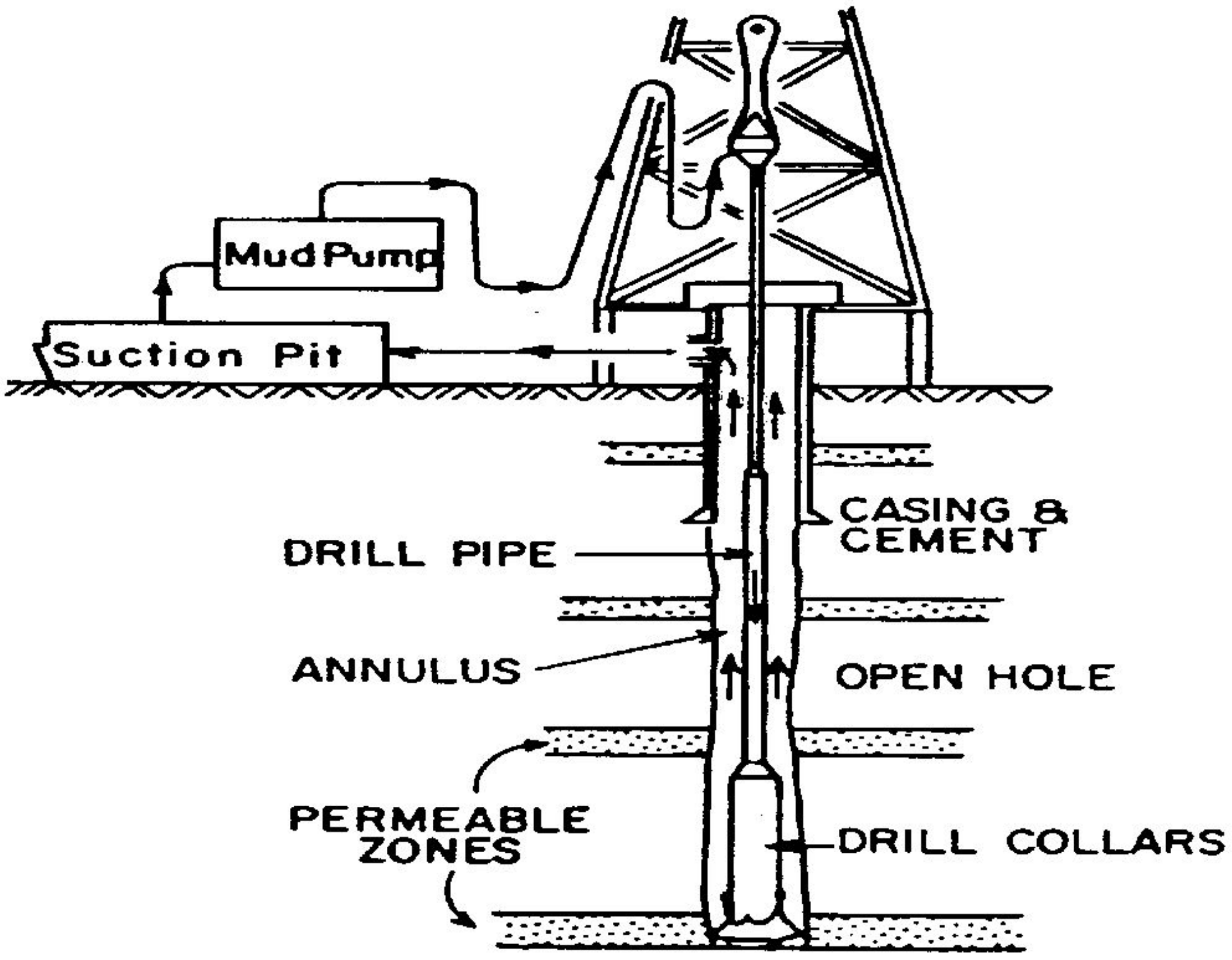
Drilling Engineering – PE 311

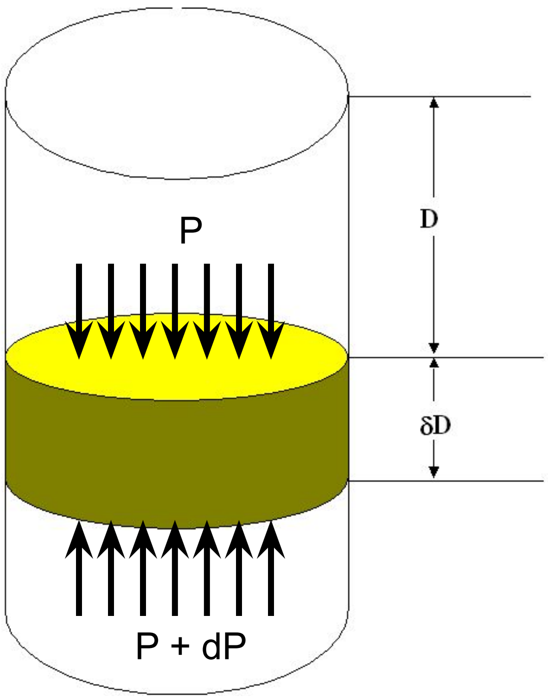
Hydraulics of Drilling Fluids

Prepared by: Tan Nguyen

Circulating System



Hydrostatic Pressure in Liquid Column



$$P_T A + W = (P_T + dP)A$$

$$dP \times A = W$$

$$dP \times A = \rho \times A \times dh \times g$$

$$dP = \rho \times g \times dh$$

For incompressible fluids, the specific weight of the liquid in field unit is given by

$$p = 0.052\rho D + p_0$$

If $P_0 = 0$ then

$$p = 0.052\,\rho\,D$$

The fluid density

$$\rho = \frac{p}{0.052\,D}$$

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Hydrostatic Pressure in Liquid Column

Example: Calculate the static mud density required to prevent flow from a permeable stratum at 12,200ft if the pore pressure of the formation fluid is 8500psig.

