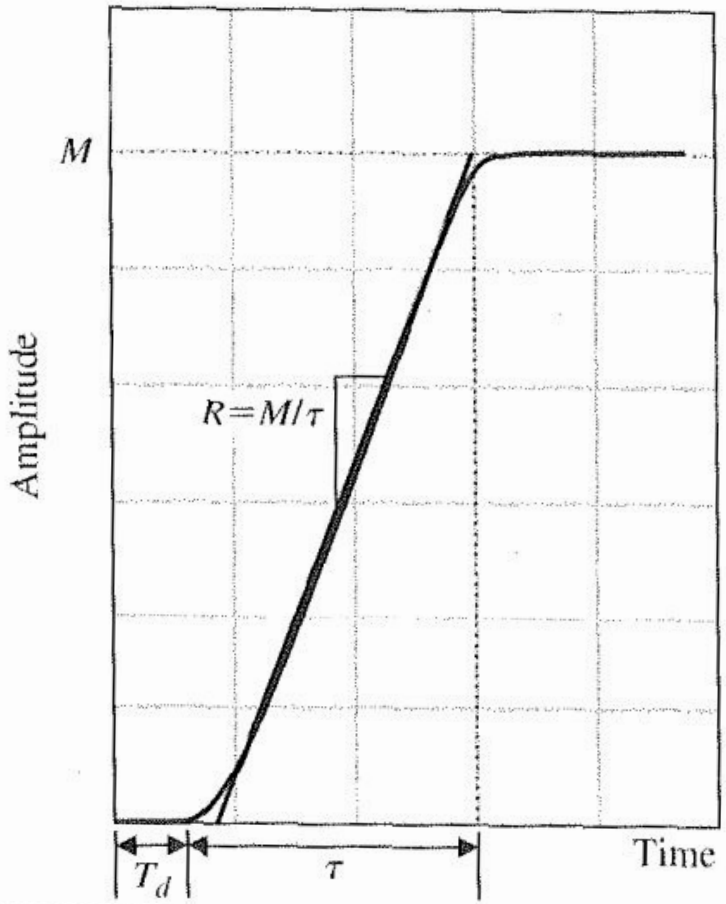


Figure 4.11 Process reaction curve



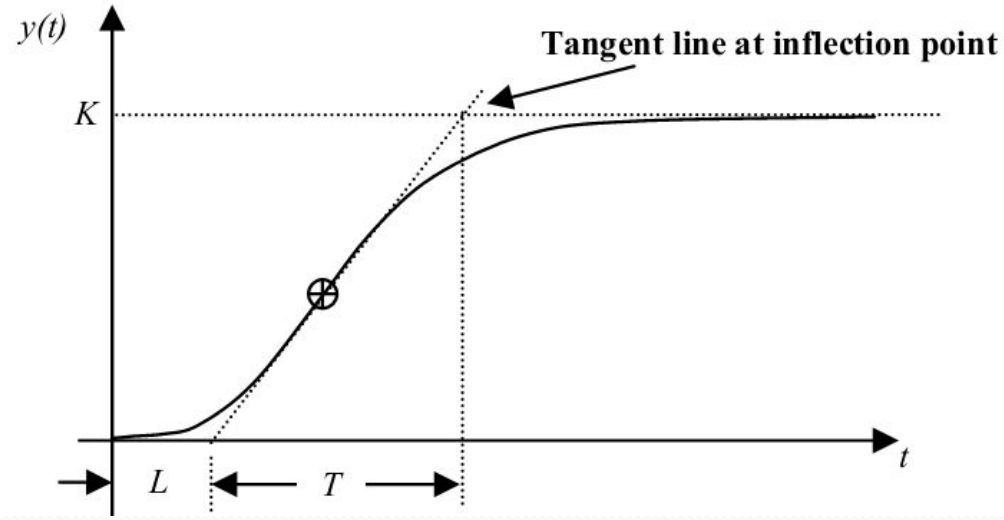


Figure 4.11 Process reaction curves (R.C.Dorf et.al and Others)

