Lecture 2.
The Time Value of Money

Olga Uzhegova, DBA
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THE TIME VALUE OF MONEY (TVM)

Money (a dollar or a yen, or any other currency) in hand today is worth more than the expectation of the same amount to be received in the future.

WHY?

- You can invest it, earn interest, and end up with more in the future;
- The purchasing power of money can change over time because of inflation.
- The receipt of money expected in the future is, in general, uncertain.

TIME IS MONEY

COMPOUNDING VS. DISCOUNTING

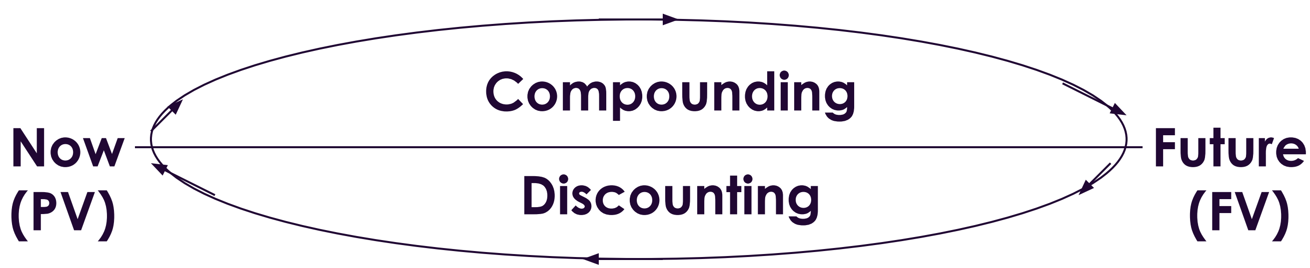
Compounding

The process of going from today's value, or present value (PV) to future value (FV)

Discounting

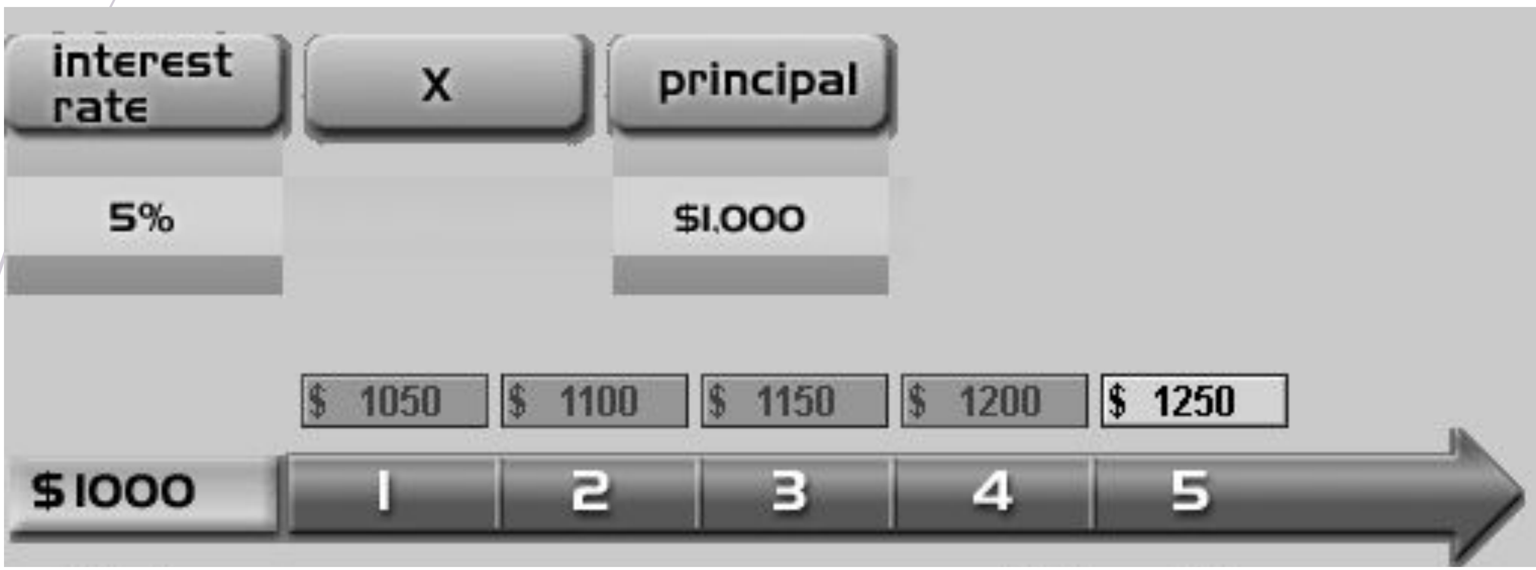
The process of going from future value (FV) to today's value, or present value (PV)

Future value is the value of an asset in the future that is equivalent in value to a specific amount today.



