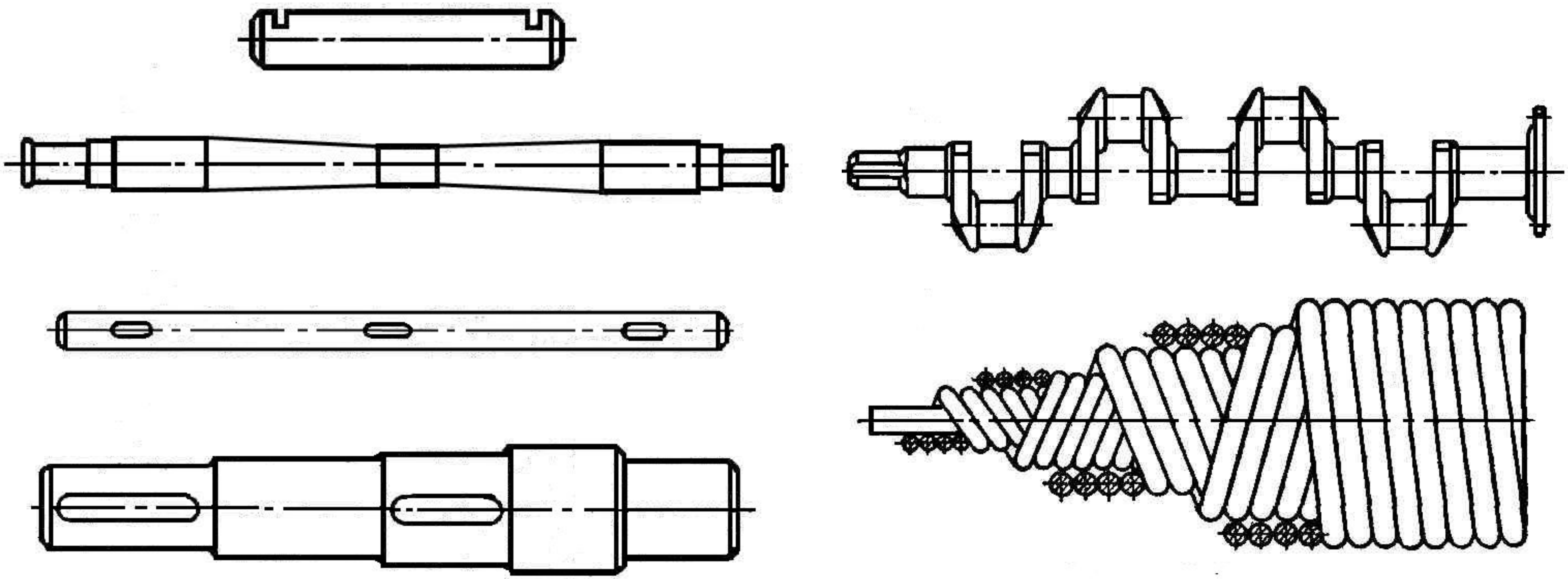
AXLES AND SHAFTS

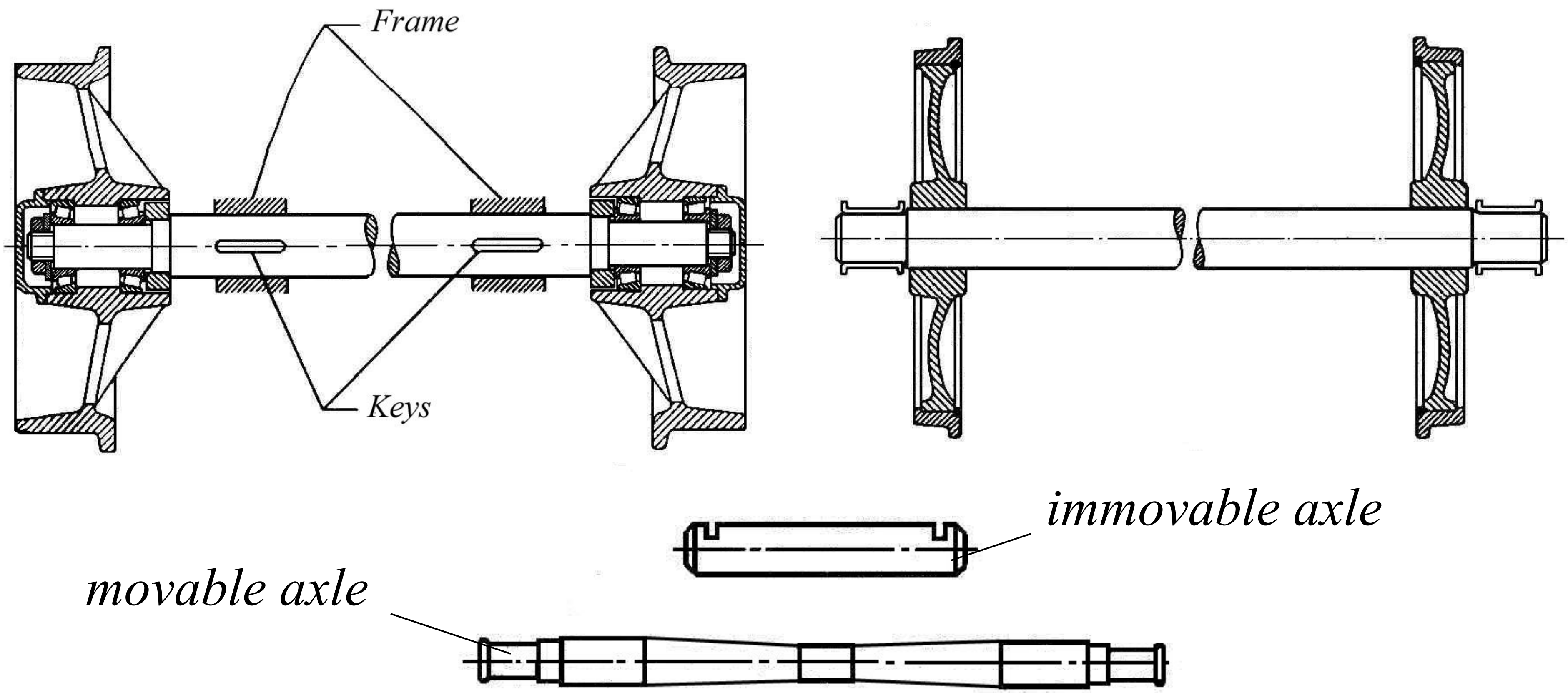
AXLES AND SHAFTS

Links intended to carry rotating elements (pulleys, sprockets, pinions, gears, half-couplings, etc.) are called as axles or shafts.



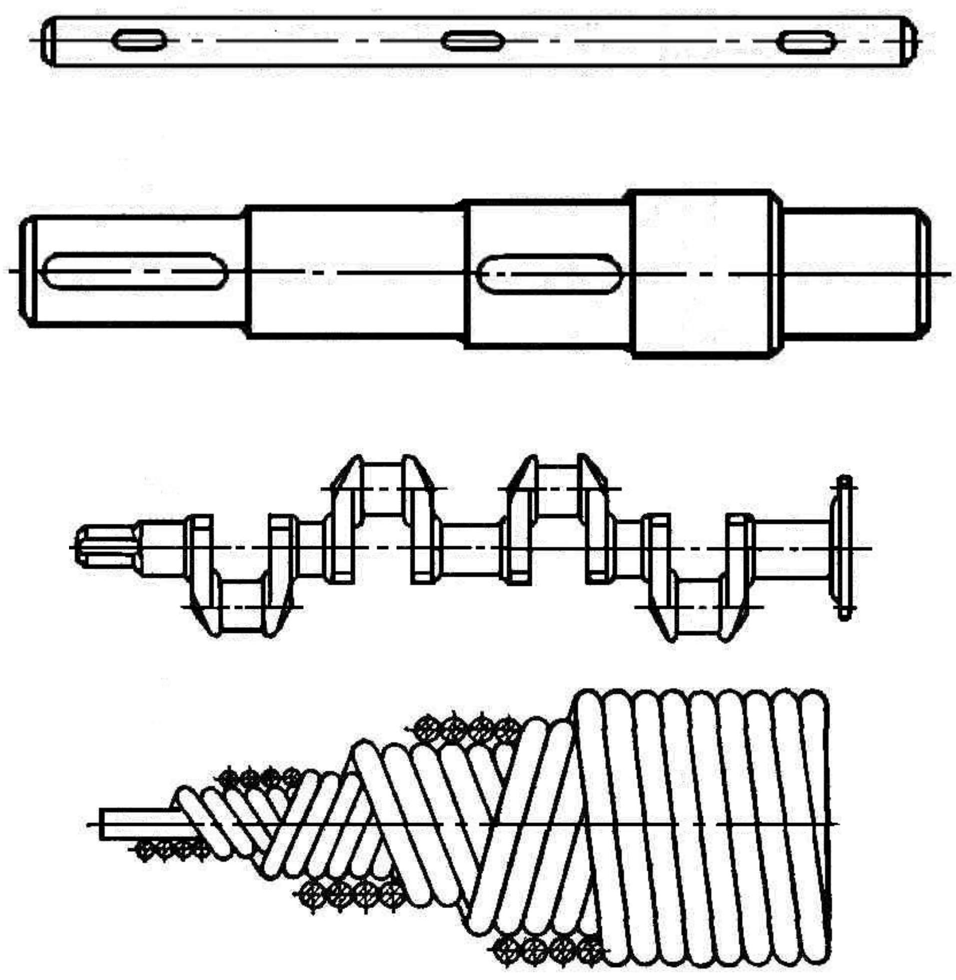
AXLES

Axles are intended to support rotating parts that do not transmit torques and are subjected to bending only.



SHAFTS

Shafts are designed to carry links which transmit torques and experience both bending and torsion.



CLASSIFICATION OF SHAFTS

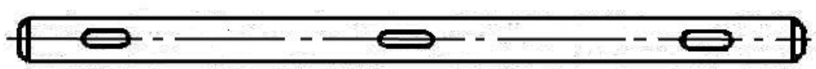
1. According to purpose

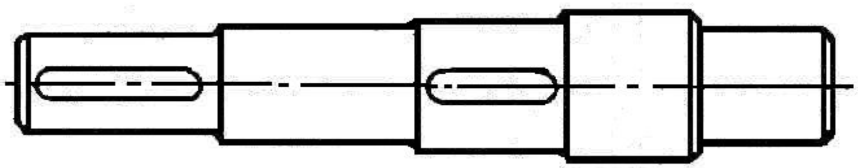
- Shafts of various drives (gear drives, belt drives, chain drives and so on);
- Main shafts of mechanisms and machines whose function is to carry not only drive elements but other elements that do not transmit torques such as rotors, fly-wheels, turbine disks, etc.

