LECTURE 3

Three-point difference derivative

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Three-point difference derivative

We can derive the three-point difference derivative. Let us construct the scheme with the second order accuracy.

$$y_i' \approx \frac{a y_i + b y_{i+1} + c y_{i+2}}{\Delta x} + O(\Delta x^2),$$
$$i = 0, \ldots, N-2;$$

The coefficients a, b and c

$$y_i' \approx \frac{ay_i + by_{i+1} + cy_{i+2}}{\Delta x} + O(\Delta x^2),$$
$$i = 0, \dots, N-2;$$

Three-point difference derivative

the found values of a, b and c are substituted into the first formula

$$y_i' \approx \frac{-3y_i + 4y_{i+1} - y_{i+2}}{2\Delta x} + O(\Delta x^2),$$
$$i = 0, \dots, N-2;$$

c. Difference formulas for $f'(x)$:

# of pts	Formula	approx error	c
2-1	$\frac{1}{h}[f(x+h)-f(x)]$	$-\frac{f''(c)}{2!}h$	$[x,x+h]$
2-2	$\frac{1}{h}[f(x)-f(x-h)]$	$-\frac{f''(c)}{2!}h$	$[x-h,x]$
3-1	$\frac{1}{h}\left[\frac{1}{2}f(x+h)-\frac{1}{2}f(x-h)\right]$	$-\frac{f'''(c)}{6}h^2$	$[x-h,x+h]$
3-2	$\frac{1}{h}\left[-\frac{3}{2}f(x)+2f(x+h)-\frac{1}{2}f(x+2h)\right]$	$\frac{f'''(c)}{3}h^2$	$[x,x+2h]$
3-3	$\frac{1}{h}\left[\frac{1}{2}f(x-2h)-2f(x-h)+\frac{3}{2}f(x)\right]$	$\frac{f'''(c)}{3}h^2$	$[x-2h,x]$
5-1	$\frac{1}{h}\left[\frac{1}{12}f(x-2h)-\frac{8}{12}f(x-h)+\frac{8}{12}f(x+h)-\frac{1}{12}f(x+2h)\right]$	$\frac{f^{(5)}(c)}{30}h^4$	$[x-2h,x+2h]$
5-2	$\frac{1}{h}\left[-\frac{25}{12}f(x)+\frac{48}{12}f(x+h)-\frac{36}{12}f(x+2h)+\frac{16}{12}f(x+3h)-\frac{3}{12}f(x+4h)\right]$	$\frac{f^{(5)}(c)}{5}h^4$	$[x,x+4h]$

Laboratory work 2

- Choose 3 difference formulas from table
- Compare them with the exact solution of the derivative
- Find the mathematical expectation
- Conclusion