SIMPLE INTEREST

With **Simple Interest**, the interest rate each year is applied to the original investment amount.



Today (PV)

Future Years (FV)

$$\underline{FV}=\underline{PV\times(1+r\times n)}$$

PV = Present value
FV = Future value
r = interest rate, in decimal points
n = number of periods

COMPOUNDING INTEREST

With **Compounding Interest**, the interest rate each year is applied to the accumulated investment balance, not the original investment amount.

Compound interest

Simple interest

Today

Future years

$$FV = PV \times (1+r)^n$$

PV = Present value

FV = Future value

r = interest rate

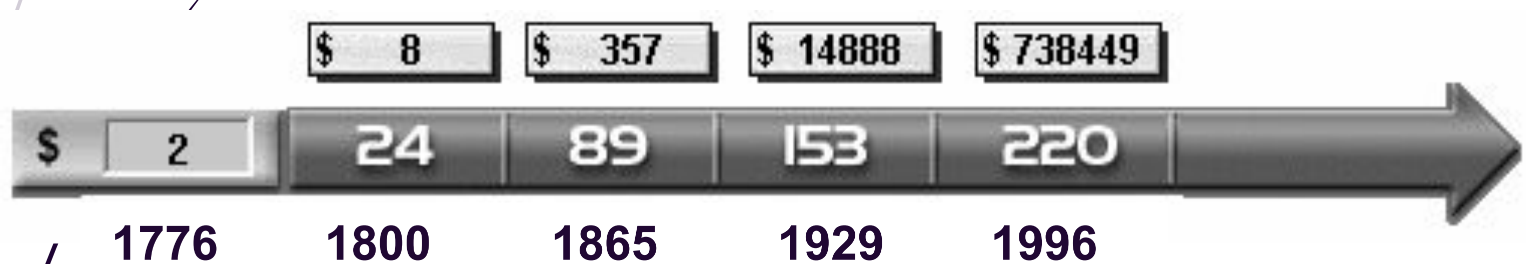
n = number of periods

FVIF – future value interest factor

COMPOUNDING INTEREST

To see how much $2 investment would have grown, compute FVs.

$$FV=PV\times(1+r)^n$$



$$\$2\times(1+0.06)^{24}=\$8$$

$$\$2\times(1+0.06)^{89}=\$357$$

$$\$2\times(1+0.06)^{153}=\$14\ 888$$

$$\$2\times(1+0.06)^{220}=\$738\ 449$$

THE TIMELINE

- A timeline is a linear representation of the timing of potential cash flows.
- Drawing a timeline of the cash flows will help you visualize the financial problem.
- Differentiate between two types of cash flows
  - Inflows are positive cash flows.
  - Outflows are negative cash flows, which are indicated with a – (minus) sign.

THE TIMELINE: EXAMPLE

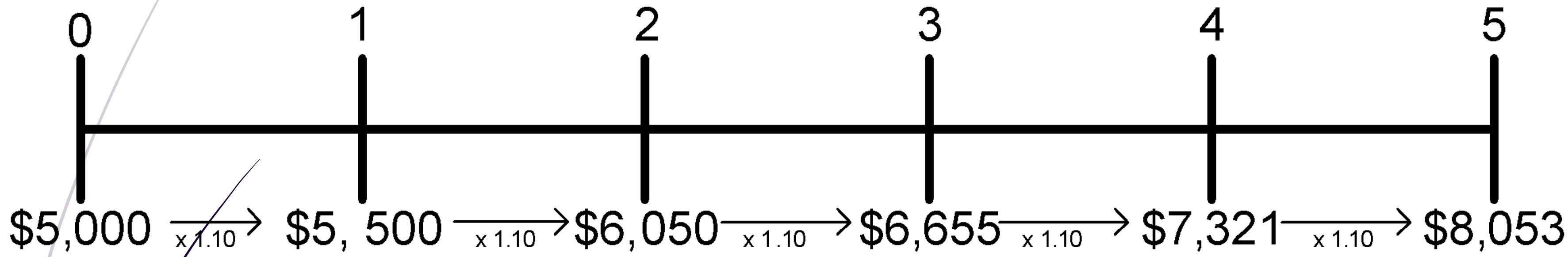
Problem

- Suppose you have a choice between receiving $5,000 today or $9,500 in five years. You believe you can earn 10% on the $5,000 today, but want to know what the $5,000 will be worth in five years.