PID Controller- Ziegler Method #1

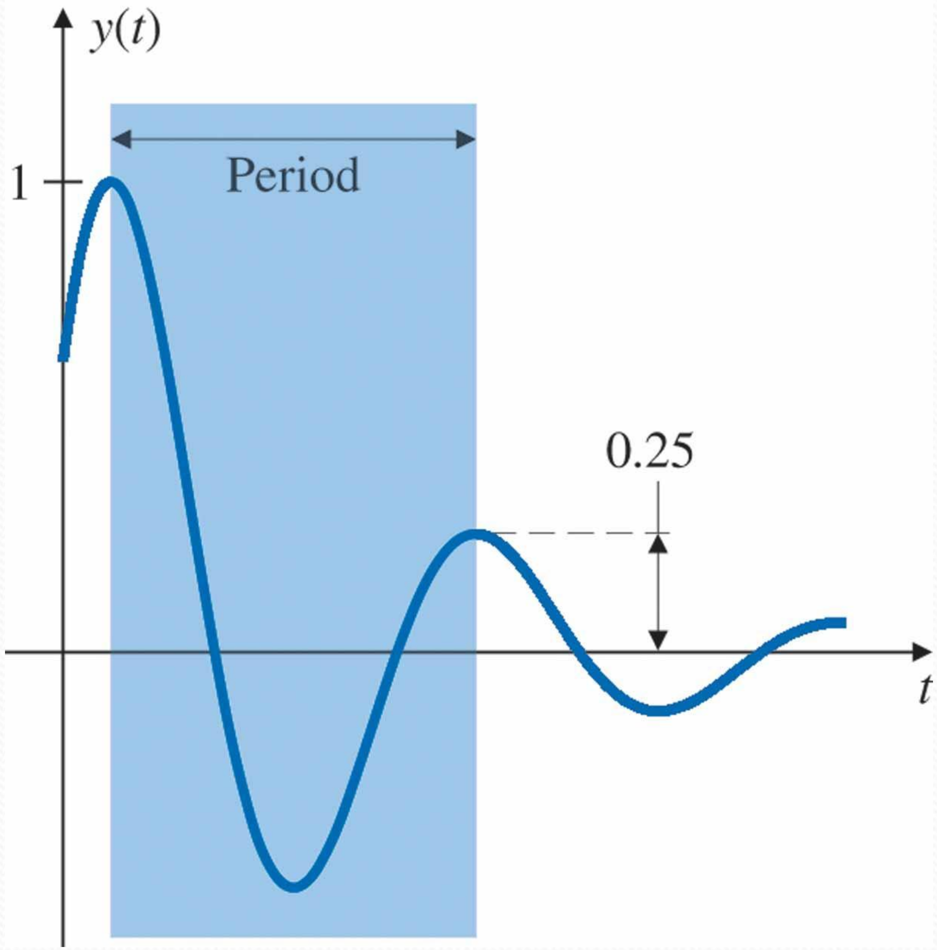


Figure 4.12 Quarter decay ratio

PID Controller (Conti…)

TABLE 4.2

Ziegler–Nichols Tuning for the Regulator $D(s) = K(1 + 1/T_I s + T_D s)$, for a Decay Ratio of 0.25

Type of Controller	Optimum Gain
P	$k_p = 1/RL$
PI	$\begin{cases} k_p = 0.9/RL \\ T_I = L/0.3 \end{cases}$
PID	$\begin{cases} k_p = 1.2/RL \\ T_I = 2L \\ T_D = 0.5L \end{cases}$

PID Controller- Ziegler Method #2

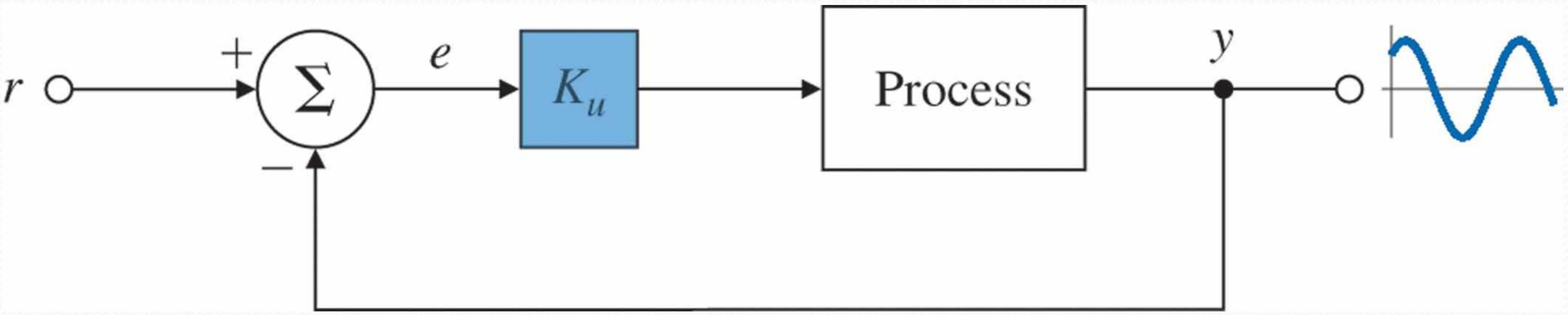


Figure 4.13 Determination of ultimate gain and period

PID Controller (Conti…)