CLASSIFICATION OF SHAFTS

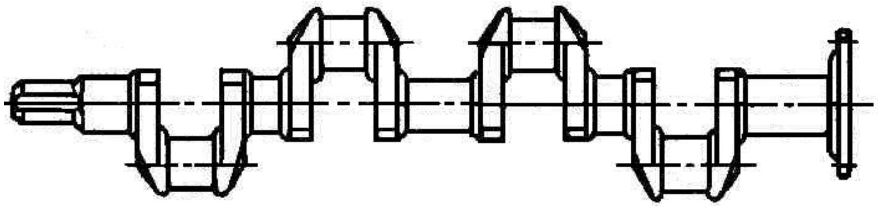
2. According to the shape

- Straight shafts;

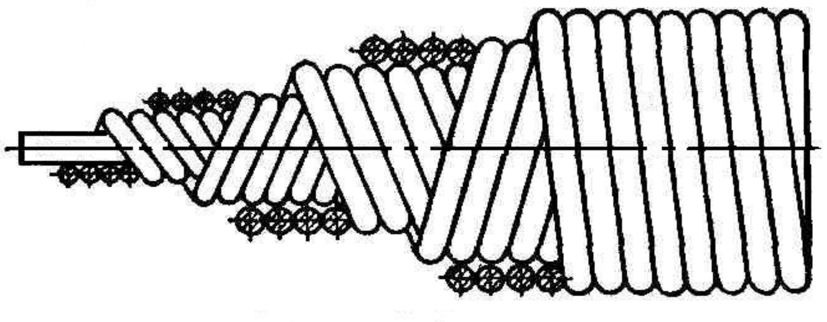




- Cranked shafts;



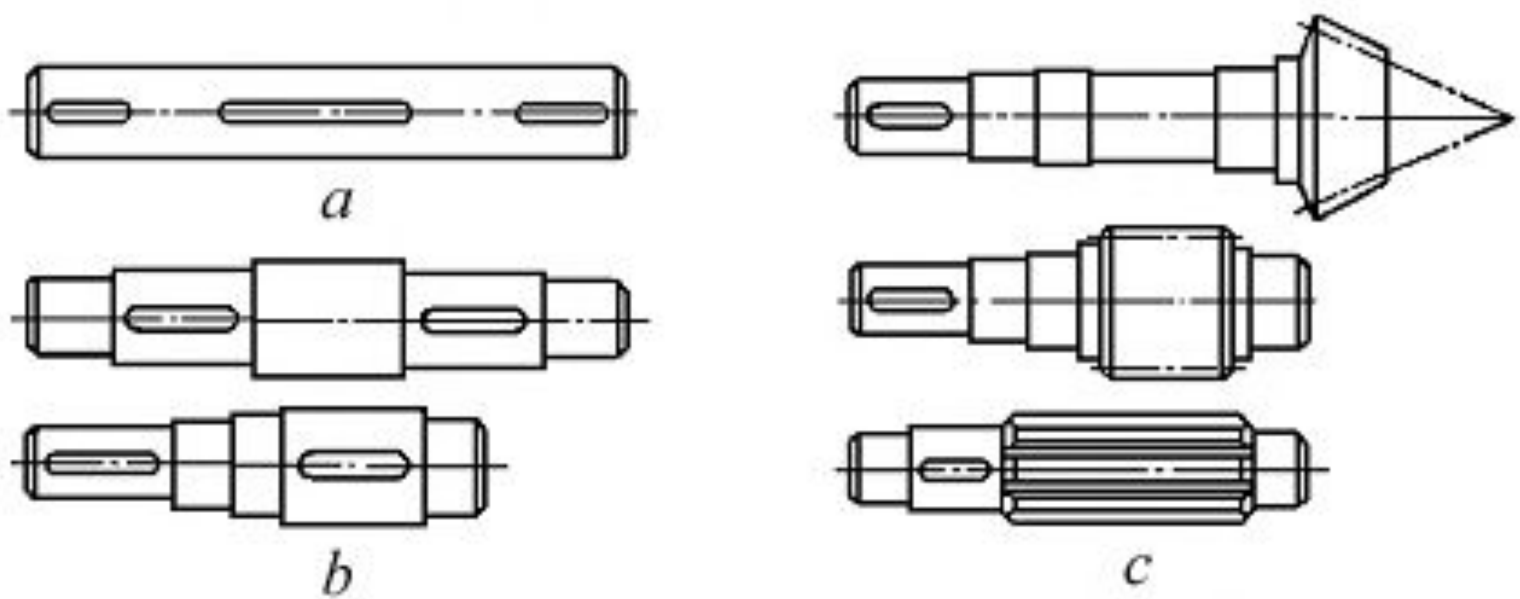
- Flexible shafts.



CLASSIFICATION OF SHAFTS

3. According to the construction

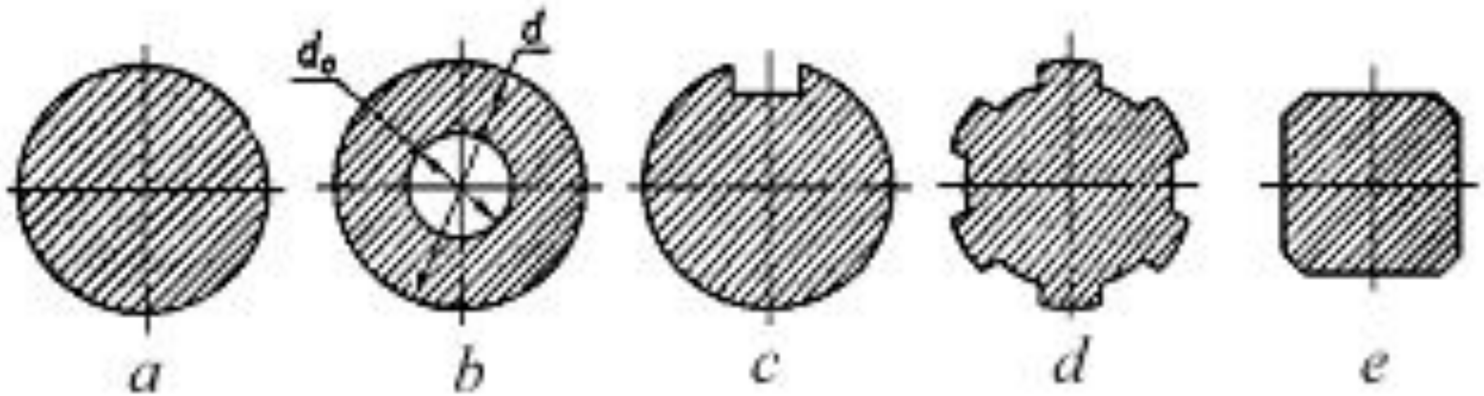
- Shafts of constant cross section (without steps);
- Shafts of variable cross section (of stepped configuration);
- Shafts made solid with gears or worms.



CLASSIFICATION OF SHAFTS

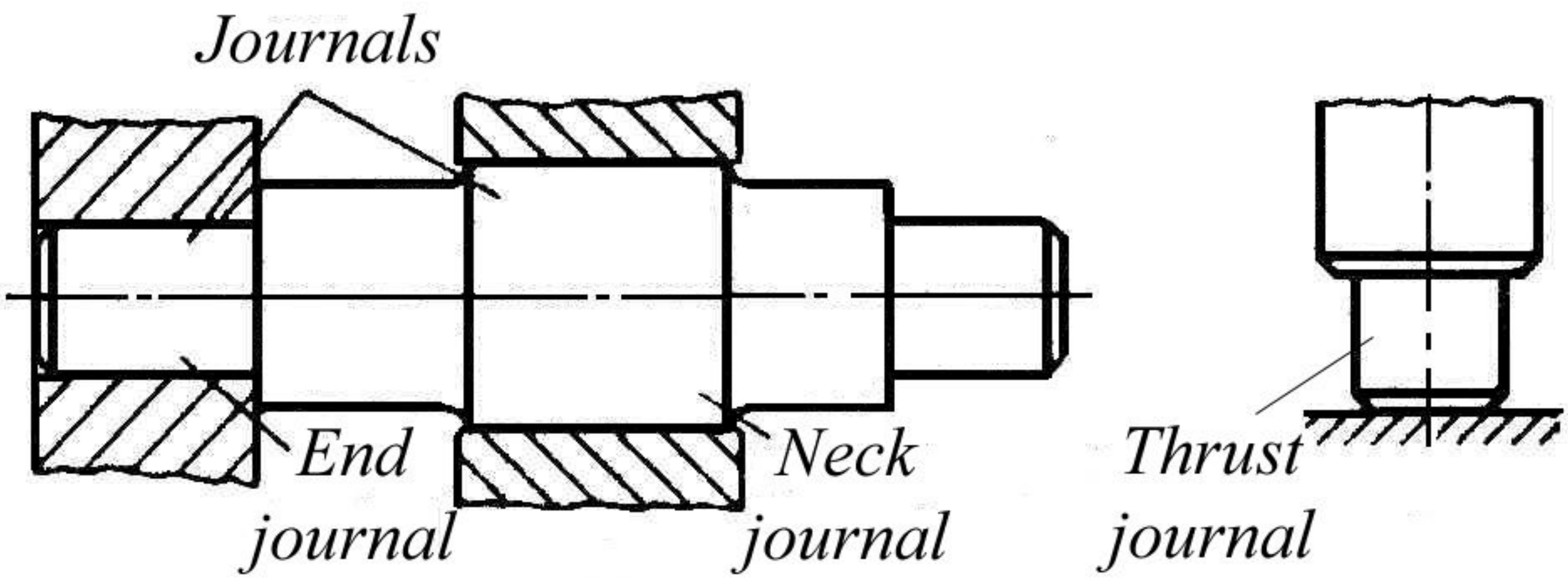
4. According to the shape of the cross section

- Shafts with solid circular cross section;
- Shafts with hollow circular cross section;
- Shafts with keyways;
- Shafts with splines;
- Shafts with rectangular cross section.



SHAFTS

Portion of the shaft which is in contact with a bearing is called ***journal***. We will distinguish between *end journal*, *neck journal* and *thrust journal*.



CALCULATION OF SHAFTS

Shafts may be calculated for:

- **Strength**;
- **Rigidity**;
- **Oscillations**.

CALCULATION OF SHAFTS FOR STRENGTH

Calculation of shafts for strength is divided into 3 stages:

1. Determination of the minimum diameter of the shaft;
2. Designing the shaft construction;
3. Strength analysis of the shaft.

DETERMINATION OF THE MINIMUM DIAMETER OF THE SHAFT

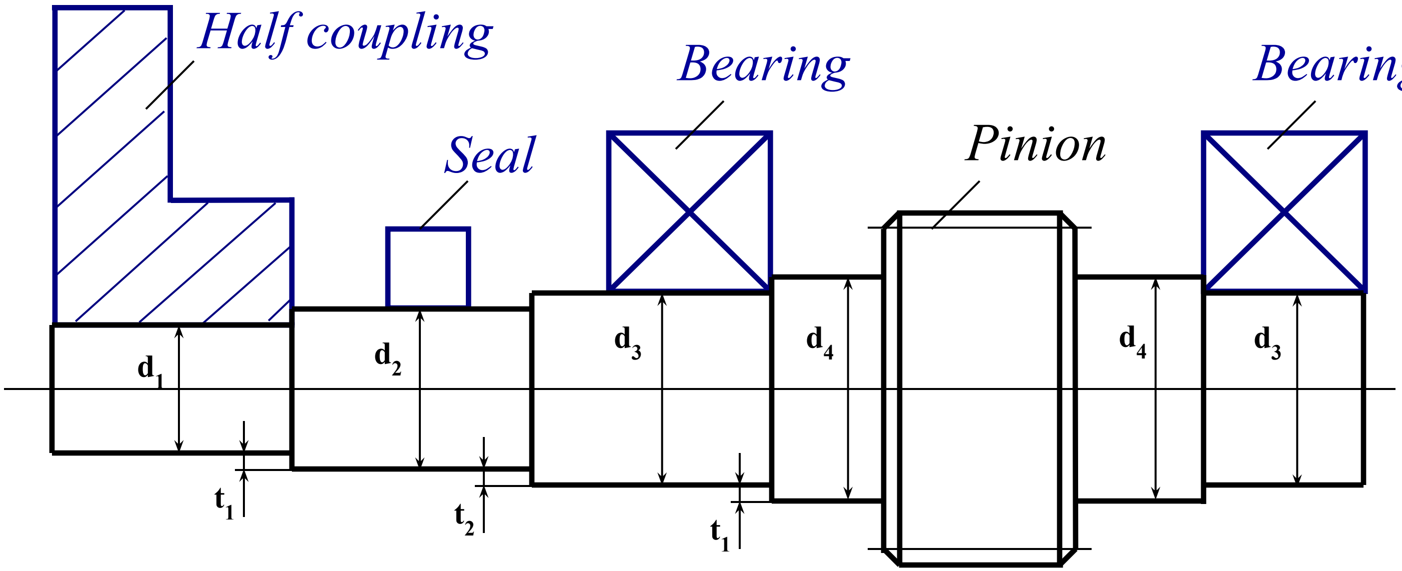
Minimum diameter of the shaft is determined taking into account torsion stresses only. In order to compensate neglect of bending stresses the allowable torsion stress is assumed as down rated ([τ]=20…40 MPa).

