# Aerodynamics I

Instructor: Hui Xu (Ch.1-4)
Wei Tian (Ch.5-7)
Email: dr.hxu@sjtu.edu.cn
Office Hours: by appointment

**School of Aeronautics and Astronautics**
**Shanghai Jiao Tong University**

**The time:**

**Start time: 12:55**
**Scheduled end time: before 2:45**

**Teaching Assistant:**

**Mr. Dongming Ding**
**Tel: 159 0215 3842**
**Email: 970243881@qq.com**

## Useful Info

# Review

## Dimensional Analysis

- **The Principle of Dimensional Homogeneity (PDH)**——a rule which is almost a self-evident axiom in physics

- Dimensional analysis is based on the obvious fact that in an equation dealing with the real physical world, each term must have the same dimensions.

- If an equation truly expresses a proper relationship between variables in a physical process, it will be *dimensionally homogeneous*; i.e. each of its additive terms will have the same dimensions.

# Review

**Buckingham pi Theorem**

Let $K$ equal the number of fundamental dimensions required to describe the physical variables. (In mechanics, all physical variables can be expressed in terms of the dimensions of *mass, length,* and *time;* hence, $K = 3$.) Let $P_1, P_2, \ldots, P_N$ represent $N$ physical variables in the physical relation

$$f_1(P_1, P_2, \ldots, P_N) = 0 \tag{1.24}$$

Then, the physical relation Equation (1.24) may be reexpressed as a relation of $(N - K)$ dimensionless products (called $\Pi$ products),

$$f_2(\Pi_1, \Pi_2, \ldots, \Pi_{N-K}) = 0 \tag{1.25}$$

where each $\Pi$ product is a dimensionless product of a set of $K$ physical variables plus one other physical variable. Let $P_1, P_2, \ldots, P_K$ be the selected set of $K$ physical variables. Then

$$\begin{aligned} \Pi_1 &= f_3(P_1, P_2, \ldots, P_K, P_{K+1}) \\ \Pi_2 &= f_4(P_1, P_2, \ldots, P_K, P_{K+2}) \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ \Pi_{N-K} &= f_5(P_1, P_2, \ldots, P_K, P_N) \end{aligned} \tag{1.26}$$

The choice of the repeating variables, $P_1, P_2, \ldots, P_K$ should be such that they include *all* the $K$ dimensions used in the problem. Also, the dependent variable [such as $R$ in Equation (1.23)] should appear in only one of the $\Pi$ products.