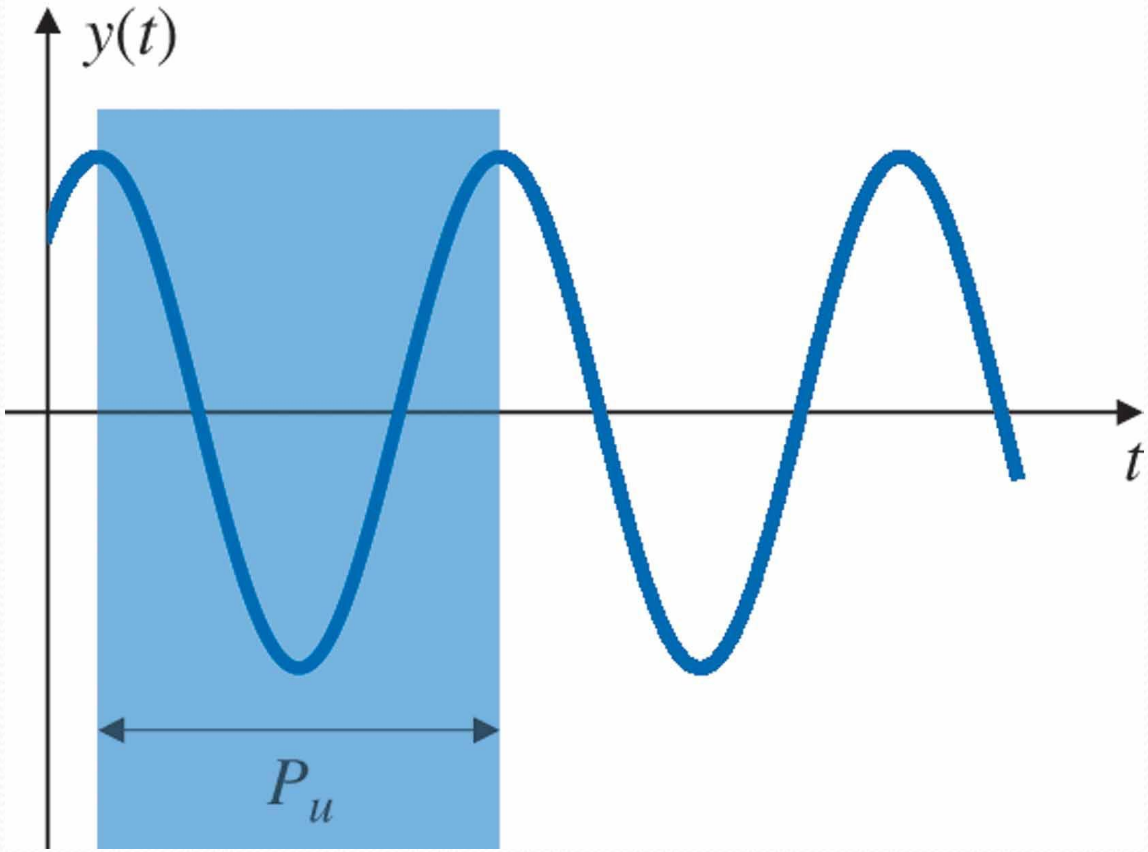


Figure 4.14 Neutrally stable system

PID Controller (Conti…)

TABLE 4.3

Ziegler–Nichols Tuning for the Regulator $D_c(s) = k_p(1 + 1/T_I s + T_D s)$, Based on the Ultimate Sensitivity Method

Type of Controller	Optimum Gain
P	$k_p = 0.5K_u$
PI	$\begin{cases} k_p = 0.45K_u \\ T_I = \dfrac{P_u}{1.2} \end{cases}$
PID	$\begin{cases} k_p = 1.6K_u \\ T_I = 0.5P_u \\ T_D = 0.125P_u \end{cases}$

Further Reading

- Franklin, et. al., Chapter 4
  - Section 4.3

  - Richard C. Dorf et.al, Chapter 6,
    Chapter 6.2