Solution:

$$\rho = \frac{p}{0.052\,D} = \frac{8500}{0.052 \times 12{,}200} = 13.4 lbm/gal$$

The mud density must be at least 13.4 lbm/gal

Hydrostatic Pressure in Gas Column

$$dp = 0.052\rho dD$$

EOS of gas:

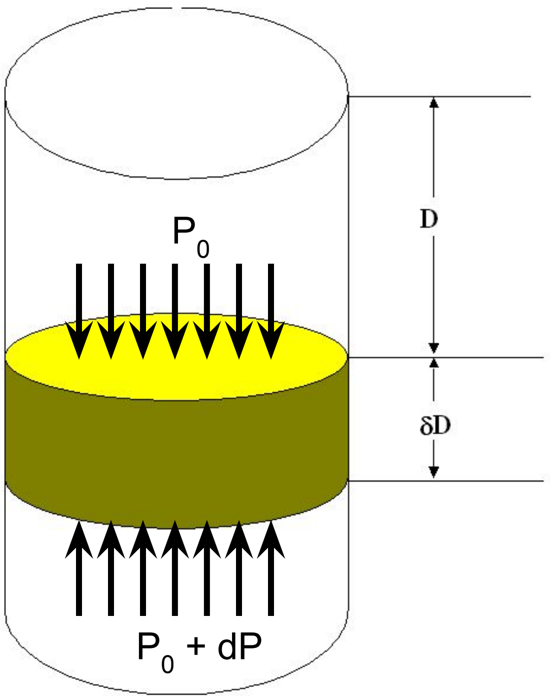
$$pV = Z\,n\,R\,T = Z\,\frac{m}{M}\,R\,T$$

$$\rho = \frac{m}{V} = \frac{pM}{ZRT} = \frac{pM}{80.3\,\mathrm{Z\,T}}$$

$$dp = \frac{0.052\,p\,M}{80.3\,\mathrm{Z\,T}}\;dD$$

$$\int_{p_0}^{p}\frac{dp}{p} = \frac{M}{1544\,\mathrm{Z\,T}}\int_{D_0}^{D} dD$$

$$p = p_0\,e^{\frac{M\,(D-D_0)}{1544\,\mathrm{Z\,T}}}$$



Hydrostatic Pressure in Gas Column

A well contains tubing filled with methane gas (MW = 16) to a vertical depth of 10000ft. The annular space is filled with a 9.0 lbm/gal brine. Assuming ideal gas behavior, compute the amount by which the exterior pressure on the tubing exceeds the interior tubing pressure at 10,000ft if the surface tubing pressure is 1000 psia and the mean gas temperature is 140F. If the collapse resistance of the tubing is 8330 psi, will the tubing collapse due to the high external pressure?

Hydrostatic Pressure in Gas Column

The pressure in the annulus (external pressure) at D = 10,000 ft is

P_2 = 0.052 * 9.0 * 10,000 + 14.7 = 4,695 psia

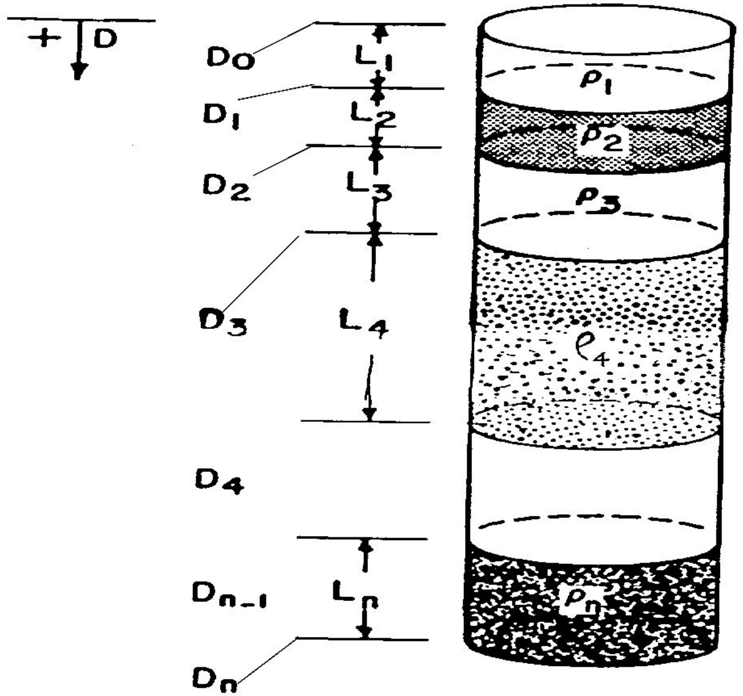
The pressure in the tubing (internal pressure) at D = 10,000ft

$$p = p_0\, e^{\frac{M(D - D_0)}{1544\, Z\, T}} = 1000 e^{\frac{16*10000}{1544*(460+140)}} = 1188\, psia$$