Basic Theorem

# Review

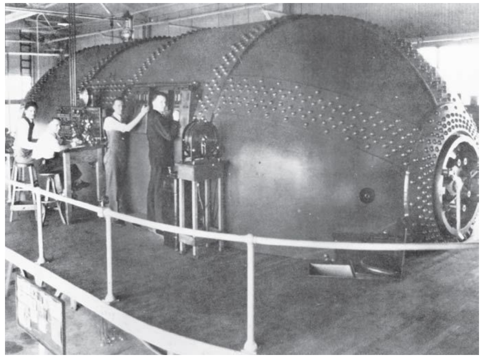
## Flow similarity

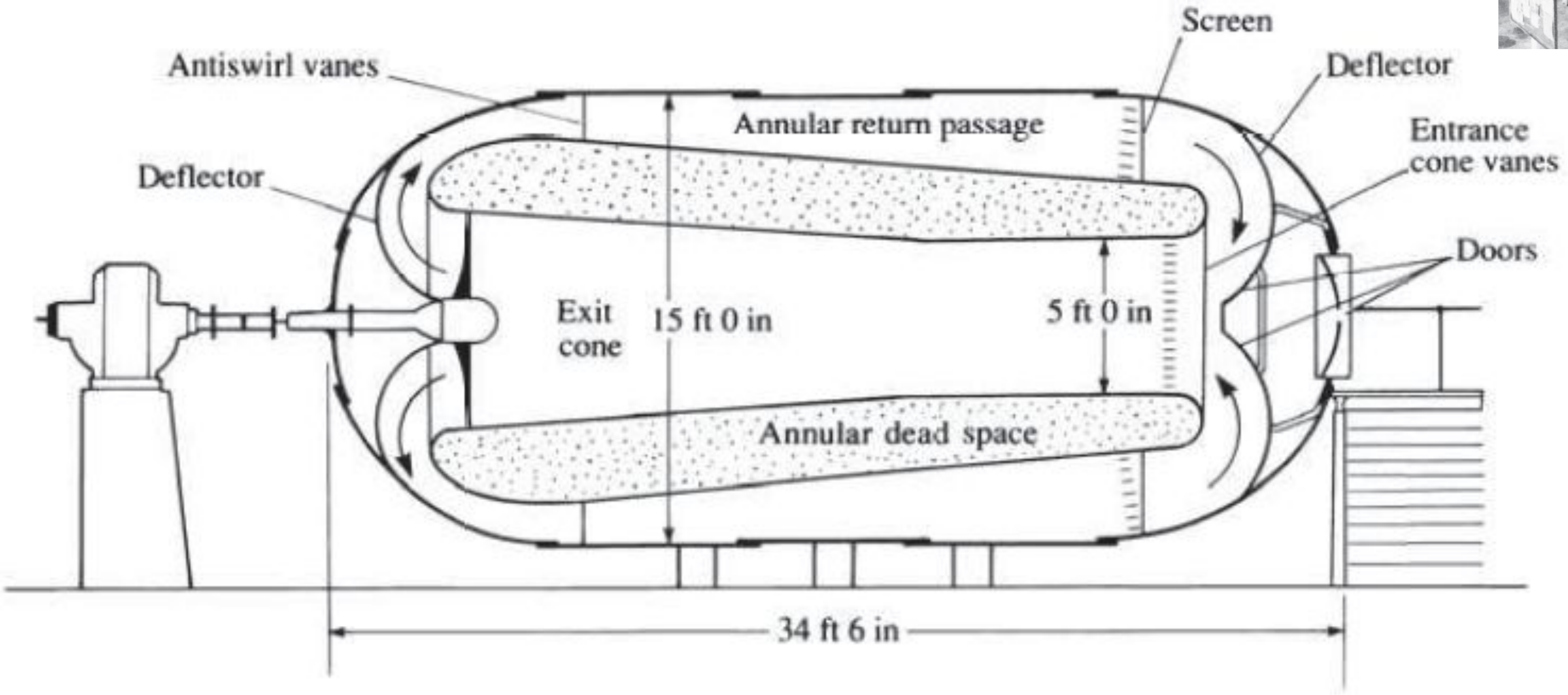
By definition, different flows are *dynamically similar* if:

1. The streamline patterns are geometrically similar.
2. The distributions of $V/V_\infty$, $p/p_\infty$, $T/T_\infty$, etc., throughout the flow field are the same when plotted against common non-dimensional coordinates.
3. The force coefficients are the same.

# Chapter 1: Introduction

Wind-tunnel test






Schematic of the variable density tunnel (VDT) (From Baals, D. D. and Carliss, W. R., Wind Tunnels of NASA, NASA SP-440, 1981).

# Review

**Fluid static dynamics: Buoyance**

Consider the forces along *y* axis: Positive upward, then—

$$-\frac{dp}{dy}(dx\,dy\,dz) - g\rho(dx\,dy\,dz) = 0$$

$$\boxed{dp = -g\rho\,dy}$$

Hydrostatic equation — differential equation, which connects the pressure variation in fluid with the vertical height.

# Review

## Type of flow

- Continuum flow

If the orders of magnitude of $\lambda$ is smaller than the specific scale of body(such as D) relatively, and then the fluid behaves as a continuous medium. The fluid presents a macro-behaving property, why?

- Free molecular flow

If $\lambda$ is the same order of the specific scale of body, the fluid presents individual molecules' behaving.

- Intermediate state

The flow features are between the above two — "Low density flow"

- Most practical aerodynamic applications involve continuum flow.