THE TIMELINE: EXAMPLE

Solution

The time line looks like this:



In five years, the $5,000 will grow to:

$\$5{,}000 \times (1.10)^5 = \$8{,}053$

The future value of $5,000 at 10% for five years is $8,053.

You would be better off forgoing the $5,000 today and taking the $9,500 in five years.

COMPOUNDING INTEREST

	deposit	interest rate	FV (6 years)
Emma	\$ 1,000	4%	\$ 1,265 = 1000 $(1+.04)^6$
Catherine	\$ 975	5%	\$ 1,307 = 975 $(1+.05)^6$

$$FV=PV\times(1+r)^n$$

FREQUENCY OF COMPOUNDING

	deposit	interest rate	FV (3 Year)
Bank A	$ 10,000	8% annually	$ 12,597 = 10,000 $(1+.08)^3$
Bank B	$ 10,000	2% quarterly	$ 12,682 = 10,000 $(1+.02)^{12}$

$$FV=PV\times(1+r)^n$$

There are 12 compounding events in Bank B compared to 3 offered by Bank A over 3 years.

The more frequent the compounding, the larger the cumulative effect.

DISCOUNTING

Discounting is a process of converting values to be received or paid in the future into the values today.

The present value is determined from the amount to be received in a future and interest rate.

Interest rates used in the present value calculations are called **"discount rates"**.

$$PV = FV \times \frac{1}{(1+r)^n} = \frac{FV}{(1+r)^n}$$

PV = Present value

FV = Future value

r = discount rate

n = number of time periods

PVIF – present value interest factor

DISCOUNTING

$$PV = \frac{FV}{(1+r)^n}$$

	In 2 Years	In 4 Years	In 2 Years
Future Value	$5,000	$5,000	$5,000
Discount Value	5 %	5 %	7%
Present Value	$4,535	$4,114	$4,367
PV Formula	$5000/(1+.05)^2$	$5000/(1+.05)^4$	$\$5000/(1+.07)^2$

If you can earn 5% interest compounded annually what do you need to put on savings today to get an amount you need?

If you can earn 7% interest compounded annually what do you need to put on savings today to get an amount you need?

If you go to Europe 4 years from now with $5000 to spent and you can earn 5% interest compounded annually, what do you need to put on savings today?

UNKNOWN VARIABLES

Any time value problem involving lump sums -- i.e., a single outflow and a single inflow--requires the use of a single equation consisting of 4 variables, i.e.,*PV, FV, r, n*

If 3 out of 4 variables are given, we can solve for the unknown one.

$$FV = PV \times (1+r)^n$$

- solving for future value

$$PV = \frac{FV}{(1+r)^n}$$

- solving for present value

$$r = \left(\frac{FV}{PV}\right)^{1/n} - 1$$

- solving for unknown rate

$$n = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+r)}$$

- solving for # of periods

EXAMPLE: UNKNOWN RATE

Problem

Bank A offers to pay you a lump sum of $20,000 after 5 years if you deposit $9,500 with them today.

Bank B, on the other hand, says that they will pay you a lump sum of $22,000 after 5 years if you deposit $10,700 with them today.

Which offer should you accept, and why?

EXAMPLE: UNKNOWN RATE

Solution

To answer this question, you have to calculate the rate of return that will be earned on each investment and accept the one that has the higher rate of return.

Bank A's Offer:

Rate = $(FV/PV)^{1/n}$ - 1 = $(\$20,000/\$9,500)^{1/5}$ – 1 = 1.16054 - 1 = ***16.054%***

Bank B's Offer:

Rate = $(FV/PV)^{1/n}$ - 1 = $(\$22,000/\$10,700)^{1/5}$ – 1 = 1.15507 - 1 = ***15.507%***

You should accept Bank A's offer, since it provides a higher annual rate of return i.e 16.05%.

EXAMPLE: UNKNOWN № OF PERIODS

You have decided that you will sell off your house, which is currently valued at $300,000, at a point when it appreciates in value to $450,000.

If houses are appreciating at an average annual rate of 4.5% in your neighborhood, for approximately how long will you be staying in the house?

Solution

$$n = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+r)}$$

n =[ln(450,000/(300,000])/[ln(1.045)]

= .40547 / .04402 = 9.21 years

RULE OF 72

The number of years it takes for a sum of money to double in value (the "doubling time") is approximately equal to the number of 72 divided by the interest rate expressed in % per year:

$$\textbf{Doubling time} = \frac{\textbf{72}}{\textbf{Interest rate}}$$

If you start with $1000 and r=10%, you will have $2000 after 7.2 years, 4000 after 14.4 years, $8000 after 21.6 years, and so on.

