Page 93, #6

- **Indices**
  - i = input elements {s,n}
  - f = fertilizers {1,2}
- **Data**
  - REQ_{if} = lower limit of proportion of f made up of i
  - $COST_i$ = cost/lb of input i
  - $PRICE_f$ = price/lb of fertilizer f
  - $AVAIL_i$ = lbs available of fertilizer i
  - $NET_{if} = PRICE_f - COST_i$ = net profit/lb for each combination
- **Variables**
  - x_{if} = lbs of i used to make f
- **Objective**

$$\max z = NET_{s,1} * x_{s,1} + NET_{s,2} * x_{s,2} + NET_{n,1} * x_{n,1} + NET_{n,2} * x_{n,2}$$

- **Constraints**

$$x_{s,1} \geq REQ_{s,1} * (x_{s,1} + x_{n,1})$$
$$x_{n,1} \geq REQ_{n,1} * (x_{s,1} + x_{n,1})$$
$$x_{s,2} \geq REQ_{s,2} * (x_{s,2} + x_{n,2})$$
$$x_{n,2} \geq REQ_{n,2} * (x_{s,2} + x_{n,2})$$
$$x_{s,1} + x_{s,2} \leq AVAIL_s$$
$$x_{n,1} + x_{n,2} \leq AVAIL_n$$
$$x_{if} \geq 0 \text{ for all } i,f$$

$$\max z = \sum_{i,f} NET_{if} * x_{if}$$

(Algebraic)

$$x_{if} \geq REQ_{if} * \sum_{i'} x_{i'f} \text{ for all } i,f$$
$$\sum_{f} x_{if} \leq AVAIL_i \text{ for all } i$$
$$x_{if} \geq 0 \text{ for all } i,f$$

Page 93, #10

- **Indices**
  - m = mines {1-3}
  - c = customers {1-4}
  - e = elements {ash, sulfur}
- **Data**
  - $PROD_m$ = production cost/ton ($) of coal from mine m
  - CAP_m = production capacity of mine m
  - $PROP_{em}$ = proportion of e per ton in mine m coal
  - LIM_e = maximum proportion of e in all coal shipped
  - $COST_{mc}$ = cost/ton ($) to ship from m to c
  - $DEMAND_c$ = tons demanded by customer c
  - TOT_{mc} = $PROD_m$ + $COST_{mc}$ = total production plus shipping cost
- **Variables**
  - x_{mc} = tons of coal shipped from m to c
- **Objective**

$$\min z = TOT_{1,1} * x_{1,1} + TOT_{1,2} * x_{1,2} + TOT_{1,3} * x_{1,3} + TOT_{1,4} * x_{1,4} +$$

$$\vdots$$

$$TOT_{3,1} * x_{3,1} + TOT_{3,2} * x_{3,2} + TOT_{3,3} * x_{3,3} + TOT_{3,4} * x_{3,4} \text{ (12 terms)}$$

$$\min z = \sum_{mc} TOT_{mc} * x_{mc}$$

- **Constraints**

$$\sum_{c} x_{mc} \le CAP_m \text{ for all } m$$

$$\sum_{m} x_{mc} \ge DEMAND_c \text{ for all } c$$

$$\sum_{mc} PROP_{em} * x_{mc} \le LIM_e * \sum_{mc} x_{mc} \text{ for all } e$$

$$x_{mc} \ge 0 \text{ for all } m,c$$

Page 104, #3

- **Indices**
  - m = months {1-3}
  - c = cake type {bf, ch}
- **Data**
  - $DEMAND_{cm}$ = demand for cake c in month m
  - $COST_{cm}$ = cost for cake c in month m
  - $HOLD_c$ = holding cost/month for cake c
  - CAP = max cakes baked/month
- **Variables**
  - x_{cm} = # of cakes c baked in month m
  - in_{cm} = inventory of c at the end of month m
- **Objective**

$$\min z = \left(\sum_{cm} COST_{cm} * x_{cm}\right) + \left(\sum_{cm} HOLD_c * in_{cm}\right)$$

$$= COST_{bt,1} * x_{bt,1} + COST_{bt,2} * x_{bt,2} + COST_{bt,3} * x_{bt,3} +$$

$$COST_{ch,1} * x_{ch,1} + COST_{ch,2} * x_{ch,2} + COST_{ch,3} * x_{ch,3} +$$

$$HOLD_{bt} * \left(in_{bt,1} + in_{bt,2} + in_{bt,3}\right) +$$

$$HOLD_{ch} * \left(in_{ch,1} + in_{ch,2} + in_{ch,3}\right)$$

- **Constraints**

$$x_{bt,1} + x_{ch,1} \le CAP$$
$$x_{bt,2} + x_{ch,2} \le CAP$$
$$x_{bt,3} + x_{ch,3} \le CAP$$
$$x_{bt,1} = DEMAND_{bt,1} + in_{bt,1}$$
$$x_{ch,1} = DEMAND_{ch,1} + in_{ch,1}$$
$$x_{bt,2} + in_{bt,1} = DEMAND_{bt,2} + in_{bt,2}$$
$$x_{ch,2} + in_{ch,1} = DEMAND_{ch,1} + in_{ch,2}$$
$$x_{bt,3} + in_{bt,2} = DEMAND_{bt,3} + in_{bt,3}$$
$$x_{ch,3} + in_{ch,2} = DEMAND_{ch,3} + in_{ch,3}$$
$$x_{cm} \ge 0,\ in_{cm} \ge 0 \text{ for all } c,\ m$$

$$\sum_c x_{cm} \le CAP \text{ for all } m$$
$$x_{cm} = DEMAND_{cm} + in_{cm} \text{ for all } c,\ m = 1$$
$$x_{cm} + in_{c,m-1} = DEMAND_{cm} + in_{cm} \text{ for all } c,\ m > 1$$
$$x_{cm} \ge 0,\ in_{cm} \ge 0 \text{ for all } c,\ m$$

(Algebraic)

Page 104, #4

- **Indices**
  - p = products {A,B}
  - a = assembly lines {1,2}
  - m = month {mar,apr}
- **Data**
  - $DEMAND_{pm}$ = demand for p in m
  - $HOURS_{am}$ = line hours of a available in m
  - $PRODRATE_{pa}$ = units of p produced/hour on a
  - PRODCOST = $/hour to run a line
  - CARRY = carrying cost ($)/unit/month
  - $INIT_p$ = initial inventory of p
  - END_p = ending inventory of p
- **Variables**
  - x_{pam} = number of p produced on a in m
  - in_{pm} = ending inventory of p in month m
- **Objective**

$$\min z = \left(PRODCOST * \sum_{pam} \frac{x_{pam}}{PRODRATE_{pa}} \right) + \left(HOLD * \sum_{pm} in_{pm} \right)$$

- **Constraints**

$$\sum_p \frac{x_{pam}}{PRODRATE_{pa}} \le HOURS_{am} \text{ for all } a, m$$

$$INIT_p + \sum_a x_{pam} = DEMAND_{pm} + in_{pm} \text{ for all } p, m = \text{"}mar\text{"}$$

$$\sum_a x_{pam} + in_{p,m-1} = DEMAND_{pm} + END_p \text{ for all } p, m = \text{"}apr\text{"}$$

$$x_{pam} \ge 0 \text{ for all } p, a, m$$

$$in_{pm} \ge 0 \text{ for all } p, m$$

Note: problem defines PRODRATE as hours/product, which is strange. I divide here because a rate is normally products/hour; if you use the data as given, you'd multiply