$$\tau = \frac{T}{W_p}; \qquad W_p = \frac{\pi \cdot d^3}{16}.$$

$$d_{min} = \sqrt[3]{\frac{T}{0.2 \cdot [\tau]}}.$$

DESIGNING THE SHAFT CONSTRUCTION

Input shaft



$$d_1 = d_{min};$$

$$d_2 = d_1 + 2 \cdot t_1;$$

$$d_3 = d_2 + 2 \cdot t_2;$$

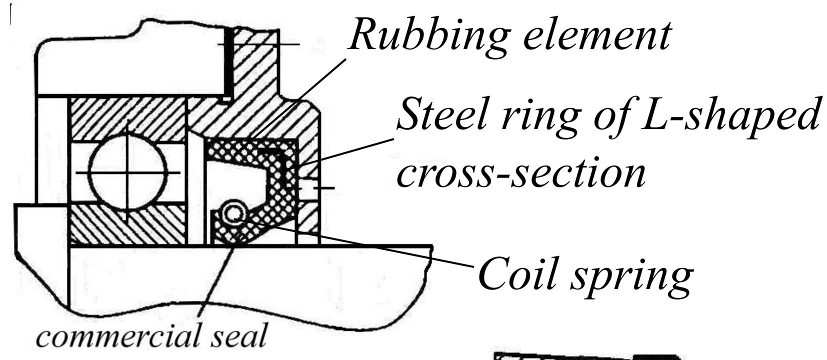
$$d_4 = d_3 + 2 \cdot t_1.$$

d, mm	20…50	55…120
t_1*, mm*	2; 2.5	5
t_2*, mm*	1; 1.5	2.5

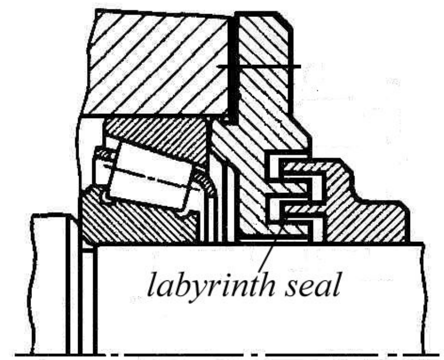
SEALS

Seals are divided into:

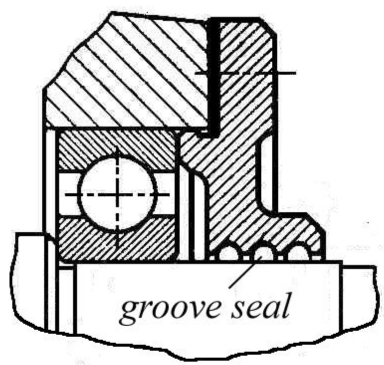
- Commercial seals (Lip-type seals);



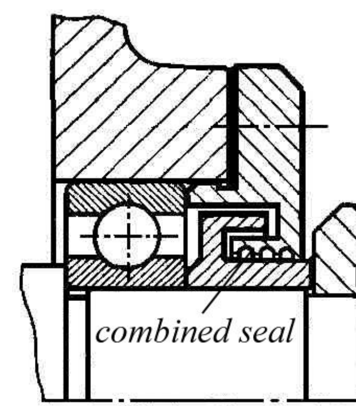
- Labyrinth seals;



- Groove seals;

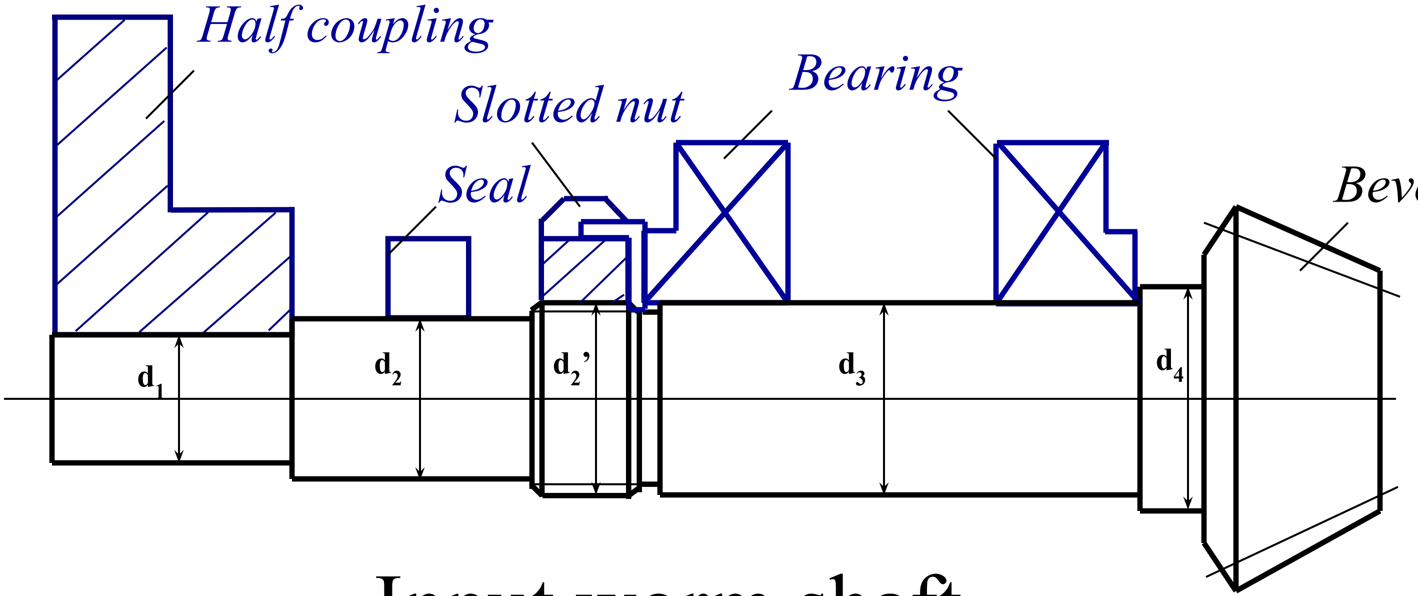


- Combined seals.