STREAM OF CASH FLOWS

ONLY VALUES AT THE SAME POINT IN TIME CAN BE COMPARED OR COMBINED

Valuing a Stream of Cash Flows

General formula for valuing a stream of cash flows:

- if we want to find the present value of a stream of cash flows, we simply add up the present values of each.
- if we want to find the future value of a stream of cash flows, we simply add up the future values of each.

Present value of a stream of cash flows

$$PV = \sum_{t=0}^{n} \frac{CF_t}{(1+r)^t}$$

where

PV = the Present Value of the Cash Flow Stream,

CF_t = the cash flow which occurs at the end of year t,

r = the discount rate,

t = the year, which ranges from zero to n, and

n = the last year in which a cash flow occurs.

Present value of a stream of cash flows

Find the Present Value of the following cash flow stream given that the interest rate is 10%.

Year	0	1	2	3	4
Cash Flow		100	200	200	300

$$PV = \frac{100}{(1+.10)^1} + \frac{200}{(1+.10)^2} + \frac{200}{(1+.10)^3} + \frac{300}{(1+.10)^4} = \$611.37$$

Future value of a stream of cash flows

$$FV_n = \sum_{t=0}^{n} CF_t (1+r)^{n-t}$$

where

FV_t = the Future Value of the Cash Flow Stream at the end of year t

CF_t = the cash flow which occurs at the end of year t

r = the discount rate

t = the year, which ranges from zero to n

n = the last year in which a cash flow occurs

Future value of a stream of cash flows

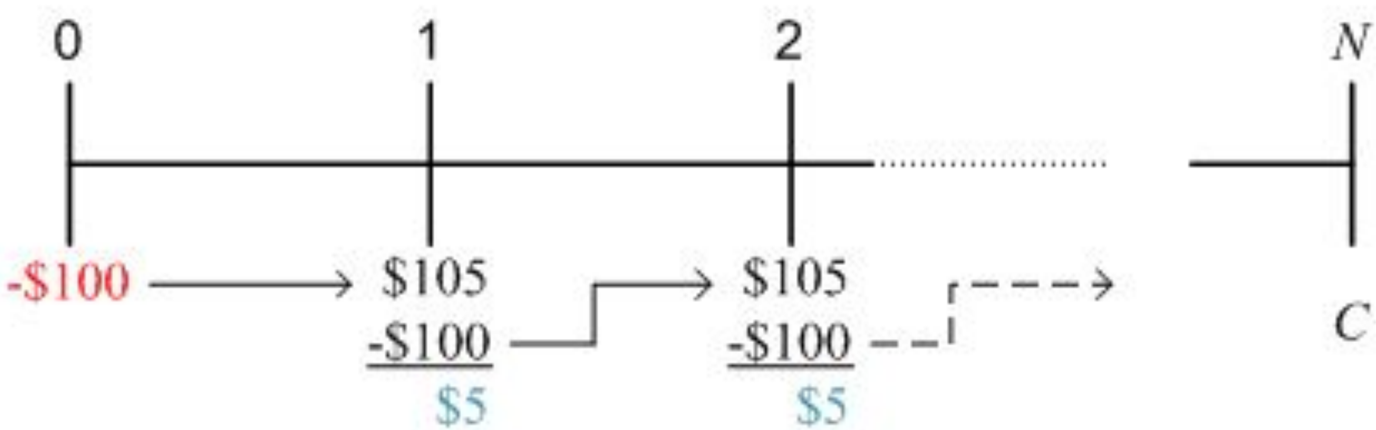
Find the Future Value at the end of year 4 of the following cash flow stream given that the interest rate is 10%.

Year	0	1	2	3	4
Cash Flow		100	200	200	300

$$FV_4 = 100(1+.10)^3 + 200(1+.10)^2 + 200(1+.10)^1 + 300 = \$895.10$$

PERPETUITIES

When a constant cash flow will occur at regular intervals forever it is called a perpetuity.



PERPETUITIES

The value of a perpetuity is simply the cash flow divided by the interest rate.