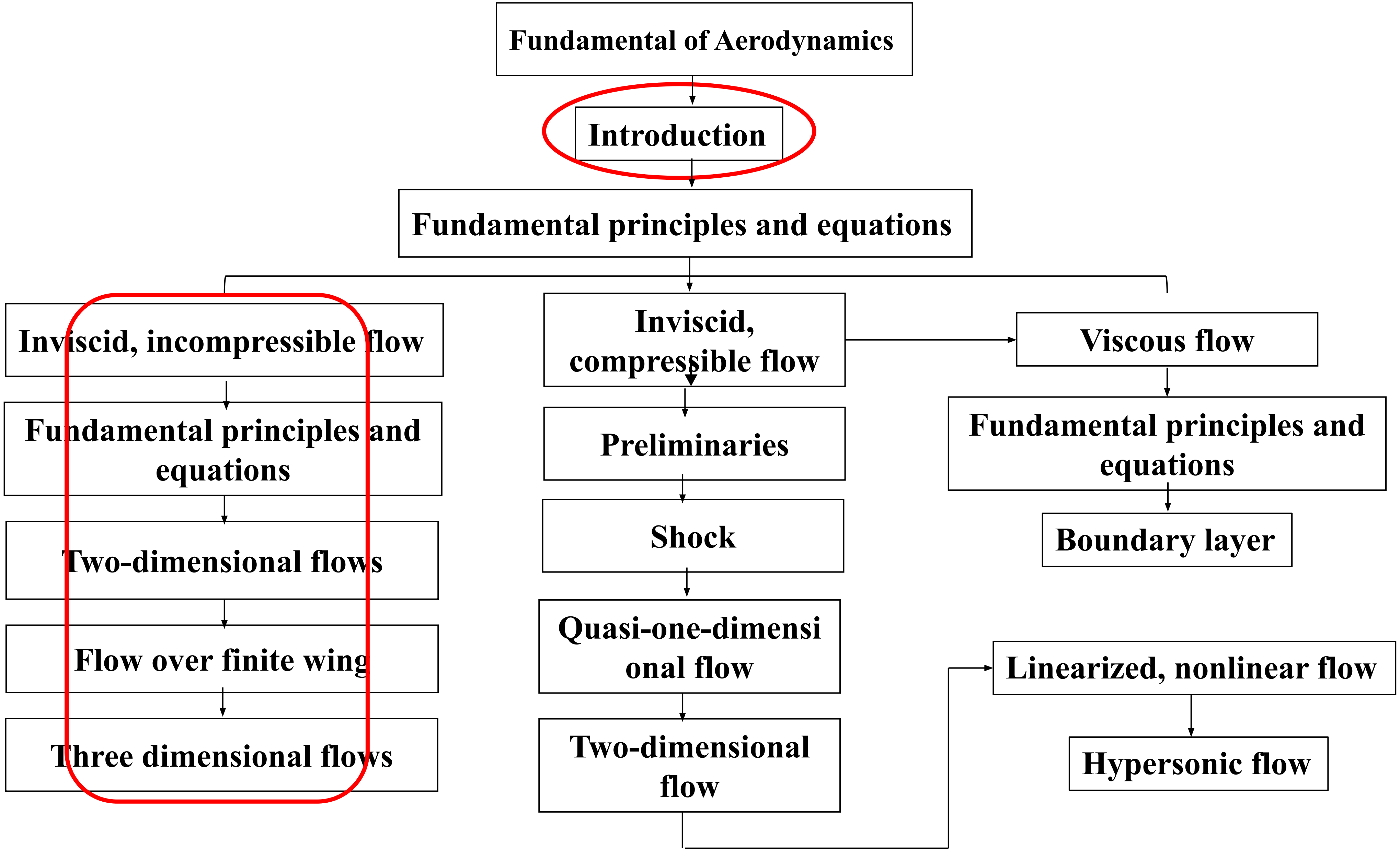
# Review

## Inviscid flow versus Viscous flow

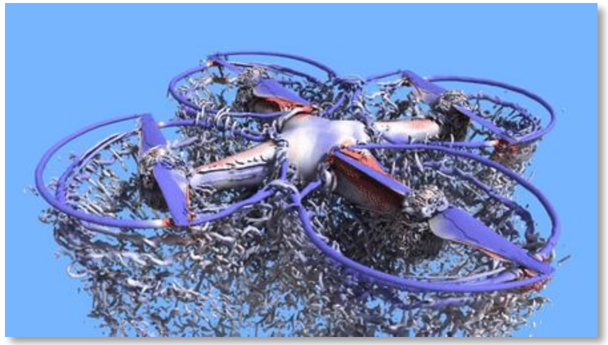
- Viscous flow:  This transport on a molecular scale gives rise to the phenomena of mass diffusion, viscosity (friction), and thermal conduction.  All real flows exhibit the effects of these transport phenomena, such flows are called viscous flows.

- Inviscid flow: Really exists? Inviscid flows do not truly exist in nature.

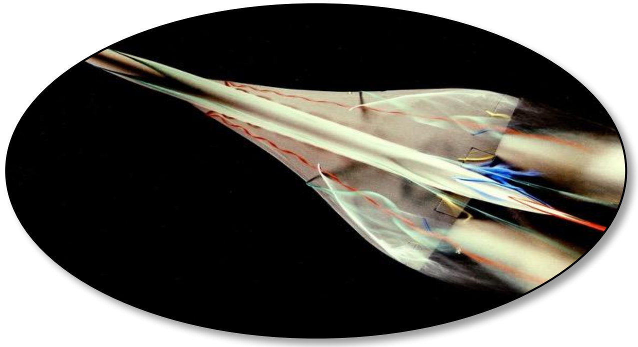
# Course Diagram



# Chapter 1



## Learning Targets



- Viscous Flow: *Boundary layers*
- Applied Aerodynamics: *The Aerodynamic Coefficients*
- Integrated Work *Challenge*: Forward-facing axial force

## 1.11 Viscous flow: Brief introduction on boundary layer

- At any point in flow field, provided that the velocity gradient exists, the shear stress does also.