Pressure difference = p_2 – p = 4695 – 1188 = 3507 < 8330 psia

The tubing will withstand the high external pressure

Hydrostatic Pressure in Complex Fluid Column

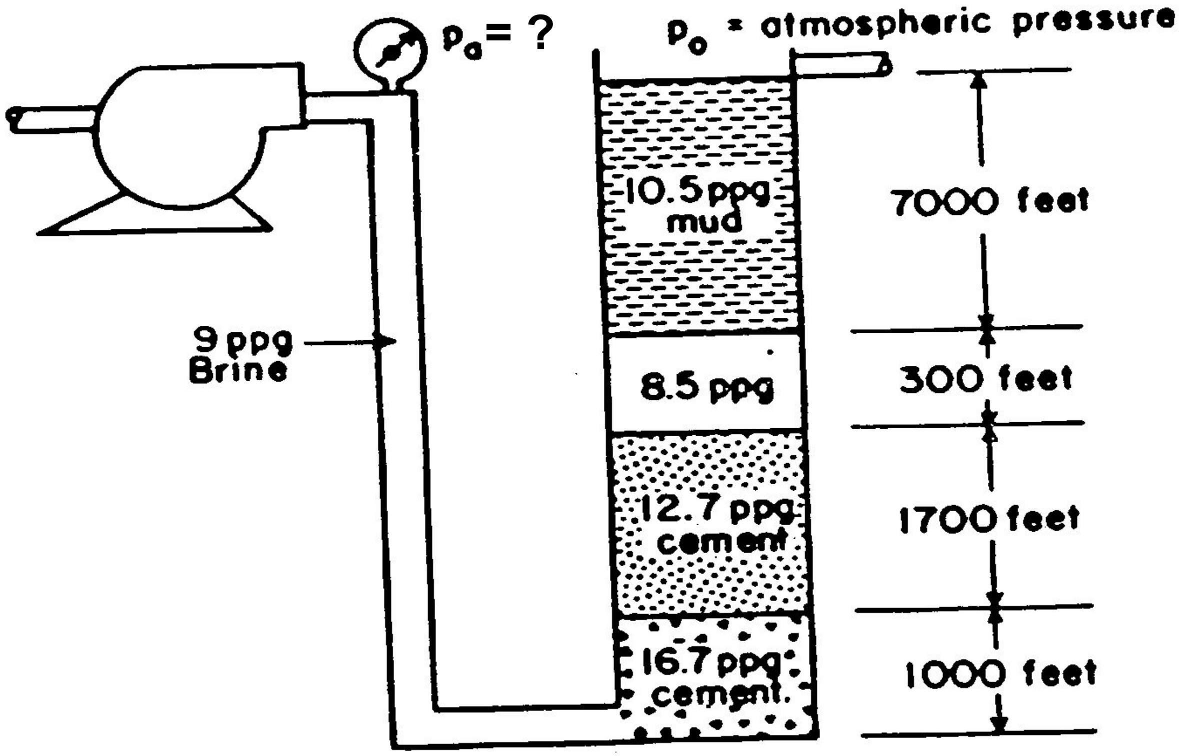


$$p_{i+1} = p_i + \Delta p = p_i + 0.052\rho_{i+1}\left(D_{i+1} - D_i\right)$$

$$p_1 = p_0 + 0.052\rho_1\left(D_1 - D_0\right)$$
$$p_2 = p_1 + 0.052\rho_2\left(D_2 - D_1\right)$$
$$p_3 = p_2 + 0.052\rho_3\left(D_3 - D_2\right)$$
$$\vdots$$
$$p_n = p_{n-1} + 0.052\rho_n\left(D_n - D_{n-1}\right)$$

$$p_n = p_0 + 0.052\sum_{i=1}^{n}\rho_i\left(D_i - D_{i-1}\right)$$

Hydrostatic Pressure in Complex Fluid Column



$$p_a = p_0 + 0.052 \times (10.5 \times 7{,}000 + 8.5 \times 300 + 12.7 \times 1700 + 16.7 \times 1{,}000 - 9.0 \times 10{,}000) = 0 + 1266 = 1266\,psig$$

Equivalent Circulating Density (ECD)

The effective density exerted by a circulating fluid against the formation that takes into account the pressure drop in the annulus above the point being considered.

The ECD is calculated as:

$$\rho = \frac{P}{0.052D}$$

ρ – mud density, ppg

P – Sum of the hydrostatic pressure and the frictional pressure drop in the annulus between the depth D and surface, Psig

D – the true vertical depth, ft

Equivalent Circulating Density (ECD)

Example: A 9.5-PPG drilling fluid is circulated through the drill pipe and the annulus. The frictional pressure losses gradient in the annulus is 0.15. Calculate the equivalent circulating density in PPG.

Solution:

ρ = 9.5 + P/0.052 = 9.5 + 0.15 / 0.052 = 12.4 PPG

Buoyancy

$$W_e = W - (P_b - P_T)A$$

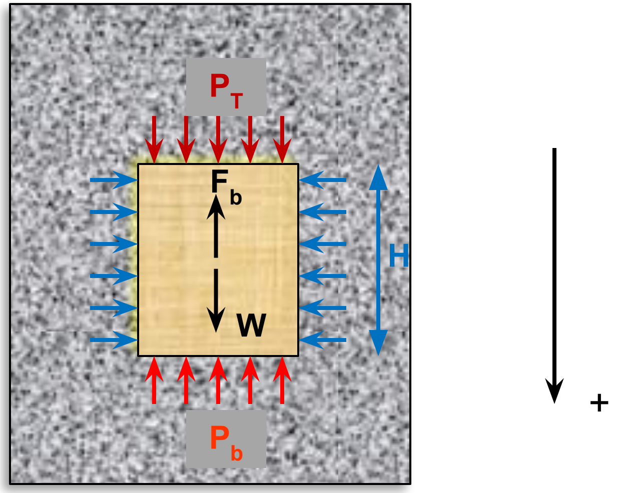
$$W_e = \rho_o g V_o - \rho_l g H A$$

$$W_e = \rho_o g V_o - \rho_l g V_o$$

$$W_e = W - W_{bo}$$

$$W_e = \rho_o g V_o \left(1 - \frac{\rho_l g V_o}{\rho_o g V}\right)$$

$$W_e = W_o \left(1 - \frac{\rho_l}{\rho_o}\right)$$



W_e , W, W_{bo} – effective weight, weight of the object in air, and buoyant force.