Present Value of a Perpetuity:

$$PV(C \text{ in perpetuity}) = \frac{C}{r}$$

In decimal points

PERPETUITIES: EXAMPLE

Problem

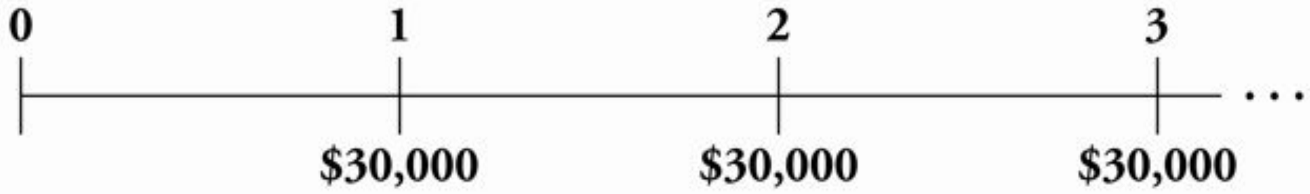
You want to donate to your University to endow an annual MBA graduation party at your alma mater. You want the event to be a memorable one, so you budget $ 30000 per year forever for the party.

If the University earns 8% per year on its investments, and the first party is in one year’s time, how much will you need to donate to endow the party?

PERPETUITIES: EXAMPLE

Solution

The timeline of the cash flows you want to provide is



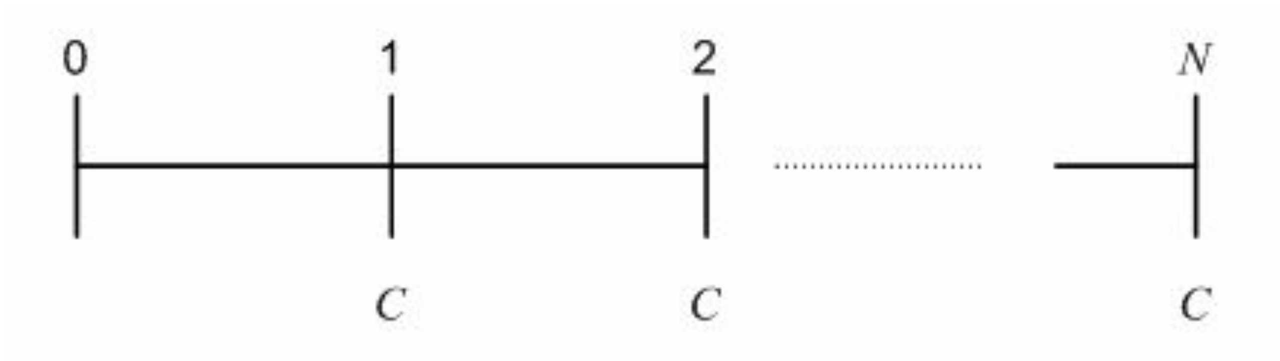
This is a standard perpetuity of $30,000 per year. The funding you would need to give the university in perpetuity is the present value of this cash flow stream. From the formula,

$$PV = C/r = \$30{,}000/0.08 = \$375{,}000 \text{ today}$$

If you donate $375,000 today, and if the university invests it at 8% per year forever, then the MBAs will have $30,000 every year for their graduation party.

ANNUITIES

When a constant cash flow will occur at regular intervals for a finite number of *N* periods, it is called an annuity.



ANNUITIES

- Annuities are equal, periodic outflows/inflows., e.g. rent, lease, mortgage, car loan, and retirement annuity payments.
- An annuity stream can begin at the start of each period (annuity due) as is true of rent and insurance payments or at the end of each period, (ordinary annuity) as in the case of mortgage and loan payments.
- The formula for calculating the future value of an annuity stream is as follows:

$$FV = PMT * \frac{(1+r)^n - 1}{r}$$

- where *PMT* is the term used for the equal periodic cash flow, *r* is the rate of interest, and *n* is the number of periods involved.

FUTURE VALUE INTEREST FACTOR OF AN ANNUITY (FVIFA)

$$FVIFA = \frac{(1+r)^n - 1}{r}$$

FVIFA – future value interest factor of an annuity

FUTURE VALUE OF AN ORDINARY ANNUITY STREAM

Problem

Jill has been faithfully depositing $2,000 at the end of each year for the past 10 years into an account that pays 8% per year. How much money will she have accumulated in the account?