- Only where the gradients are substantial, has the local stress as a meaningful effect on the flow.
- The flow field can be divided into two regimes—

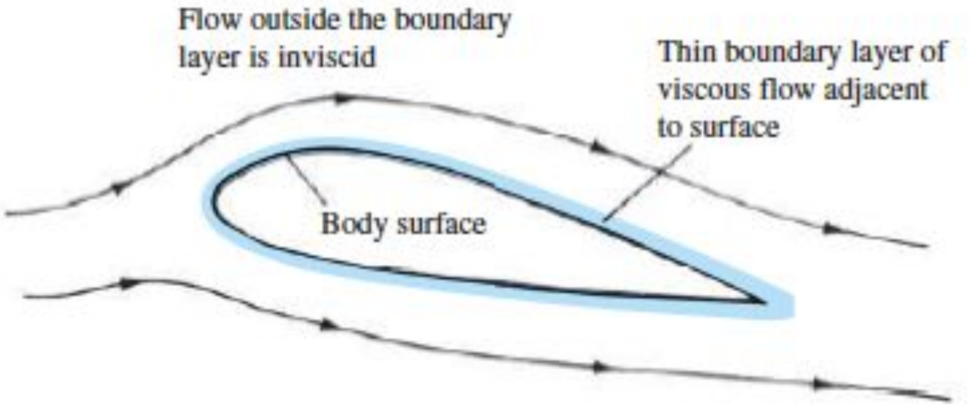



Figure 1.42 The division of a flow into two regions: (1) the thin viscous boundary layer adjacent to the body surface and (2) the inviscid flow outside the boundary layer.

1. Inside a thin layer adjacent to the body surface, the velocity gradient is large, friction play a definitive/crucial role—*Viscous flow (Boundary layer)*
2. In the region outside the thin layer, the gradient is so relatively small that the friction plays nearly no role—*Inviscid flow*

- In 1904, Prandtl put forward the concept of boundary layer: boundary layer—the region adjacent to the body surface. Two regions—viscous and inviscid, can be solved respectively.