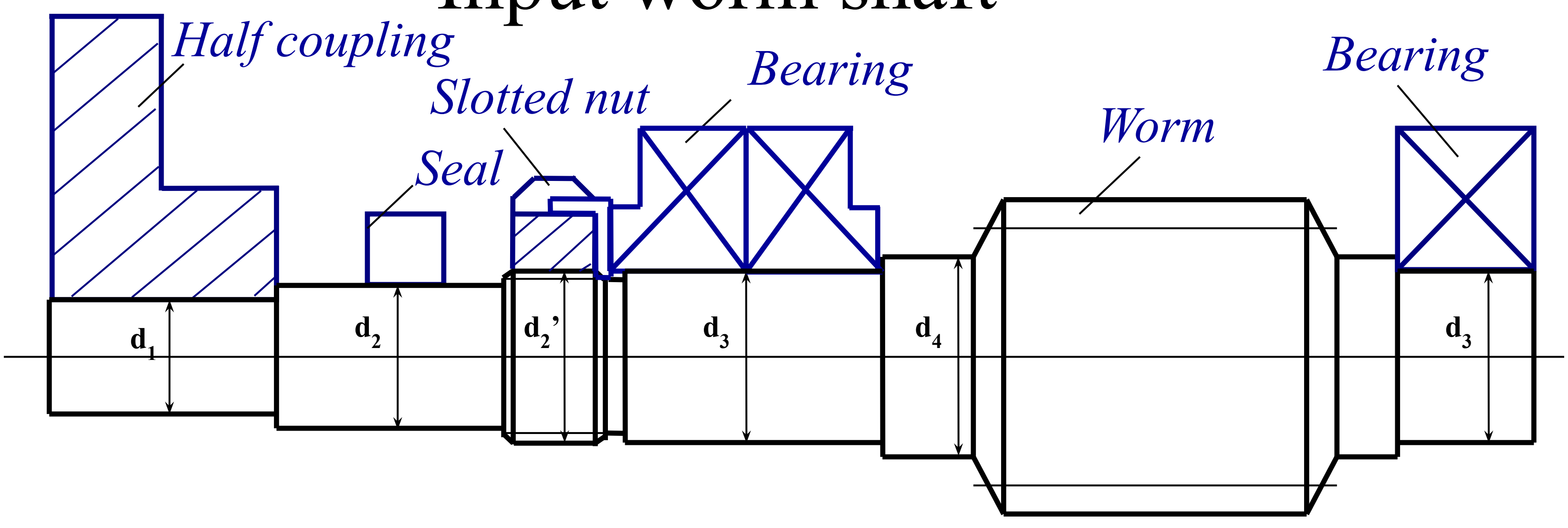


DESIGNING THE SHAFT CONSTRUCTION

Input bevel pinion shaft

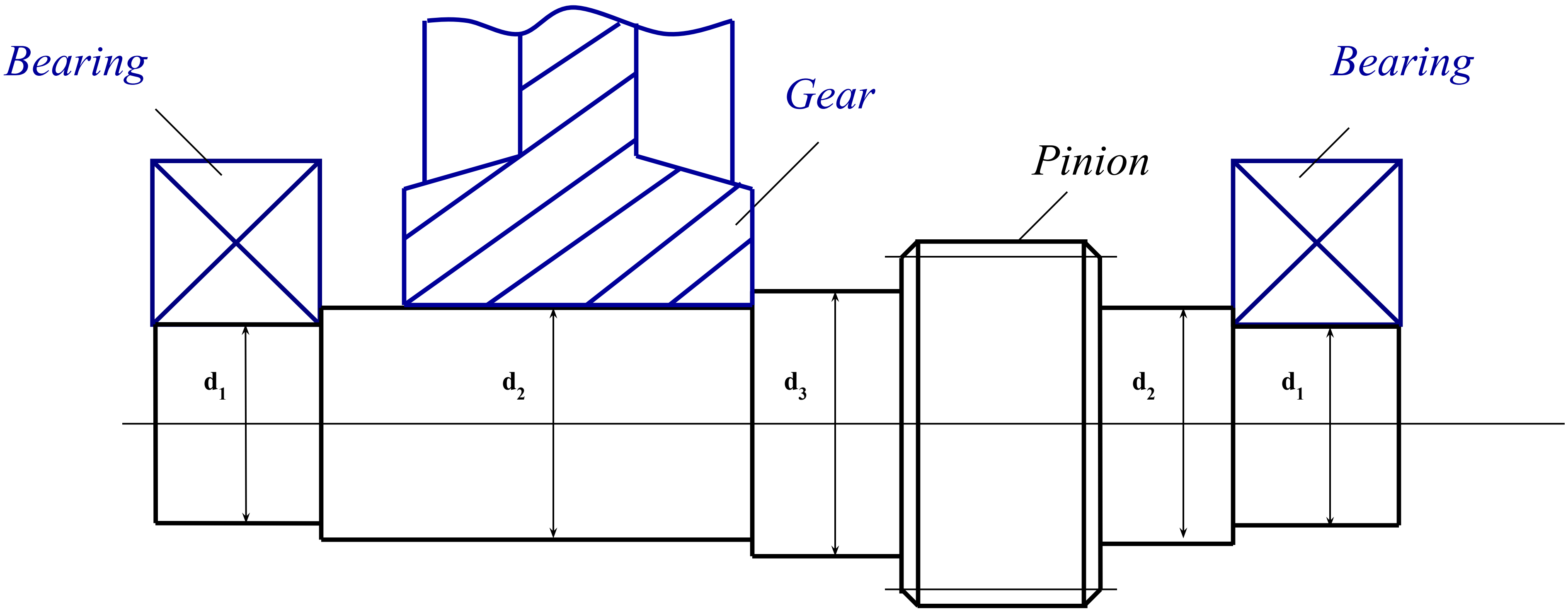


Input worm shaft



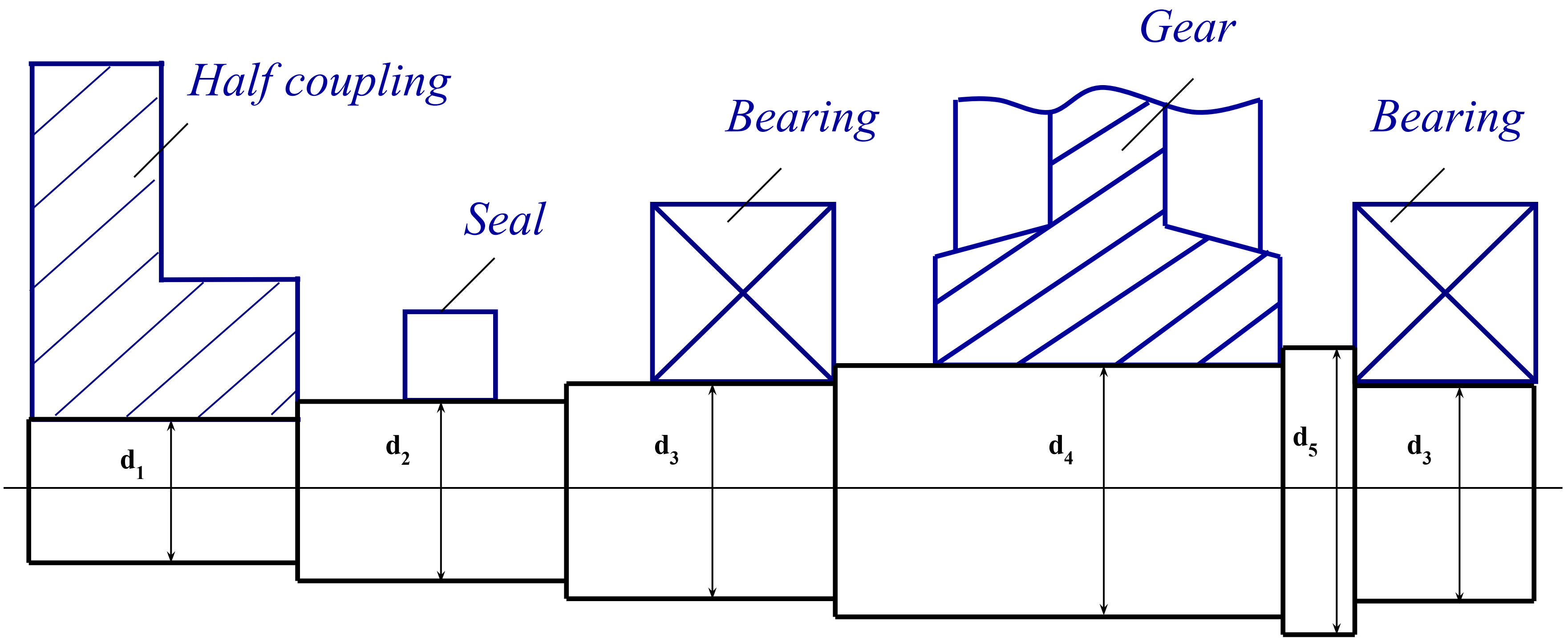
DESIGNING THE SHAFT CONSTRUCTION

Intermediate shaft

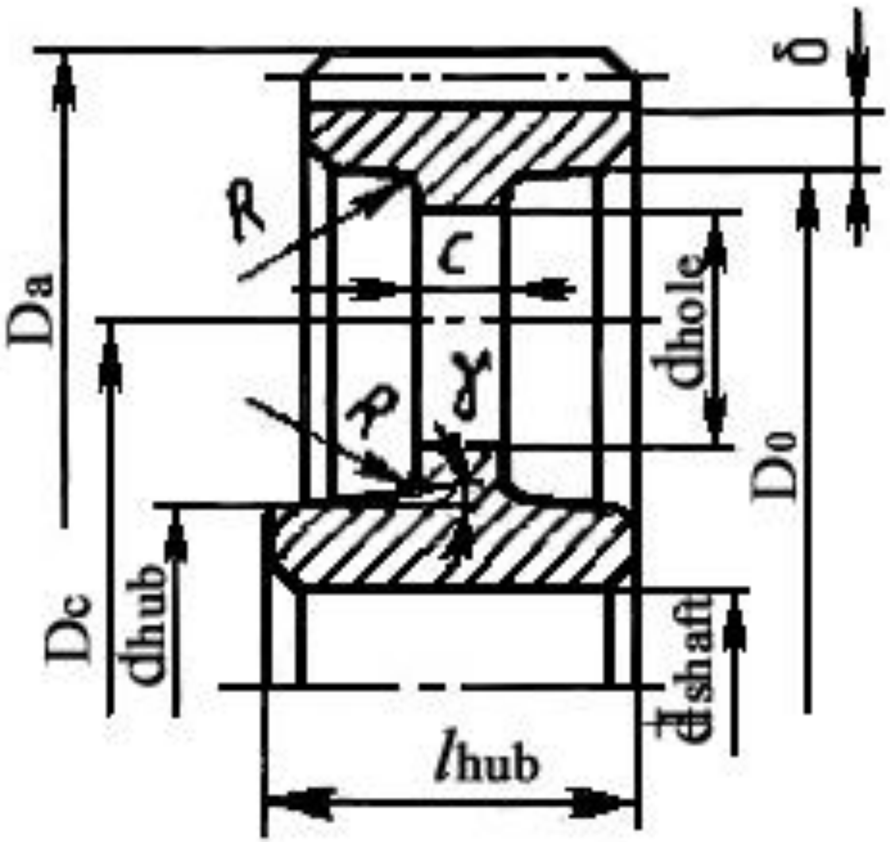


DESIGNING THE SHAFT CONSTRUCTION

Output shaft

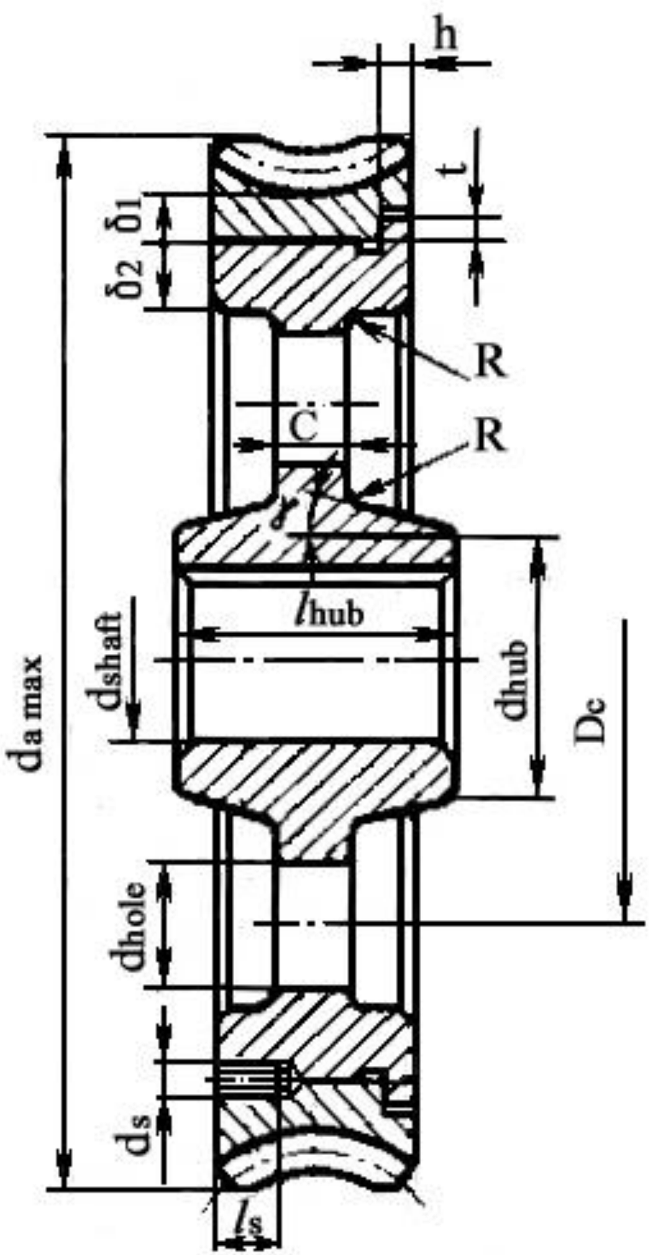


SPUR GEAR



- Thickness of the rim
  $\delta = (3 \ldots 4) \cdot m$;
- Thickness of the web
  $C = (0.2 \ldots 0.3) \cdot b^g$;
- Diameter of the hub
  $d_{hub} = (1.5 \ldots 1.7) \cdot d_{shaft}$;
- Length of the hub
  $l_{hub} = (1.2 \ldots 1.5) \cdot d_{shaft}$;
- Diameter of the hole
  $d_{hole} = (D_0 - d_{hub})/4$;
- Diameter of the hole centre line
  $D_c = (D_0 + d_{hub})/4$; ;
- Fillet radii $R \geq 6$ mm;
- Angle $\gamma \geq 7^o$.