ρ_l and ρ_o - densities of liquid and the object

Buoyancy

10,000 ft of 19.5-lbm/ft drillpipe and 600 ft of 147 lbm/ft drill collars are suspended off bottom in a 15-lbm/gal mud. Calculate the effective hook load that must be supported by the derrick. Density of steel is 65.5 lbm/gal

Solution:

W = 19.5 * 10000 + 147 * 600 = 283200 lbm

We = W(1 - ρ_f/ρ_s) = 283200 * (1 - 15/65.5) = 218300 lbm

(density of steel = 65.5 lbm/gal = 490lbm/cu ft)

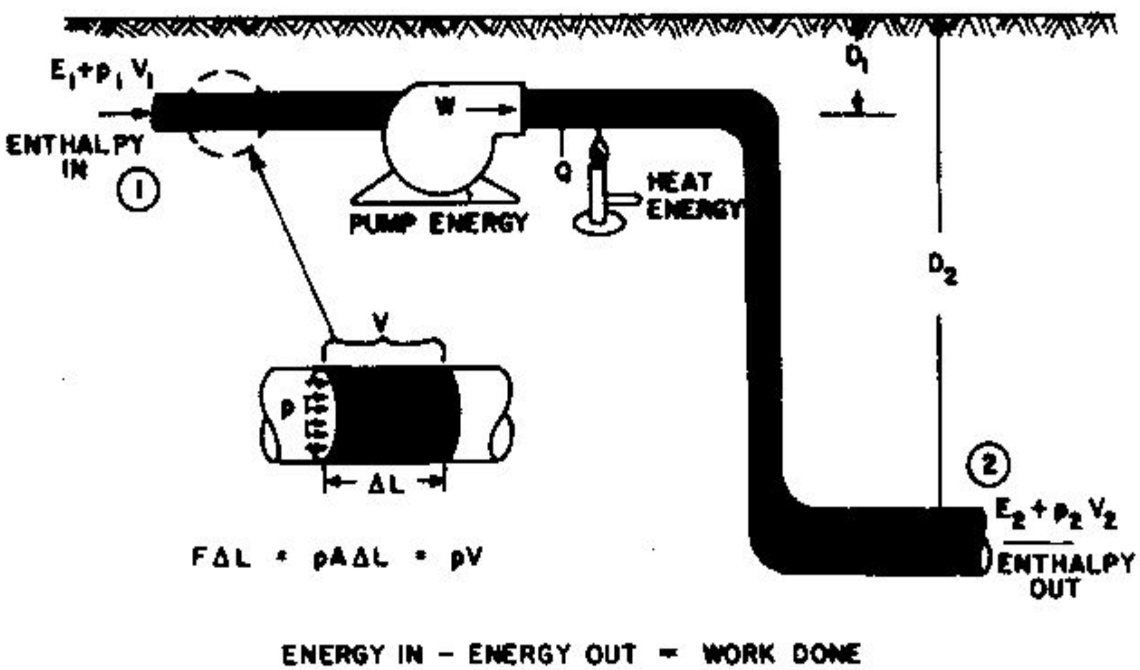
Flow Through Jet Bits

Energy balance:

$$p_2 - p_1 = 0.052\ \rho\ \left(D_2 - D_1\right) - 8.074 * 10^{-4} \rho\ \left(v_2^2 - v_1^2\right) + \Delta P_p - \Delta p_f$$

ΔP_p is heat entering the system

ΔP_f is heat loss due to friction



Flow Through Jet Bits

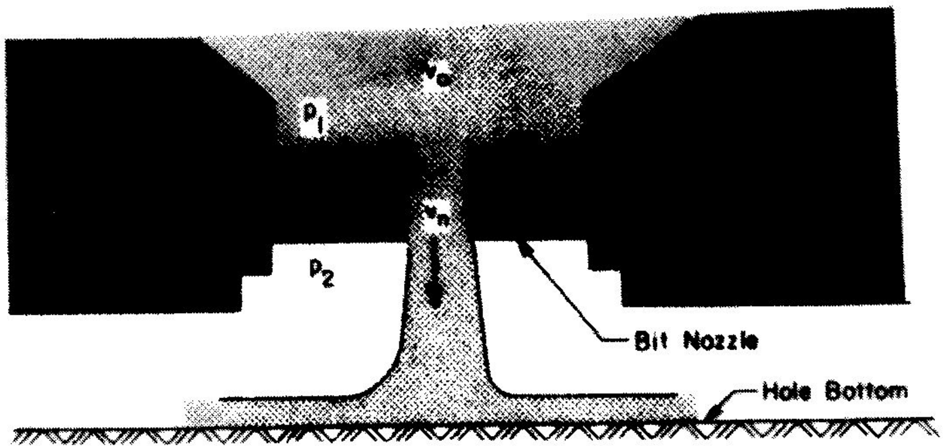
Applying the energy equation for a flow through a nozzle with neglecting:

(1)effects of elevation: D_2 - D_1 = 0

(2)effects of uptream velocity v_o = 0

(3)Heat entering the system ΔP_p = 0 and friction loss ΔP_f = 0

$$p_2 = p_1 + 0.052\ \rho\ (D_2 - D_1) - 8.074 * 10^{-4}\ \rho\ \left(v_2^2 - v_1^2\right) + \Delta P_p - \Delta p_f$$