- Flow separation — Pressure distribution will be changed severely—thin, but important
  ✔ *Pressure drag*

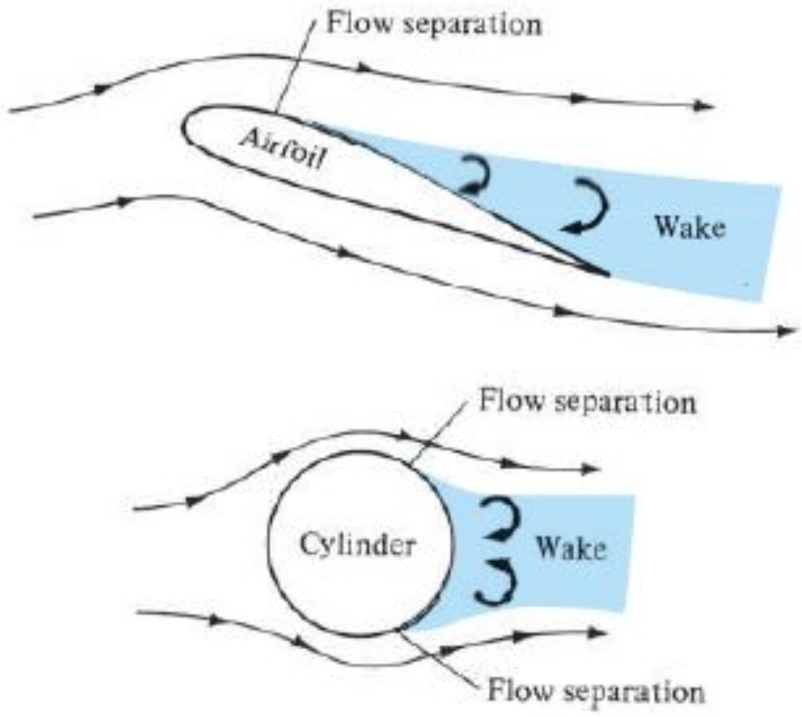



**Figure 1.43** Examples of viscous-dominated flow.

# Chapter 1: Introduction

- Question: Why are the gradients so large inside the layer?
- Inviscid, no friction, the surface is streamline. In fact, adjacent to the surface, the molecules' velocities are zero—no-slip condition, but the increasing gradient must be?
- The boundary layer theory: shear stress $\tau_w$ and layer thickness $\delta$.
- Indicated by researches: The pressure is nearly constant along the normal direction to the surface in the layer.

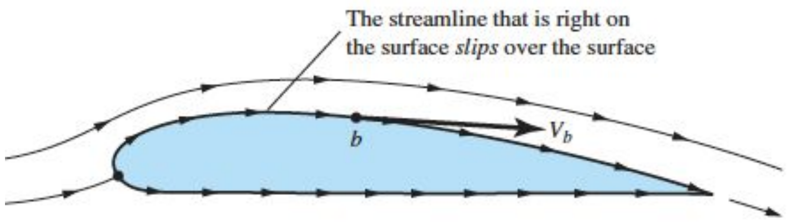



Figure 1.46 Inviscid (frictionless) flow.

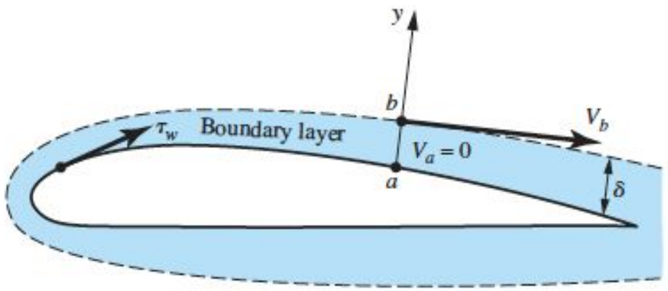



Figure 1.47 Flow in real life, with friction. The thickness of the boundary layer is greatly overemphasized for clarity.

- Question: What does the above suggest?
- The pressure at the outside edge obtained by inviscid theory has no any change across the layer (the change can be neglected);
- The distribution is very close to the real one(very high accuracy);
- The above conclusions have preconditions: thin layer and no separation.