FUTURE VALUE OF AN ORDINARY ANNUITY STREAM

Solution

Future Value of Payment One = $\$2,000 \times 1.08^9$ = \$3,998.01

Future Value of Payment Two = $\$2,000 \times 1.08^8$ = \$3,701.86

Future Value of Payment Three = $\$2,000 \times 1.08^7$ = \$3,427.65

Future Value of Payment Four = $\$2,000 \times 1.08^6$ =\$3,173.75

Future Value of Payment Five = $\$2,000 \times 1.08^5$ = \$2,938.66

Future Value of Payment Six = $\$2,000 \times 1.08^4$ = \$2,720.98

Future Value of Payment Seven = $\$2,000 \times 1.08^3$ = \$2,519.42

Future Value of Payment Eight = $\$2,000 \times 1.08^2$ = \$2,332.80

Future Value of Payment Nine = $\$2,000 \times 1.08^1$ = \$2,160.00

Future Value of Payment Ten = $\$2,000 \times 1.08^0$ = \$2,000.00

Total Value of Account at the end of 10 years \$28,973.13

FUTURE VALUE OF AN ORDINARY ANNUITY STREAM

$$FV = PMT * \frac{(1+r)^n - 1}{r}$$

where, PMT = \$2,000; r = 8%; and n=10.

FVIFA ➔ $[((1.08)^{10} - 1)/.08] = 14.486562$,

FV = \$2000*14.486562 ➔ \$28,973.13

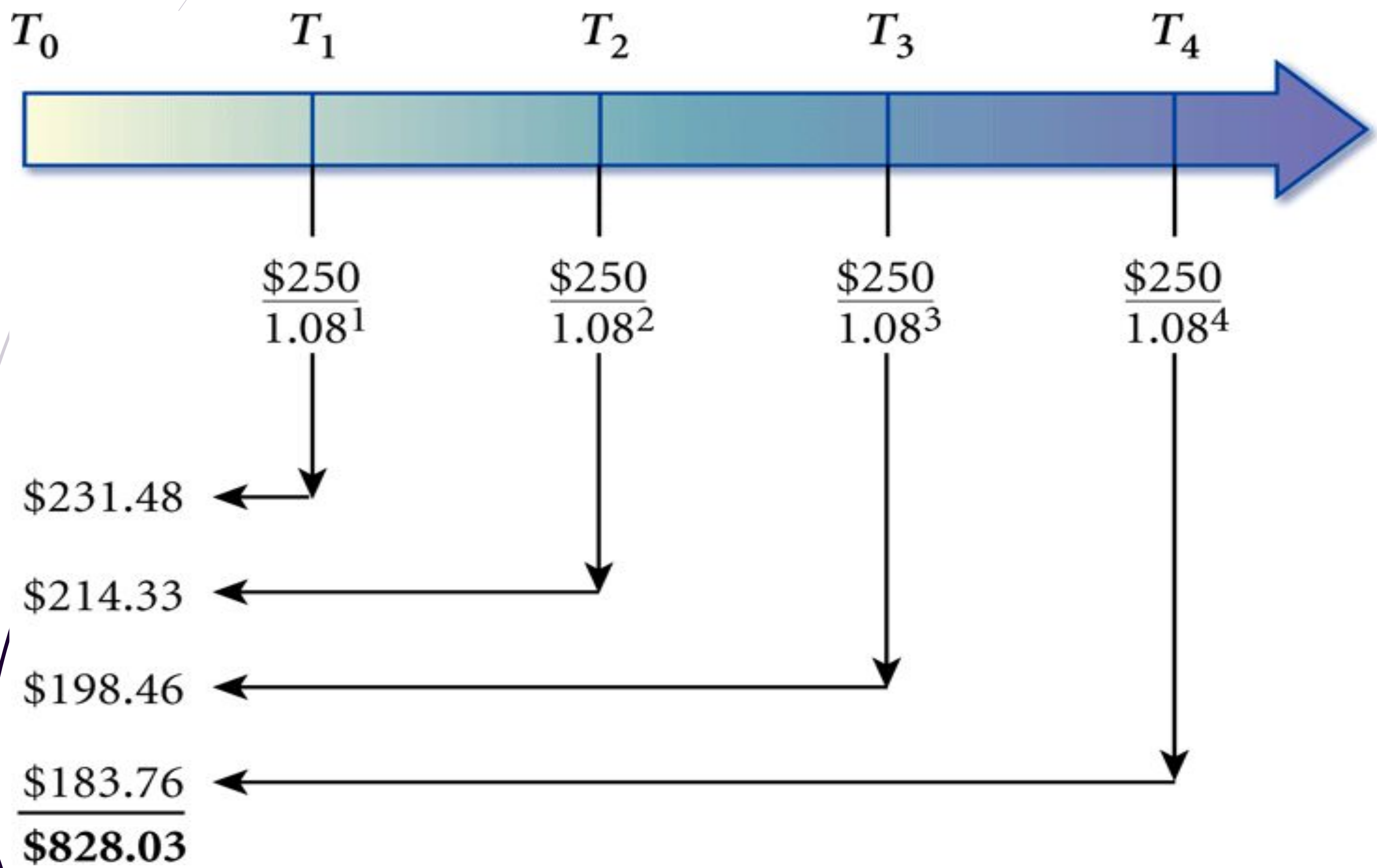
PRESENT VALUE OF AN ANNUITY

To calculate the value of a series of equal periodic cash flows at the current point in time, we can use the following simplified formula:

$$PV = PMT \times \frac{\left[1 - \left(\frac{1}{(1+r)^n}\right)\right]}{r}$$

PVIFA – present value interest factor of an annuity

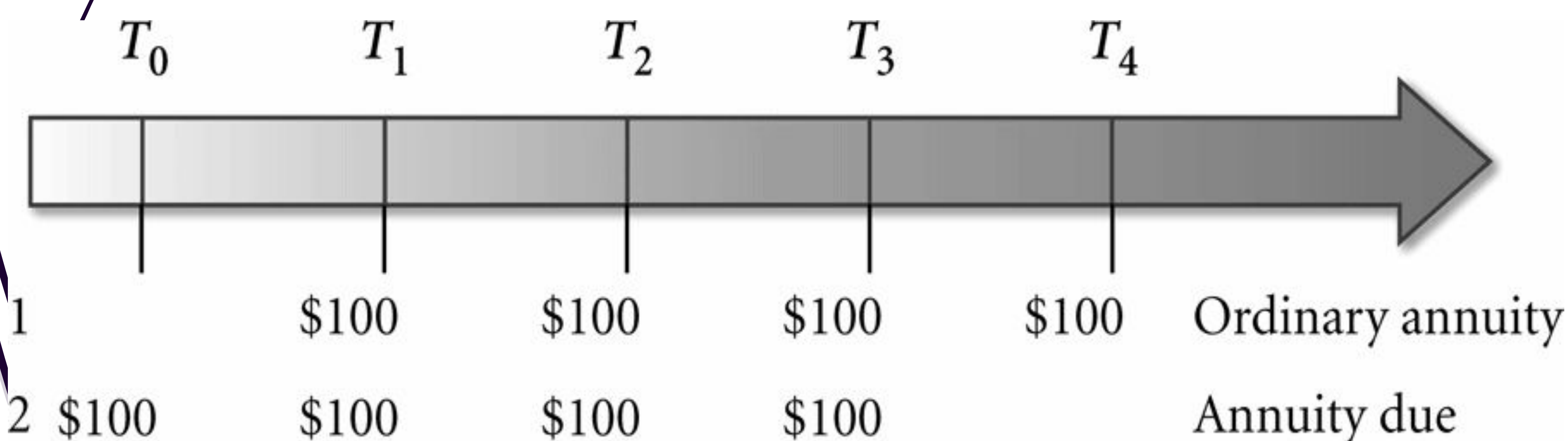
TIME LINE OF PRESENT VALUE OF ANNUITY STREAM



ANNUITY DUE VS ORDINARY ANNUITY

A cash flow stream such as rent, lease, and insurance payments, which involves equal periodic cash flows that begin right away or at the beginning of each time interval, is known as an ***annuity due.***

Equal periodic cash flows that begin at the end of each time interval, is known as an ***ordinary annuity.***



ANNUITY DUE VS ORDINARY ANNUITY

PV annuity due = PV ordinary annuity x (1+r)

FV annuity due = FV ordinary annuity x (1+r)

PV annuity due > PV ordinary annuity

FV annuity due > FV ordinary annuity

ANNUITY DUE VS ORDINARY ANNUITY

Problem:

Let's say that you are saving up for retirement and decide to deposit $3,000 each year for the next 20 years into an account that pays a rate of interest of 8% per year. By how much will your accumulated nest egg vary if you make each of the 20 deposits at the beginning of the year, starting right away, rather than at the end of each of the next twenty years?

ANNUITY DUE VS ORDINARY ANNUITY

Given information: PMT = $3,000; n=20; i= 8%.

$$FV = PMT \times \left[\frac{(1+r)^n - 1}{r}\right]$$

FVIFA – future value interest factor of an annuity

FV of ordinary annuity= $3,000 * [((1.08)^{20} - 1)/.08]
= $3,000 * 45.76196
= $137,285.89

FV of annuity due = FV of ordinary annuity * (1+r)

FV of annuity due = $137,285.89*(1.08) = $148,268.76

TYPES OF LOAN REPAYMENTS

There are 3 basic ways to repay a loan:

Discount loans: pay off the **principal** and all the **interest** at one time at the maturity date of the loan.