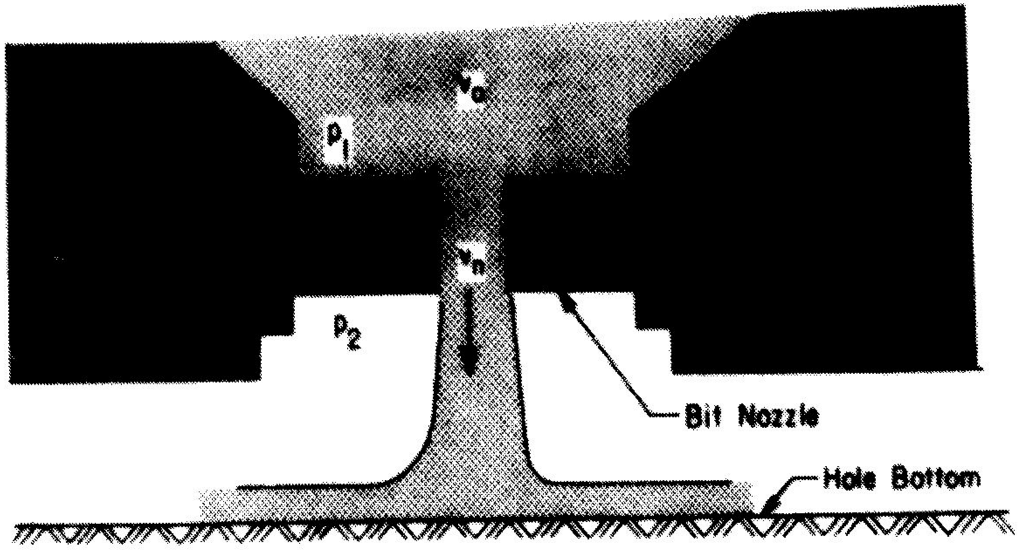
Flow Through Jet Bits

$$p_2 = p_1 - 8.074 * 10^{-4} \rho v_n^2$$

$$v_n = \sqrt{\frac{\Delta p_b}{8.074 \times 10^{-4} \rho}}$$

$$v_n = C_d \sqrt{\frac{\Delta p_b}{8.074 \times 10^{-4} \rho}}$$

$$C_d --- correction\ \ factor$$



Flow Through Jet Bits

Flow Through Parallel Nozzles

Assuming a constant ΔP_b

through all the nozzles

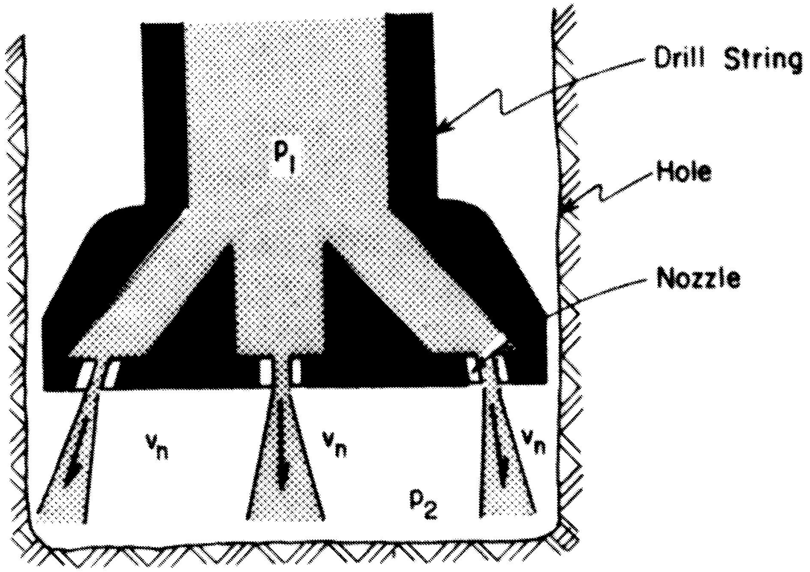
$$v_n = Cd\sqrt{\frac{\Delta p_b}{8.074\times 10^{-4}\rho}}$$

$$v_n = \frac{q_1}{A_1} = \frac{q_2}{A_2} = \frac{q_3}{A_3} = ... = \frac{q_n}{A_n} = \frac{\sum q_i}{\sum A_i} = \frac{q}{A_t}$$

Pressure drop across the bit

$$\Delta p_{bit} = \frac{8.311 * 10^{-5}\rho\ q^2}{C_d^2\ A_t^2}$$

ρ – lbm/gal ; q – gpm ; A_t - in^2



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Flow Through Jet Bits

Hydraulic Impact Force

The purpose of the jet nozzles is to improve the cleaning action of the drilling fluid at the bottom of the hole. Since the fluid is traveling at a vertical velocity v_n , before reaching to the hole and traveling at zero vertical velocity after striking the hole bottom hence all the fluid momentum is transferred to the hole bottom.

Force is time rate of change of momentum, hence:

$$F_j = \frac{\Delta(mv)}{\Delta t} = \left(\frac{m}{\Delta t}\right)\Delta v = \frac{\rho \; q \; v_n}{32.17 * 60}$$

Substitute v_n to the equation above gives

$$F_j = 0.01823 \; c_d \; q\sqrt{\rho \; \Delta p}$$

Where F_j is the hydraulic impact force given in pounds.

Flow Through Jet Bits

Flow Through Parallel Nozzles

Example: A 12.0 lbm/gal drilling fluid is flowing through a bit containing three 13/32 in nozzles at a rate of 400 gal/min. Calculate the pressure drop across the bit and the impact force developed by the bit.