WORM GEAR



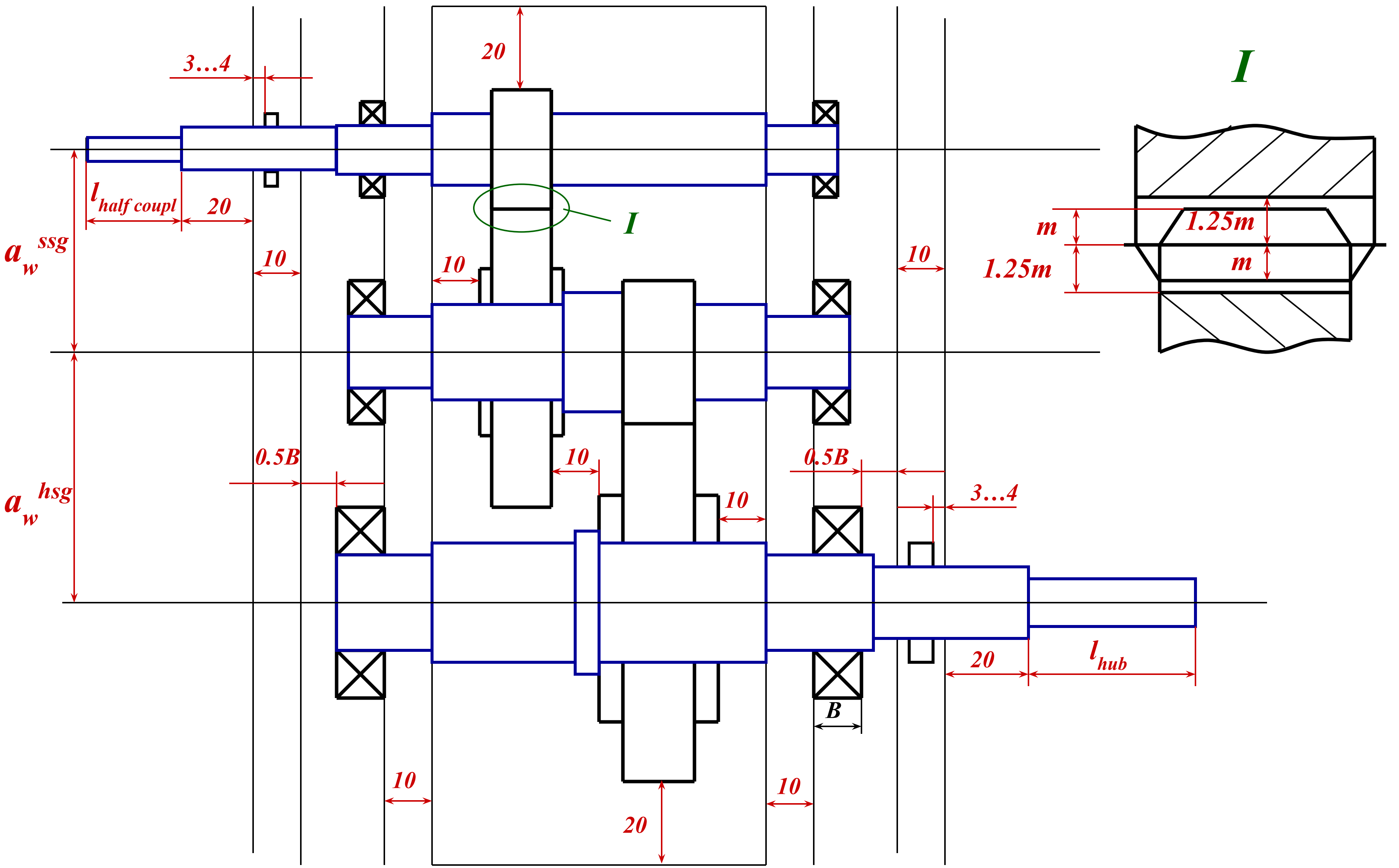
- Thickness of the bronze ring $\delta_1 = 2 \cdot m$;
- Thickness of the steel rim $\delta_2 = 2 \cdot m$;
- Thickness of the web $C = 0.2 \ldots 0.3) \cdot b^g$;
- Diameter of the hub $d_{hub} = (1.5 \ldots 1.7) \cdot d_{shaft}$;
- Length of the hub $l_{hub} = (1.2 \ldots 1.5) \cdot d_{shaft}$;
- Diameter of the screw $d_s = (1.2 \ldots 1.4) \cdot m$;
- Length of the screw $ls = (0.3 \ldots 0.4) \cdot b^g$;
- Diameter of the hole $d_{hole} = (D_0 - d_{hub})/4$;
- Diameter of the hole centre line

  $D_c = (D_0 + d_{hub})/4$;
- Width and height of the collar

  $h = 0.15 \cdot b^g$; $t = 0.8 \cdot h$;
- Fillet radii $R \geq 6$ mm
- Angle $\gamma \geq 7^o$.