Note: the above two are reasonable for the layer attached on the surface all the way, but not appropriate for the separated flow shown by Fig.1.43.

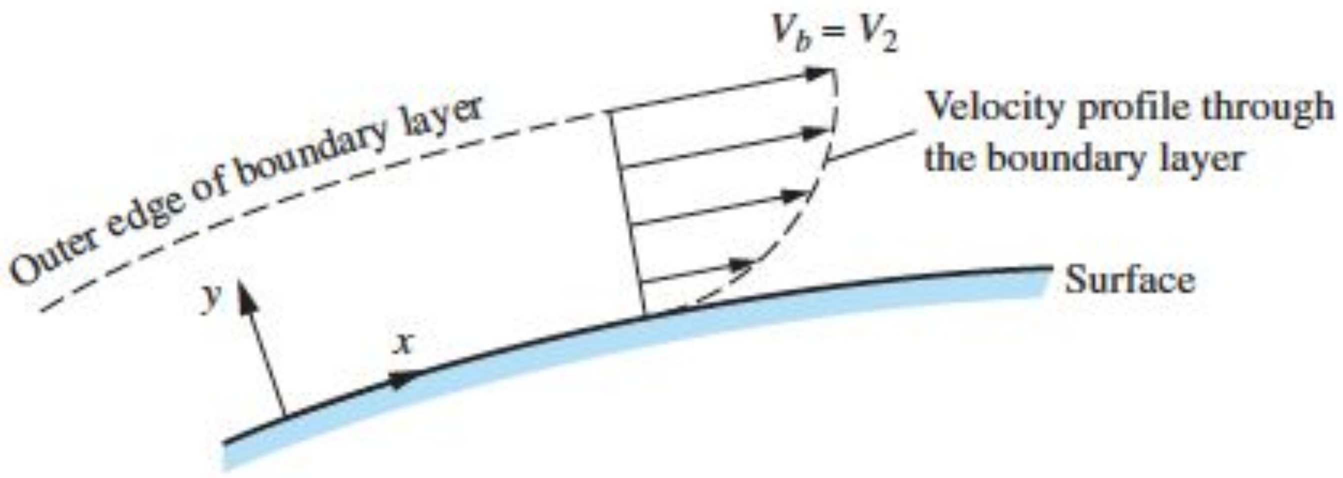



Figure 1.48 Velocity profile through a boundary layer.

- Velocity profile inside boundary layer: The curve (vector end curve) of the velocity inside the boundary layer as the function of the normal axis to the local body surface. Function of *x*.

Temperature profile through the layer: The curve of the temperature through the boundary layer as the function of the local normal axis. Dominated by (1) Thermal conduction——heat mixing by molecules random motion; (2) Frictional dissipation——frictional shear stress results in energy conversion(kinetic- internal). The shear stress on the wall skin is determined by the slope rate of the velocity profile adjacent to the skin:

$$\tau_w = \mu \left(\frac{dV}{dy}\right)_{y=0}$$

where, $\mu$ is the absolute viscous coefficient ($M/LT$).