Solution: Assume C_d = 0.95

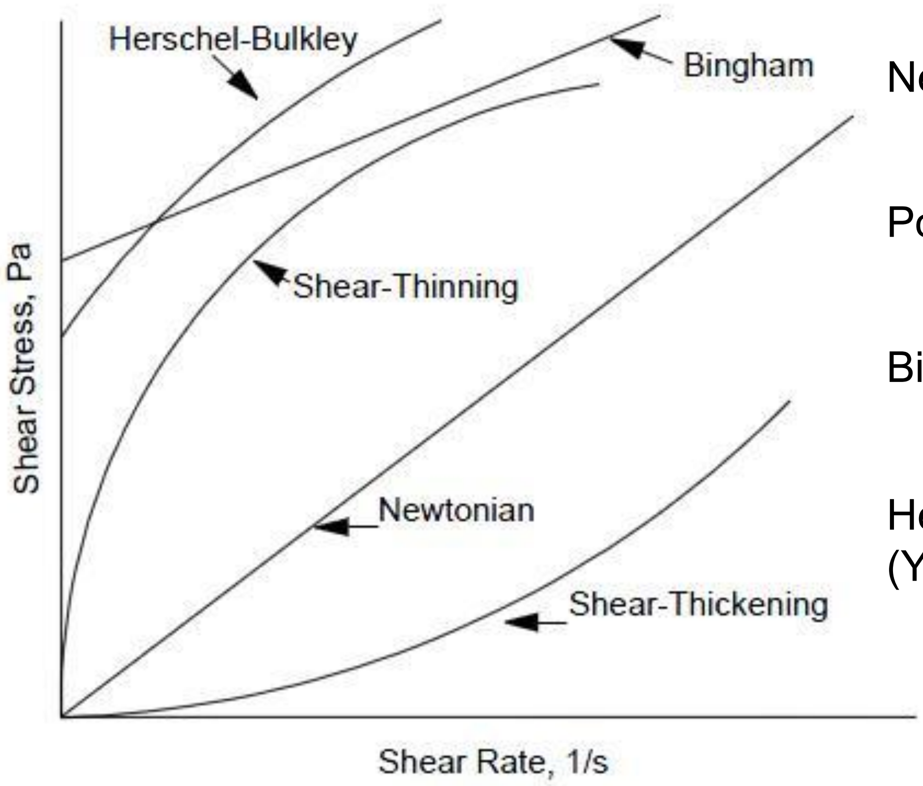
$$A_t = 3 \times \frac{\pi}{4}\left(\frac{13}{32}\right)^2 = 0.3889 in^2$$

$$\Delta p_{bit} = \frac{8.311 * 10^{-5} \rho \ q^2}{C_d^2 \ A_t^2} = \frac{8.311 * 10^{-5} * 12 * 400^2}{0.95^2 * 0.3889^2} = 1169 \, psi$$

Hydraulic impact force:

$$F_j = 0.01823 \ c_d \ q \sqrt{\rho \ \Delta p} = 0.01823 \ 0.95 \ 400 \sqrt{12 \ * 1{,}169} = 820 \ lbf$$