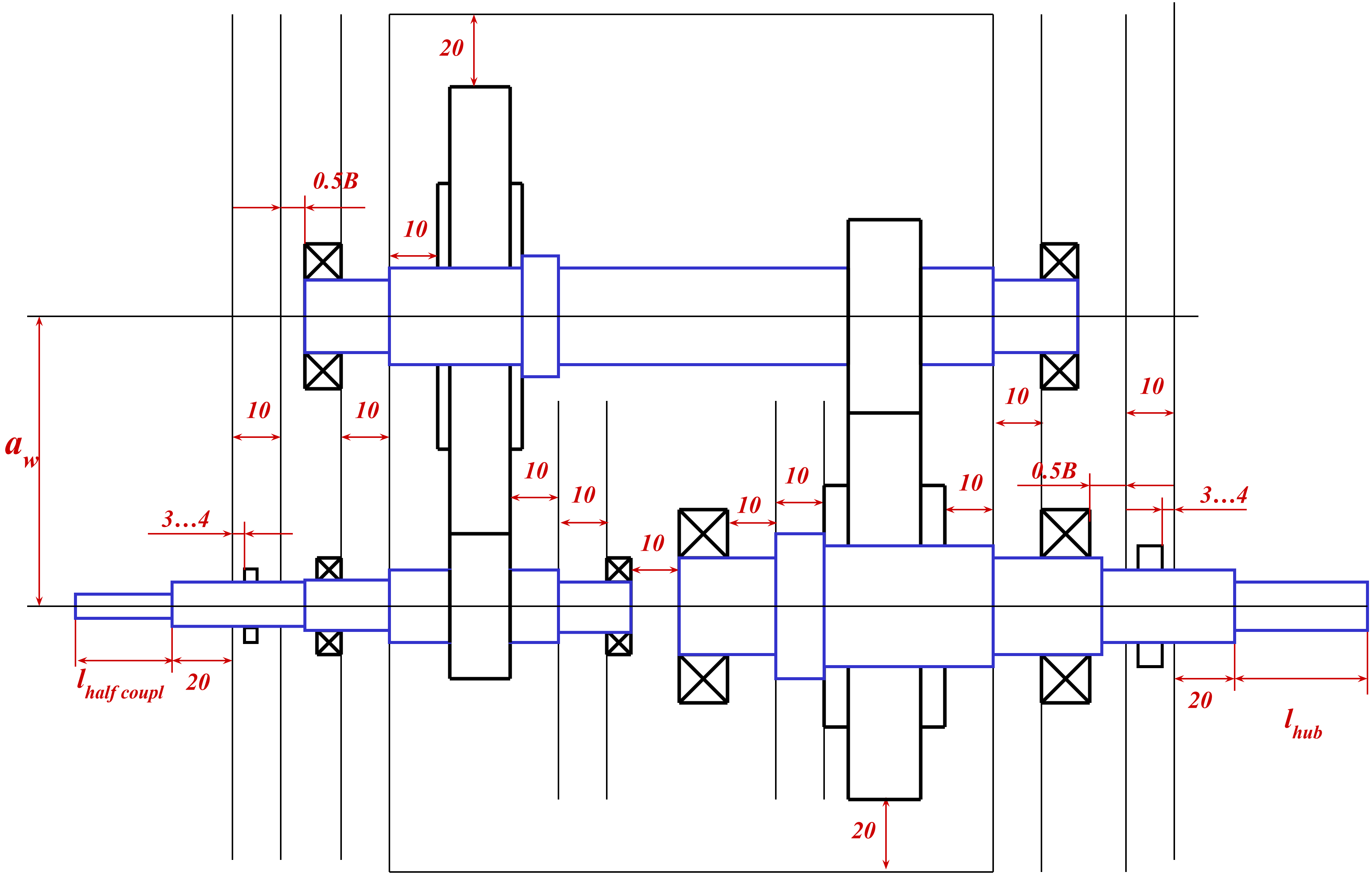
SKETCH LAYOUT

Double stage spur gear speed reducer



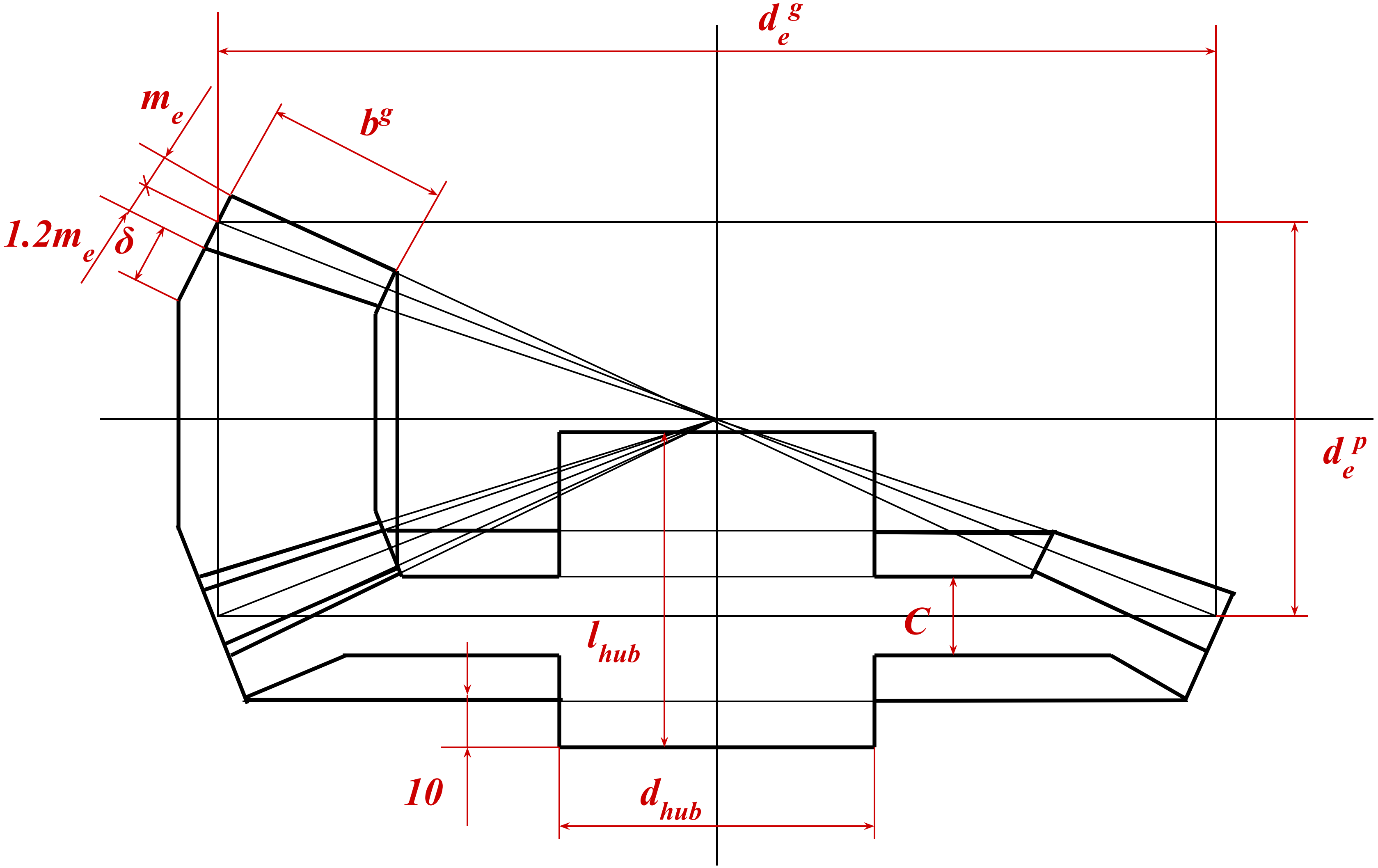
SKETCH LAYOUT

Double stage coaxial spur gear speed reducer



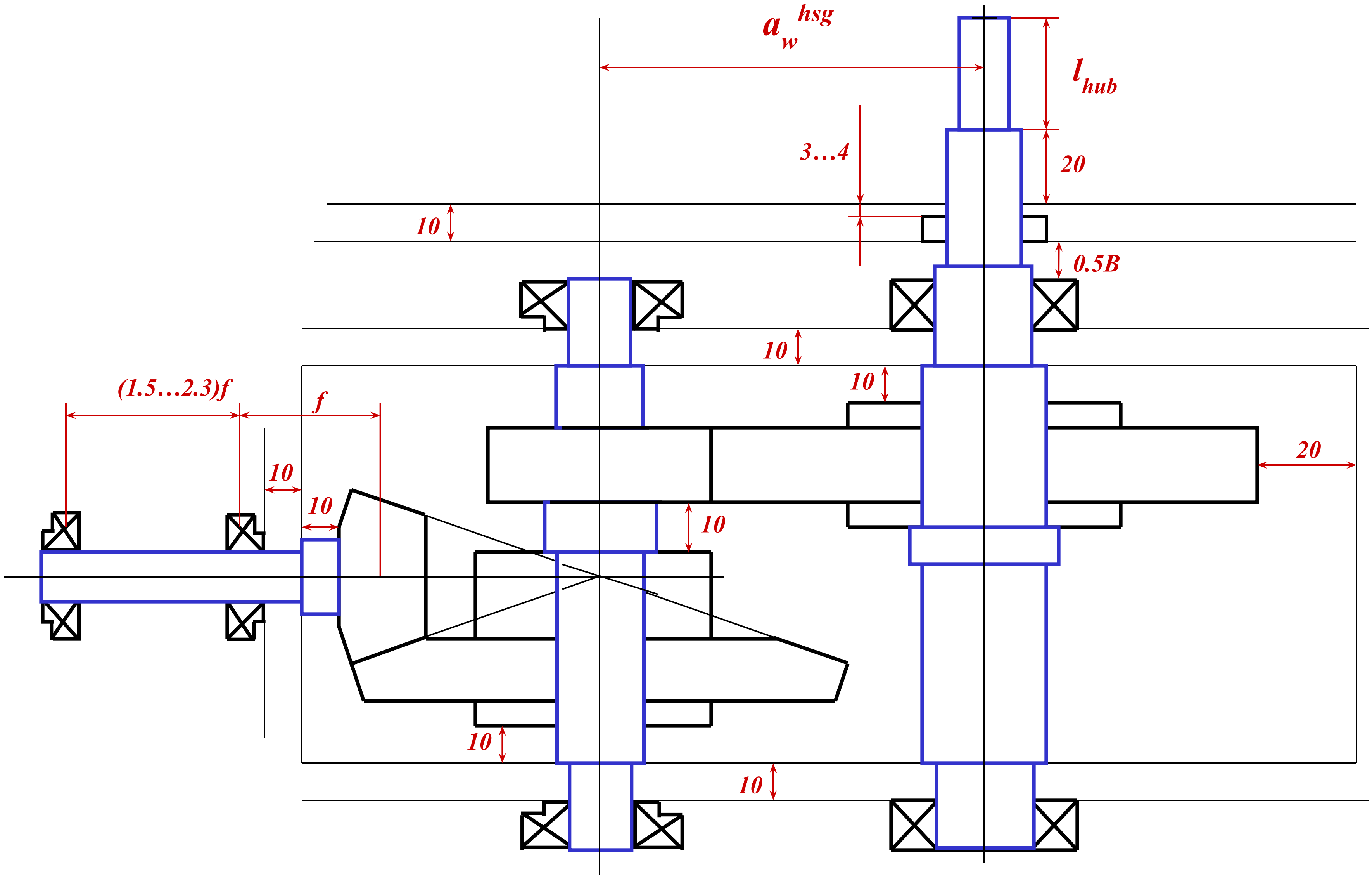
SKETCH LAYOUT

Bevel gears

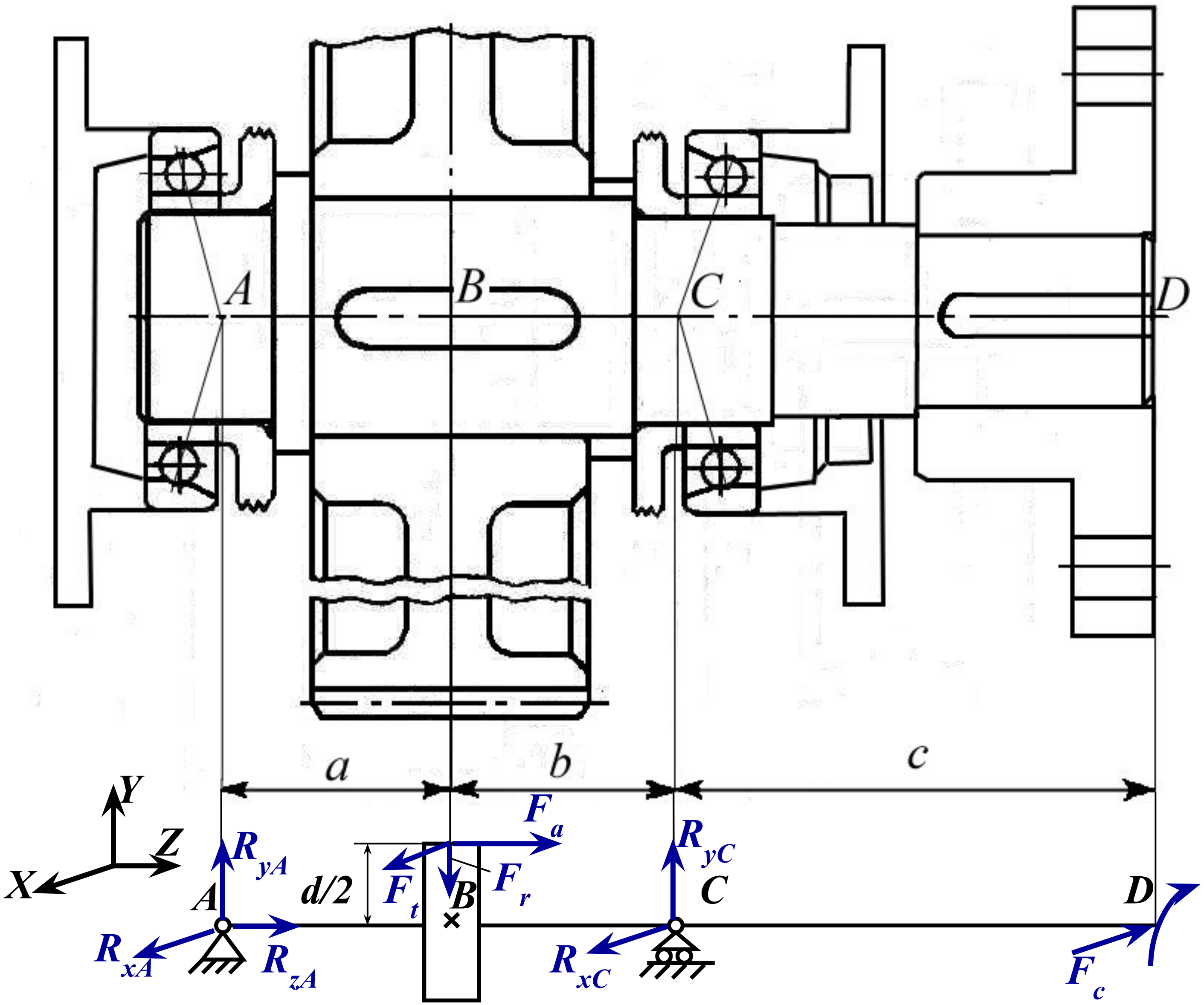


SKETCH LAYOUT

Double stage bevel and spur gear speed reducer



STRENGTH ANALYSIS OF THE SHAFT



For ***single*** stage speed reducers

$$F_c = 125\sqrt{T};$$

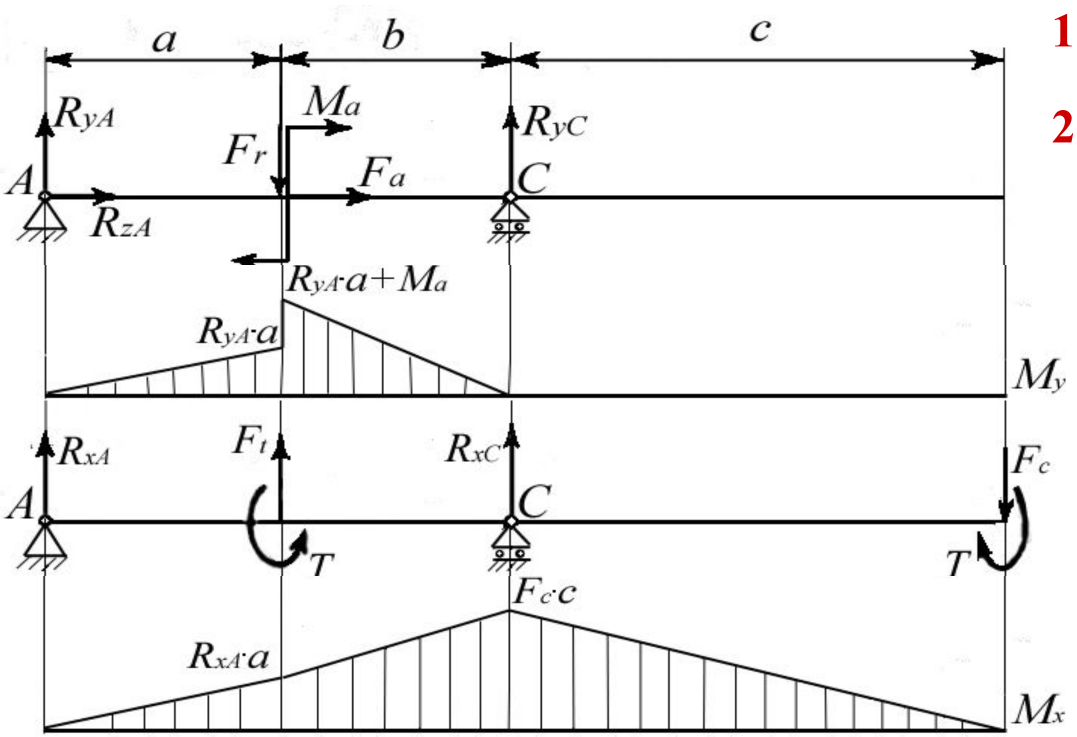
For ***double*** stage speed reducers

$$F_c = 250\sqrt{T}.$$

STRENGTH ANALYSIS OF THE SHAFT

1. Draw the analytical model in the vertical plane and transfer all forces to the shaft;
2. Determine vertical support reactions R_{yA} and R_{yC}. For this purpose we set up equations of moments relative to points A and C. For checking we will write equation of forces that are parallel to Y axis;
3. Plot the bending moment diagram in the vertical plane;
4. Draw the analytical model in the horizontal plane and transfer all forces to the shaft;
5. Determine horizontal support reactions R_{xA} and R_{xC}. For that we set up equations of moments relative to points A and C. For checking we write equation of forces that are parallel to X axis;
6. Plot the bending moment diagram in the horizontal plane;
7. Plot the total bending moment diagram $(M_{\Sigma} = \sqrt{M_x^2 + M_y^2})$;
8. Plot the twisting moment diagram;
9. Plot the reduced moment diagram $(M_{red} = \sqrt{M_t^2 + 0.75 \cdot T^2})$.