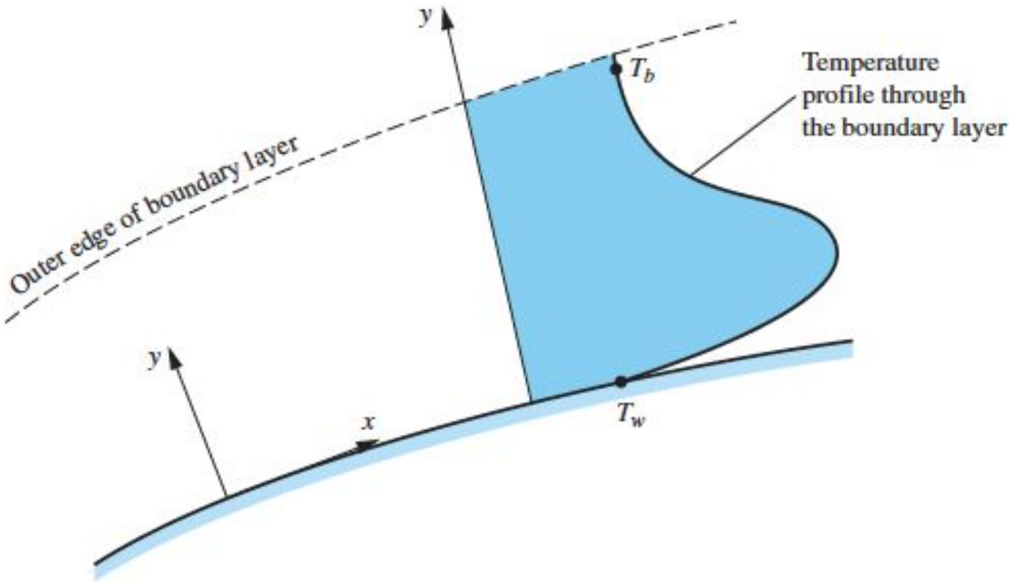



Figure 1.49 Temperature profile through a boundary layer.

# Chapter 1: Introduction

- $\mu$ variation with temperature: liquid (olive ; soybean- edible oil)—T increase, $\mu$ decrease; Gas, the contrary. Sea level, standard atmospheric air (right figure: $\mu = \mu(T)$)

$$\mu = 1.7894 \times 10^{-5} kg/(m)(s)$$

- The slope of the temperature profile on the wall indicates whether if the wall is aero-heated or cooled. Aero-heating rate is:

$$\dot{q}_w = -k \left( \frac{dT}{dy} \right)_{y=0}$$

Where, $k$ is the thermal conductivity of the gas. The negative sign ?

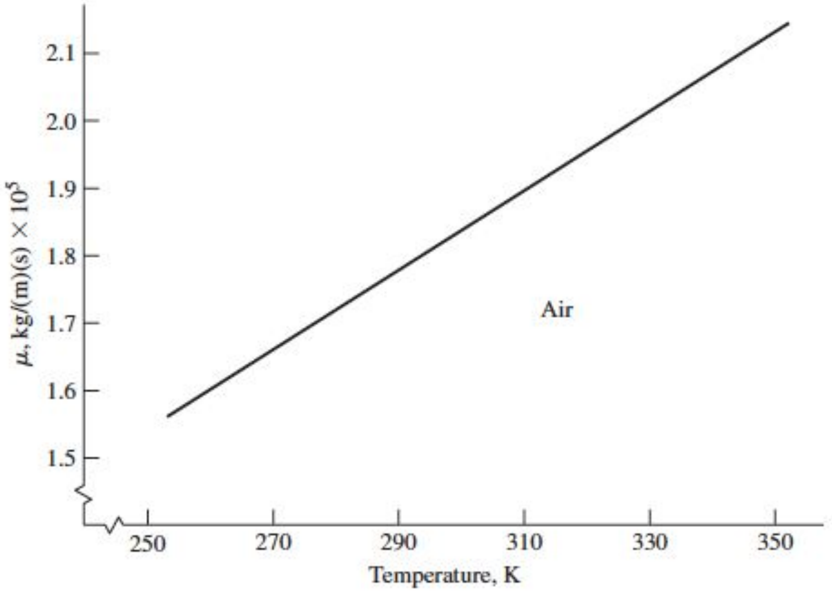



Figure 1.50 Variation of viscosity coefficient with temperature.

# Chapter 1: Introduction

- At standard sea level temperature:

$$k=2.53\times10^{-2} J/(m)(s)(K)$$

- Proportional to the viscous coefficient essentially, *i.e*

$$k=(const)\times\mu$$

- "Convective heat transfer": the air flow over a body surface heats or cools the surface(thermal conduction).
- "Aerodynamic heating": the heat transfer between boundary layer and body surface. Dominates the supersonic flow especially hypersonic one.

- *Re* effect on boundary layer:
- Figure below: Development of boundary layer on plate

  Local Reynolds number at *x* away from the leading edge(characteristic length is *x*):

$$\mathrm{Re}_x = \frac{\rho_\infty V_\infty x}{\mu_\infty}$$

- "∞" dictates the parameter of the far forward flow relative to plate.
- The skin friction(shear stress) and the thickness of boundary layer at *x* on the plate are the function of $\mathrm{Re}_x$ .