Rheological Model



Flow curves of time-independent fluids

Newtonian fluids: $\tau = \mu\gamma$

Power law fluids: $\tau = K\gamma^{n}$

Bingham fluids: $\tau = \tau_{y} + \mu_{p}\gamma$

Herschel-Bulkley (Yield power law fluids) $\tau = \tau_{y} + K\gamma^{n}$

Rheological Model

Newtonian Model $\tau = \mu \dot{\gamma}$

Non-Newtonian Model

Bingham-plastic model

$$\tau = \mu_p \dot{\gamma} + \tau_y ; \quad \tau > \tau_y$$

$$\dot{\gamma} = 0; \qquad -\tau_y \leq \tau \leq +\tau_y$$

$$\tau = \mu_p \dot{\gamma} - \tau_y ; \quad \tau < -\tau_y$$

Power Law model: $\tau = K \left(\dot{\gamma} \right)^n$

Yield power law model: $\tau = \tau y + K \left(\dot{\gamma} \right)^n$

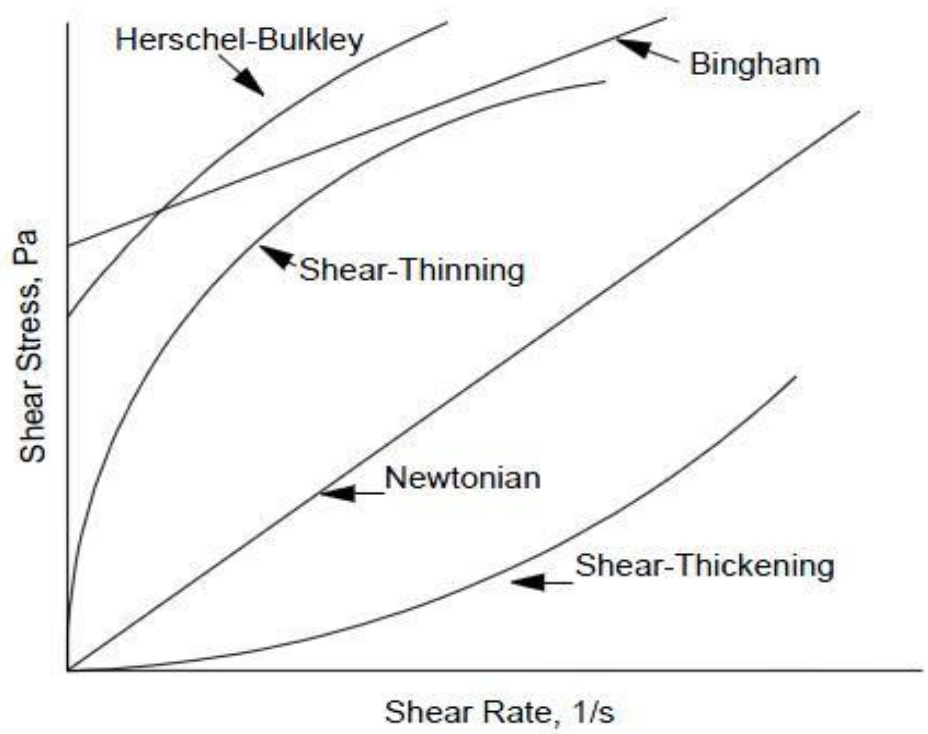
Classification of Drilling Fluids

Pseudoplastic *(Time-independent shear thinning fluids)*

If the apparent viscosity decreases with increasing shear rate

Dilatant *(Time-independent shear thickening fluids)*

If the apparent viscosity increases with increasing shear rate



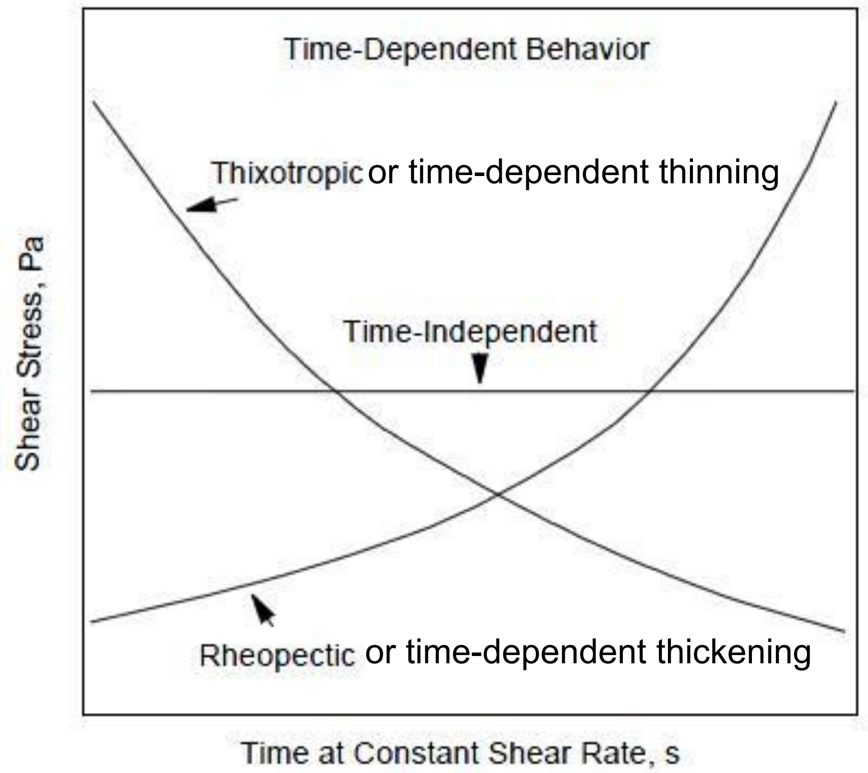
Flow curves of time-independent fluids

Classification of Drilling Fluids

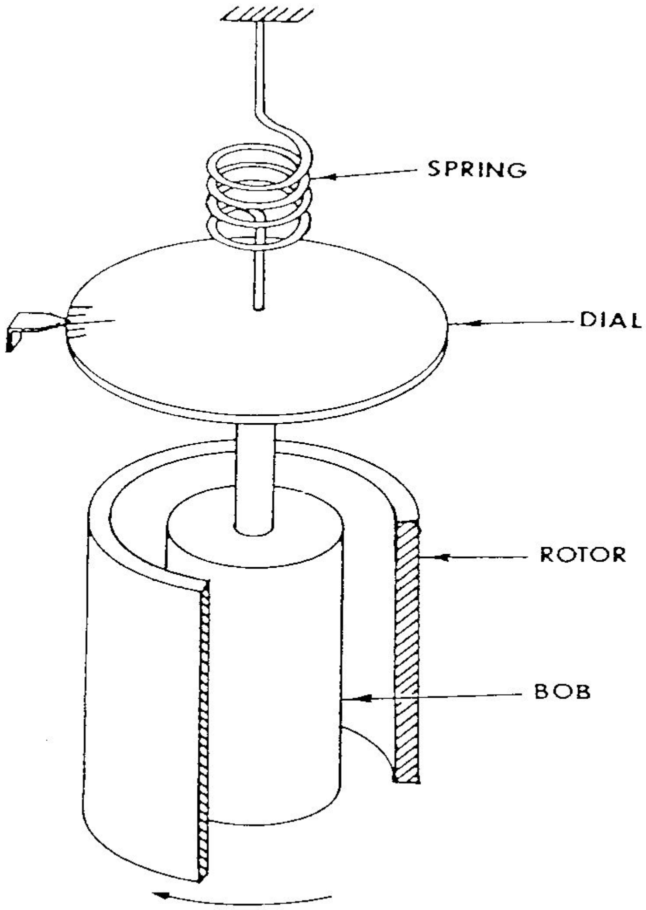
Thixotropic *(Time-dependent shear thinning fluids):* If the apparent viscosity decreases with time after the shear rate is increased to a new constant value

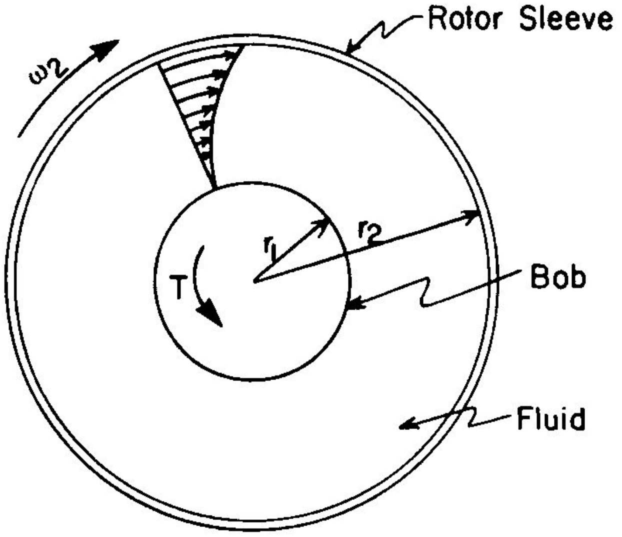
R*heopectic* *(Time-dependent shear thickening fluids):* If the apparent viscosity increases with time after the shear rate is increased to a new constant value