STRENGTH ANALYSIS OF THE SHAFT



1. $M_a = F_a \cdot \dfrac{d}{2}.$

2. $\sum M_A = 0: \ -F_{\epsilon}\cdot a - M_a + R_y \ \cdot (a+b) = 0;$

$$R_{yC} = \frac{F_r \cdot a + M_a}{a+b};$$

$\sum M_c = 0: \ -R_{yA}\cdot(a+b) + F_r \cdot b - M_a = 0;$

$$R_{yA} = \frac{F_r \cdot b - M_a}{a+b};$$

Checking: $\sum F_{yi} = 0: \ R_{yA} - F_r + R_{yC} = 0.$

3. $0 \le x \le a; \quad M_y = R_{yA}\cdot x;$

$M_y(0) = 0; \quad M_y(a) = R_{yA}\cdot a;$

$a \le x \le a+b;$

$M_y = R_{yA}\cdot x + M_a - F_r\cdot(x-a);$

$M_y(a) = R_{yA}\cdot a + M_a; \ M_y(a+b) = 0.$

4. $T = F_t \cdot \dfrac{d}{2}.$

5. $\sum M_A = 0: \ F_C\cdot a + R_x \ \cdot (a_c + b) - F \ \cdot (a+b+c) = 0;$

$$R_{xC} = \frac{-F_t\cdot a + F_c\cdot(a+b+c)}{a+b};$$

$\sum M_c = 0: \ -R_{xA}\cdot(a+b) - F_t\cdot b - F_c\cdot c = 0;$

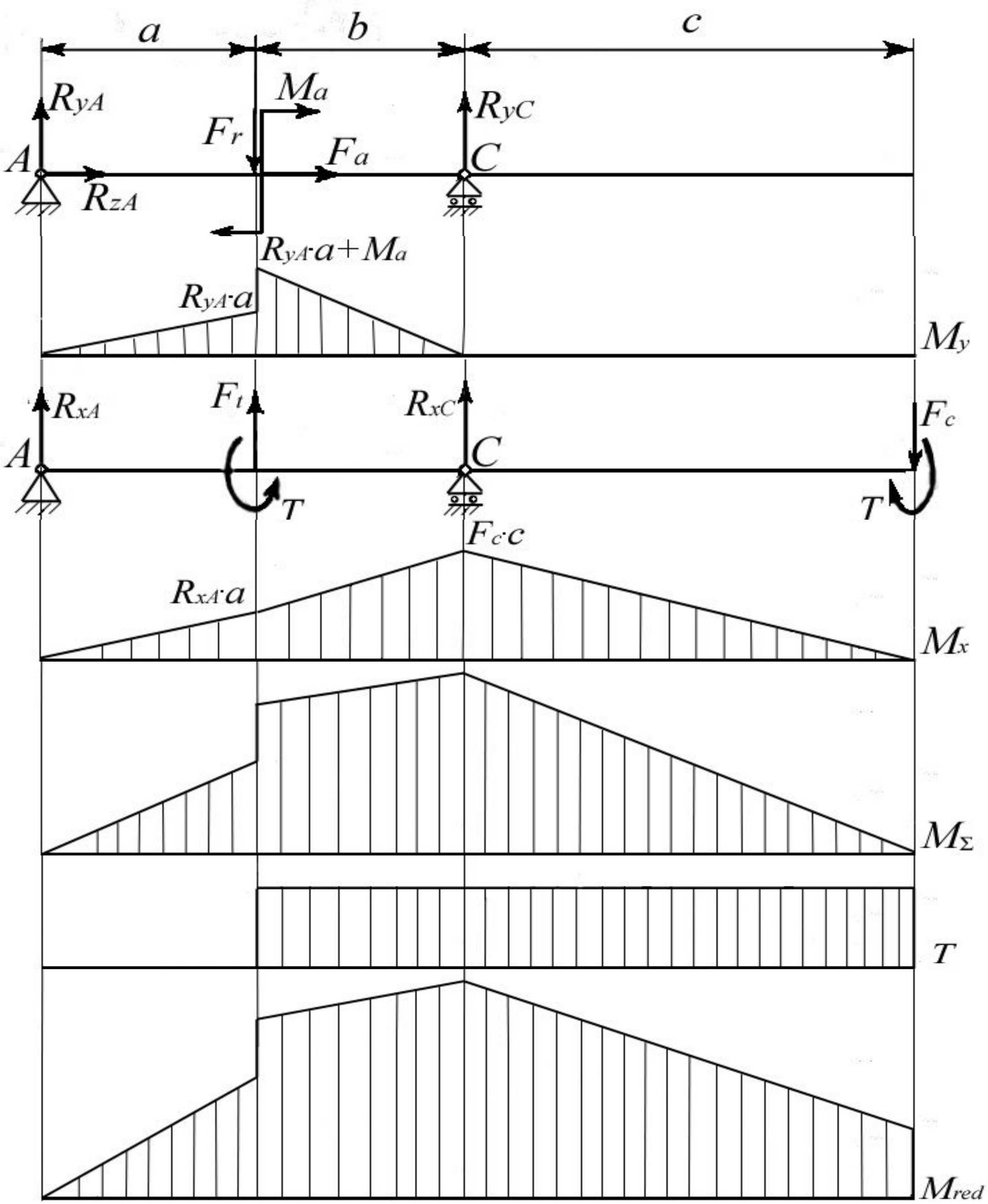
$$R_{xA} = \frac{-F_t\cdot b - F_c\cdot c}{a+b};$$

Checking: $\sum F_{xi} = 0: \ R_{xA} + F_t + R_{xC} - F_c = 0.$

6. $0 \le x \le a; \ M_x = R_{xA}\cdot x; \ M_x(0) = 0; \ M_x(a) = R_{xA}\cdot a;$

$0 \le x \le c; \ M_x = F_c\cdot x; \ M_x(0) = 0; \ M_x(c) = F_c\cdot c.$

STRENGTH ANALYSIS OF THE SHAFT



7. $M_{\Sigma} = \sqrt{M_x^2 + M_y^2}$;

8. T

9. $M_{red} = \sqrt{M_t^2 + 0.75 \cdot T^2}$;

Calculation for static strength

$$\sigma_b = \frac{M}{W} \leq [\sigma_b];$$

$$M = M_{red\ max}; \quad W = \frac{\pi \cdot d^3}{32};$$

$$\sigma_b = \frac{M_{red\ max}}{0.1 \cdot d^3} \leq [\sigma_b],$$ where

$M_{red\ max}$ is the reduced moment at the critical section;

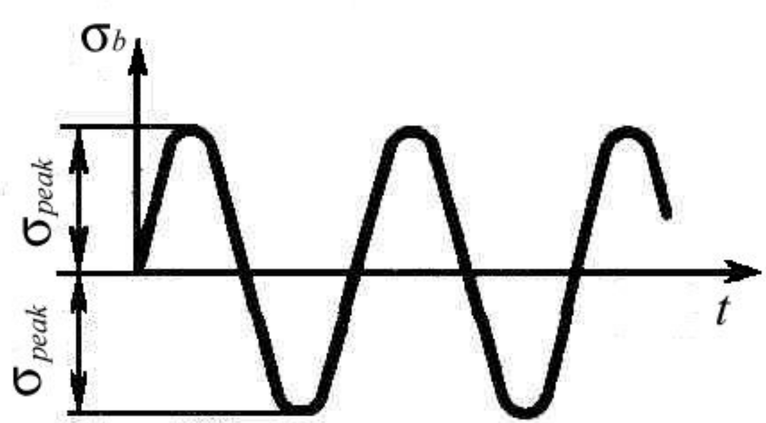
d is diameter of the shaft at the critical section;

$[\sigma_b] = 100…120$ MPa.

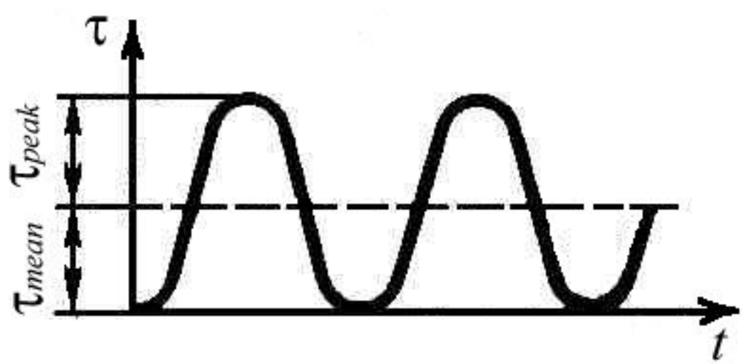
STRENGTH ANALYSIS OF THE SHAFT

Calculation of the shaft for fatigue strength

Changing of bending stresses



Changing of torsion stresses



Safety factor

$$S = \frac{S_\sigma \cdot S_\tau}{\sqrt{S_\sigma^2 + S_\tau^2}} \geq [S] = 1.5...2.5.$$

Safety factor for bending

$$S_\sigma = \frac{\sigma_{lim}}{\frac{K_\sigma}{K_d \cdot K_F} \cdot \sigma_{peak} + \psi_{mean} \cdot \sigma};$$

Safety factor for torsion

$$S_\tau = \frac{\tau_{lim}}{\frac{K_\tau}{K_d \cdot K_F} \cdot \tau_{peak} + \psi_{mean} \cdot \tau}.$$

STRENGTH ANALYSIS OF THE SHAFT

Calculation of the shaft for fatigue strength

$$S_{\sigma} = \frac{\sigma_{lim}}{\frac{K_{\sigma}}{K_d \cdot K_F} \cdot \sigma_{peak} + \psi_{mean} \cdot \sigma}; \qquad S_{\tau} = \frac{\tau_{lim}}{\frac{K_{\tau}}{K_d \cdot K_F} \cdot \tau_{peak} + \psi_{mean} \cdot \tau}.$$

σ_{lim}, τ_{lim} – limit of endurance in bending and in torsion

$\sigma_{lim} = 0.43 \cdot \sigma_{ult}$ - for carbon steels;

$\sigma_{lim} = 0.35 \cdot \sigma_{ult} + 120$ - for alloy steels;

$\tau_{lim} = (0.2...0.3) \cdot \sigma_{ult}$.

σ_{peak}, τ_{peak} – variable (peak) components of bending and torsion stresses

$$\sigma_{peak} = \frac{\sigma_{max} - \sigma_{min}}{2} = \sigma_{max} = \frac{M_{\Sigma}}{W} = \frac{M_{\Sigma}}{0.1 \cdot d^3};$$

$$\tau_{peak} = \frac{\tau_{max} + \tau_{min}}{2} = \frac{\tau_{max}}{2} = 0.5 \cdot \frac{T}{W_p} = 0.5 \cdot \frac{T}{0.2 \cdot d^3}.$$

STRENGTH ANALYSIS OF THE SHAFT

Calculation of the shaft for fatigue strength

$$S_\sigma = \frac{\sigma_{lim}}{\frac{K_\sigma}{K_d \cdot K_F} \cdot \sigma_{peak} + \psi_{mean} \cdot \sigma}; \qquad S_\tau = \frac{\tau_{lim}}{\frac{K_\tau}{K_d \cdot K_F} \cdot \tau_{peak} + \psi_{mean} \cdot \tau}.$$

$\sigma_{mean}, \tau_{mean}$ – constant (mean) components of bending and torsion stresses

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2} = 0;$$

$$\tau_{mean} = \frac{\tau_{max} + \tau_{min}}{2} = \frac{\tau_{max}}{2} = 0.5 \cdot \frac{T}{W_p} = 0.5 \cdot \frac{T}{0.2 \cdot d^3}.$$

ψ_σ, ψ_τ – factors of constant components of bending and torsion stresses

$\psi_\sigma = 0.1; \ \psi_\tau = 0.05$ – for carbon steels;

$\psi_\sigma = 0.15; \ \psi_\tau = 0.1$ – for alloy steels.

K_σ, K_τ – effective stress concentration factors;

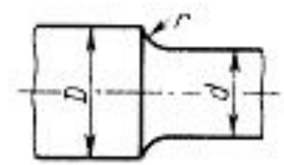
K_d – scale factor;

K_F - surface roughness factor.