Interest-only loans: make periodic interest payments and then pay the principal and final interest payment at the maturity date

Amortized loans: pay both principal and interest as they go by making equal payments each period

LOAN REPAYMENTS: EXAMPLE

Problem:

The Corner Bar & Grill is in the process of taking a five-year loan of $50,000 with First Community Bank. The bank offers the restaurant owner his choice of three payment options:

1) Pay all of the interest (8% per year) and principal in one lump sum at the end of 5 years;
2) Pay interest at the rate of 8% per year for 4 years and then a final payment of interest and principal at the end of the 5th year;
3) Pay 5 equal payments at the end of each year inclusive of interest and part of the principal.

Under which of the three options will the owner pay the least interest and why?

LOAN REPAYMENTS: EXAMPLE

Solution:

Under Option 1: Principal and Interest Due at the end.

Payment at the end of year 5 = $FV_n = PV \times (1 + r)^n$

$FV_5 = \$50,000 \times (1+0.08)^5$

$= \$50,000 \times 1.46933$

$= \$73,466.5$

Interest paid = Total payment - Loan amount

Interest paid = $73,466.5 - $50,000 = $23,466.50

LOAN REPAYMENTS: EXAMPLE

Solution:

Under Option 2: Interest-only Loan

Annual Interest Payment (Years 1-4)

= $50,000 x 0.08 = $4,000

Year 5 payment = Annual interest payment + Principal payment

= $4,000 + $50,000 = $54,000

Total payment = $16,000 + $54,000 = $70,000

Interest paid = $20,000

LOAN REPAYMENTS: EXAMPLE

Solution:

Under Option 3: Amortized Loan

To calculate the annual payment of principal and interest, we can use the PV of an ordinary annuity equation and solve for the PMT value using n = 5; I = 8%; PV=$50,000

PMT $12,522.82

Total payments = 5*$12,522.82 = $62,614.11

Interest paid = Total Payments - Loan Amount

= $62,614.11-$50,000

Interest paid = $12,614.11

LOAN REPAYMENTS: EXAMPLE

Comparison of total payments and interest paid under each method:

Loan Type	Total Payment	Interest Paid
Discount Loan	$73,466.50	$23,466.50
Interest-only Loan	$70,000.00	$20,000.00
Amortized Loan	$62,614.11	$12,614.11

So, the amortized loan is the one with the lowest interest expense, since it requires a higher annual payment, part of which reduces the unpaid balance on the loan and thus results in less interest being charged over the 5-year term.

AMORTIZATION SCHEDULES

Amortization schedule contains the following information:

- Beginning principal;
- Total periodic payments;
- Periodic interest expense;
- Reduction of principal amount in each period;
- Remaining principal.

AMORTIZATION SCHEDULES

Problem

$ 25,000 loan being paid off at 8% annual interest rate within 6 years by equal installments.

Solution

Beginning balance: $25,000.

Periodic payments:

$$PMT = \frac{\$25,000}{\dfrac{1-[1/(1+0.08)^{6}]}{0.08}} = \frac{25,000}{4.6229} = \$5,407.88$$

Interest expense for the 1st year:

$$\$25,000 \times 0.08 = \$2000$$

The principal should be reduced by:

$$\$5,407.88 - \$2000 = \$3,407.88$$

Remaining principal after 1st y.:

$$\$25,000 - 3,407.88 = \$21,592.12$$

AMORTIZATION SCHEDULES

Solution

For all consequent periods:

1. Apply step 3 to the principal amount remaining at the end of previous period

Remaining principal $_t$ = Beginning principal$_{t+1}$

2. Principal should be reduced by: **payment made during a period – interest expense at the same period**

Remaining principle $_{t+1}$= Beginning principal $_{t+1}$ – the amount by which the principal should be reduced

AMORTIZATION SCHEDULES

Year	Beginning Principal	Annual Payment	Interest Expense	Principal Reduction	Remaining Principal
1	$25,000.00	$ 5407.88	$2,000.00	$ 3,407.88	$21,592.12
2	$21,592.12	$ 5407.88	$1,727.37	$ 3,680.51	$ 17,911.61
3	$ 17,911.61	$ 5407.88	$1,432.93	$ 3,974.95	$13,936.66
4	$13,936.66	$ 5407.88	$1,114.93	$ 4,292.95	$ 9,643.71
5	$ 9,643.71	$ 5407.88	$ 771.50	$ 4,636.38	$ 5,007.33
6	$ 5,007.33	$ 5407.92	$ 400.59	$ 5,007.33	$ 0
Total		$32,447.32	$7,447.32	$25,000.00	

THE END