Drilling fluids and cement slurries are generally *thixotropic*



Rotational Viscometer





Rotational Viscometer

A rotational viscometer is used to determine type of the fluid and the rheological model of the fluid. This can be done by varying the speed of the rotor (varying the shear rate) and reading the dial reading (shear stress). To convert the speed to shear rate and dial reading to shear stress, simply use these corellations:

γ = 1.703 x rpm, 1/s

τ = 1.06 x Dial Reading

Rotational Viscometer

Newtonian Model

$$\mu_a = \frac{300}{N}\theta_N$$

$$\dot{\gamma} = \frac{5.066}{r^2}N$$

Bingham Plastic Model

$$\mu_p = \theta_{600} - \theta_{300}$$

or

$$\mu_p = \frac{300}{N_2 - N_1}(\theta_{N_2} - \theta_{N_1})$$

$$\dot{\gamma} = \frac{5.066}{r^2}N + \frac{479\tau_y}{\mu_p}\left(\frac{3.174}{r^2} - 1\right)$$

$$\tau_y = \theta_{300} - \mu_p$$

or

$$\tau_y = \theta_{N_1} - \mu_p \frac{N_1}{300}$$

$$\tau_g = \theta_{max} \text{ at 3 rpm}$$

Rotational Viscometer

Power-Law Model

$$n = 3.322 \log \left(\frac{\theta_{600}}{\theta_{300}} \right)$$

or

$$n = \frac{\log\left(\frac{\theta_{N_2}}{\theta_{N_1}} \right)}{\log\left(\frac{N_2}{N_1} \right)}$$

$$K = \frac{510\ \theta_{300}}{(511)^n}$$

or

$$K = \frac{510\ \theta_N}{(1.703\ N)^n}$$

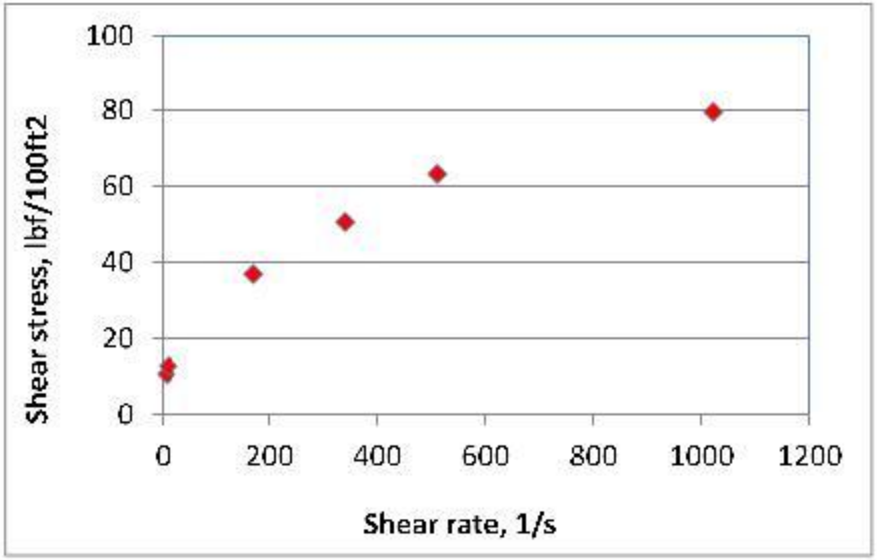
$$\dot{\gamma} = 0.2094N \frac{\frac{1}{r^{2/n}}}{n\left[\frac{1}{r_1^{\ 2/n}} - \frac{1}{r_2^{\ 2/n}} \right]}$$

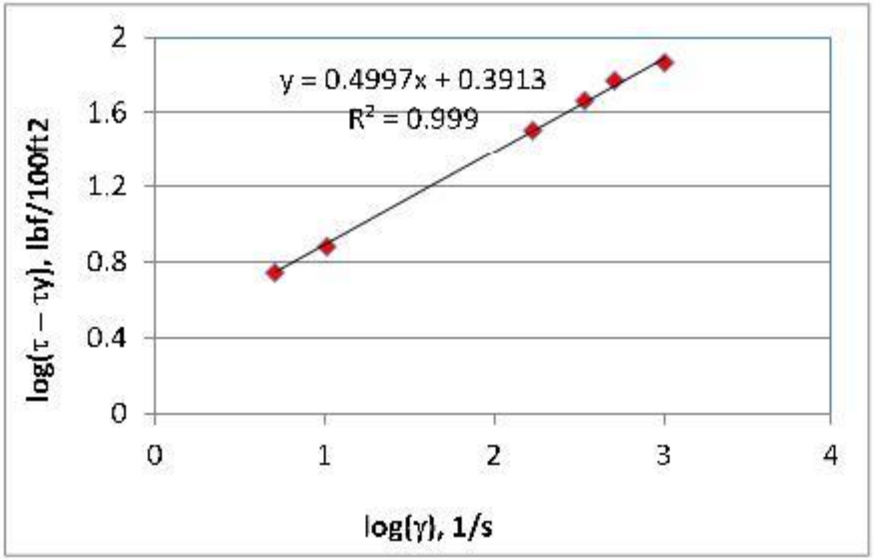
Rotational Viscometer

The data below are obtained from a rotational viscometer. Determine type of fluid and the rheological model of this fluid.

RPM	Dial Reading
3	10
6	12
100	35
200	48
300	60
600	75

Rotational Viscometer





LogK = 0.3913 --> K = 2.49

n = 0.39

τ_y = 5 lbf/100ft^2