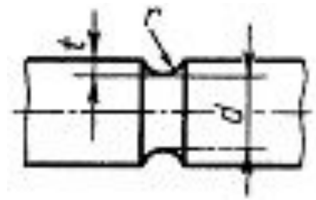
STRENGTH ANALYSIS OF THE SHAFT

The most typical stress concentrations of the shaft

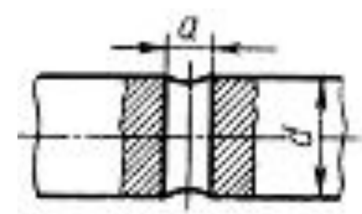
- Filleted transition regions;



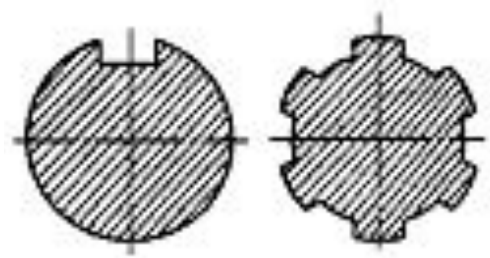
- Grooves;



- Radial holes;



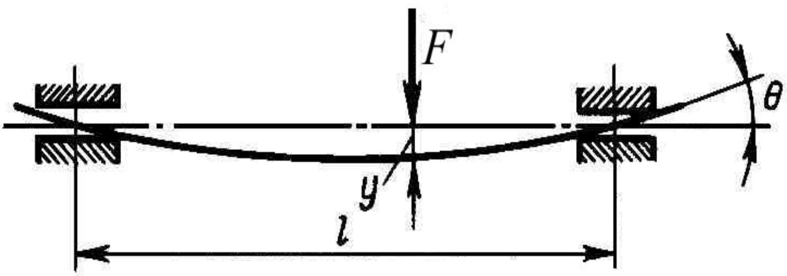
- Keyed and splined portions;



- Threaded portions;
- Interference fits.

RIGIDITY ANALYSIS OF THE SHAFT

Flexural rigidity



Basic criteria of flexural rigidity are:

Maximum deflection (sag) ***y*** of the shaft;

Angle of rotation ***θ*** of support sections.

Flexural rigidity conditions

$$y \le [y]; \qquad \theta \le [\theta],$$

where

[y] is the maximum safe sag; ***[θ]*** is the maximum safe angle of rotation.

[y]= 0.01m – for shafts of spur gears and worm gear drives;

[y]= 0.005m – for shafts of bevel gear, hypoid gear and hourglass worm gear drives;

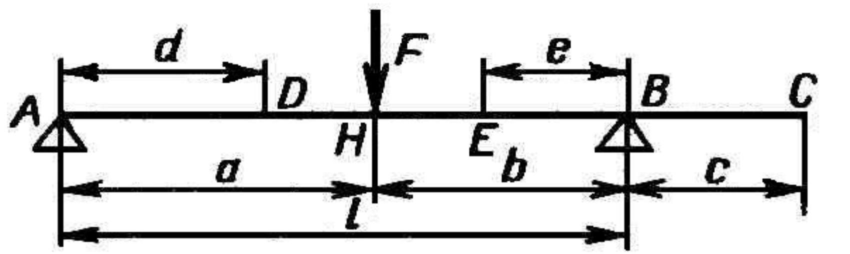
[y]= (0.0002…0.0003)l – for general purpose shafts used in machine tools;

[θ]= 0.001 rad – for shafts mounted in sliding contact bearings;

[θ]= 0.005 rad – for shafts mounted in radial ball bearings.

RIGIDITY ANALYSIS OF THE SHAFT

Flexural rigidity



$$\theta_A = \frac{F \cdot a \cdot b \cdot (l + b)}{6 \cdot E \cdot J \cdot l};$$

$$\theta_B = \frac{F \cdot a \cdot b \cdot (l + a)}{6 \cdot E \cdot J \cdot l};$$

$$\theta_C = \theta_B;$$

$$y_C = \frac{F \cdot a \cdot b \cdot c \cdot (l + a)}{6 \cdot E \cdot J \cdot l};$$

$$\theta_D = \frac{F \cdot b \cdot (l^2 - b^2 - 3d^2)}{6 \cdot E \cdot J \cdot l};$$

$$y_D = \frac{F \cdot b \cdot d \cdot (l^2 - b^2 - d^2)}{6 \cdot E \cdot J \cdot l};$$

$$\theta_E = \frac{F \cdot a \cdot (l^2 - a^2 - 3e^2)}{6 \cdot E \cdot J \cdot l};$$

$$y_E = \frac{F \cdot a \cdot e \cdot (l^2 - a^2 - e^2)}{6 \cdot E \cdot J \cdot l};$$

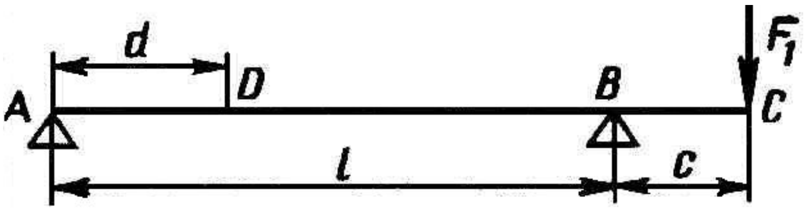
$$\theta_H = \frac{F \cdot a \cdot b \cdot (b - a)}{3 \cdot E \cdot J \cdot l}.$$

$$y_H = \frac{F \cdot a^2 \cdot b^2}{3 \cdot E \cdot J \cdot l}.$$

E is modulus of elasticity of the shaft material; ***J*** is centroidal moment of inertia.

RIGIDITY ANALYSIS OF THE SHAFT

Flexural rigidity



$$\theta_A = \frac{F_1 \cdot c \cdot l}{6 \cdot E \cdot J};$$

$$\theta_B = \frac{F_1 \cdot c \cdot l}{3 \cdot E \cdot J};$$

$$\theta_C = \frac{F_1 \cdot c \cdot (2 \cdot l + 3 \cdot c)}{6 \cdot E \cdot J};$$

$$y_C = \frac{F_1 \cdot c^2 \cdot (l + c)}{3 \cdot E \cdot J};$$

$$\theta_D = \frac{F_1 \cdot c \cdot (3 \cdot d^2 + l^2)}{6 \cdot E \cdot J \cdot l};$$

$$y_D = \frac{F_1 \cdot c \cdot d \cdot (l^2 + d^2)}{6 \cdot E \cdot J \cdot l}.$$

RIGIDITY ANALYSIS OF THE SHAFT

Torsional rigidity

Basic criterion of torsional rigidity is the angle of twist.

Torsional rigidity condition

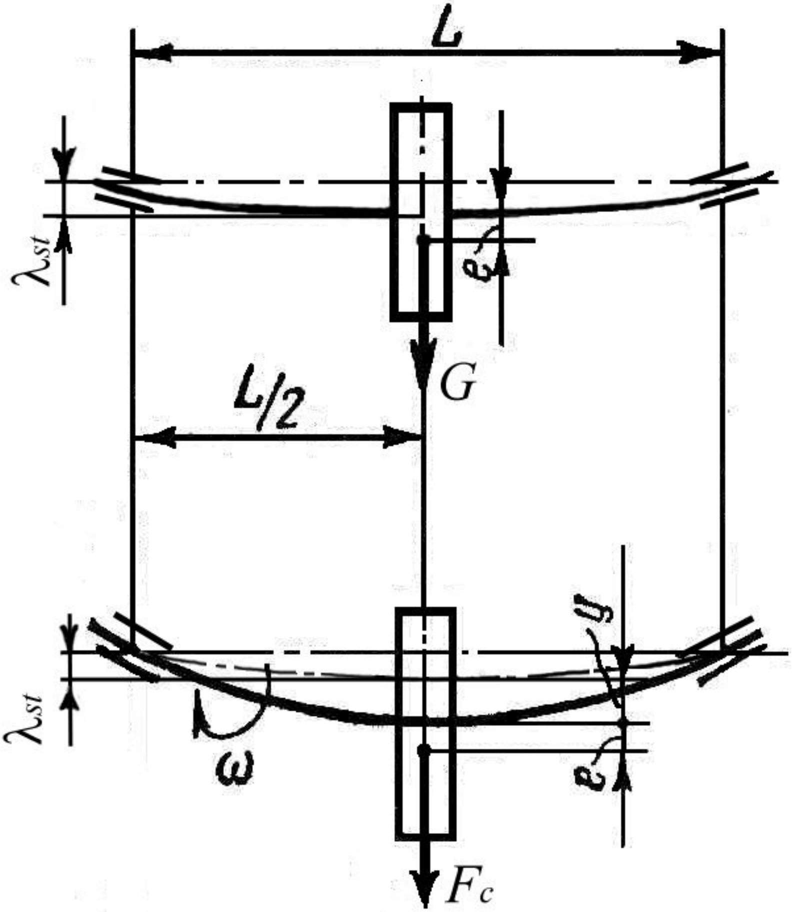
$$\varphi \leq [\varphi],$$

where ***[ϕ]*** is the maximum safe angle of twist.

$$\varphi = \frac{T \cdot l}{G \cdot J_p},$$

where ***T*** is torque; ***l*** is length of the shaft; ***G*** is shear modulus;
$J_p = \pi d^4/32$ is polar moment of inertia.

CALCULATION OF THE SHAFT FOR OSCILLATIONS



$$\lambda_{st} = \frac{G}{c}$$ - static deflection;

$$F_c = m \cdot \omega^2 \cdot (y+e);$$

$$F_{el} = c \cdot y;$$

$$F_c = F_{el};$$

$$m \cdot \omega^2 \cdot (y+e) = c \cdot y;$$

$$y = \frac{m \cdot \omega^2 \cdot e}{c - m \cdot \omega^2} = \frac{e}{\dfrac{c}{m \cdot \omega^2} - 1}$$ – dynamic deflection

$$m \cdot \omega^2 = c$$ - condition of resonance.

$$\omega_{cr} = \sqrt{\frac{c}{m}}$$ - *critical angular velocity*.

CALCULATION OF THE SHAFT FOR OSCILLATIONS

$$\omega = \frac{\pi \cdot n}{30} \Rightarrow n_{cr} = \frac{30}{\pi} \cdot \omega_{cr} = \frac{30}{\pi} \cdot \sqrt{\frac{c}{m}} = \frac{30}{\pi} \cdot \sqrt{\frac{c \cdot g}{m \cdot g}} = \frac{30}{\pi} \cdot \sqrt{\frac{g}{\lambda_{st}}};$$

$$n_{cr} = \frac{30}{\pi} \cdot \sqrt{\frac{g}{\lambda_{st}}}$$ ***- critical rotational speed,***

where $g = 9.81\ m / sec^2$ - free fall acceleration;

$\lambda_{st} = \frac{G}{c}$ - static deflection;

$c = \frac{48 \cdot E \cdot J}{L^3}$ - rigidity of the shaft;

E - modulus of elasticity of the shaft material;

L - distance between shaft supports;

$J = \frac{\pi \cdot d^4}{64}$ - shaft moment of inertia.

CALCULATION OF THE SHAFT FOR OSCILLATIONS

$$if \quad n \le 0.7 \cdot n_{cr} \quad \text{- rigid shafts;}$$

$$if \quad n \ge 1.2 \cdot n_{cr} \quad \text{- flexible shafts.}$$

$$y = \frac{e}{\frac{c}{m \cdot \omega^2} - 1}.$$

$$if \quad \omega \to \infty,$$

$$y \to -e.$$

In this case we deal with shaft self-centering.