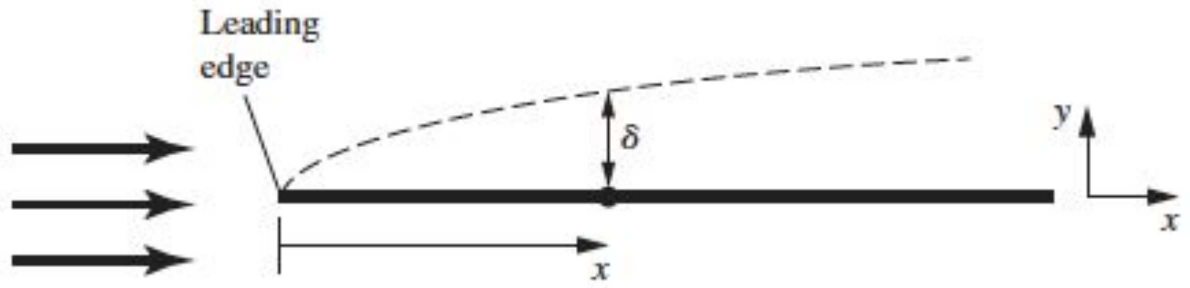



Figure 1.51 Growth of the boundary layer thickness.

# Chapter 1: Introduction

- The Reynolds number generally governs the nature of the viscous flow.
- The basic types of viscous flow:
  1. Laminar flow: the streamlines are smooth and regular , and the fluid element moves smoothly along the streamline;
  2. Turbulent flow: the streamlines break up and a fluid element moves in a random, irregular and tortuous fashion.

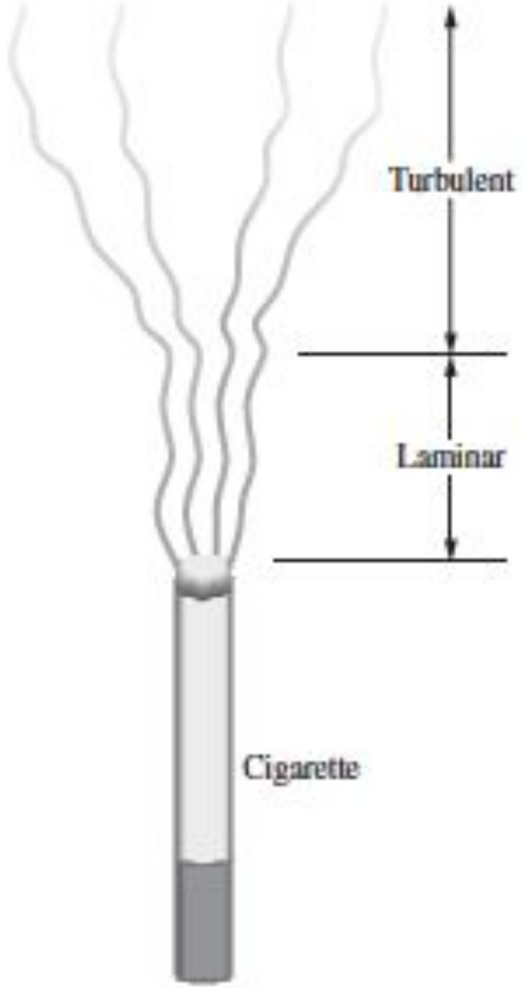



Figure 1.52 Smoke pattern illustrating transition from laminar to turbulent flow.

Fig.1.53: Comparison for the two velocity profiles——

1. Turbulent, relatively "plump";
2. Laminar, relatively "gradual".

Approaching to the wall, the gradients of the two flows are different.

The relation of the velocity gradient at the wall:

$$\left(\frac{dV}{dy}\right)_{y=0} \text{ for laminar flow} < \left(\frac{dV}{dy}\right)_{y=0} \text{ for turbulent flow}$$

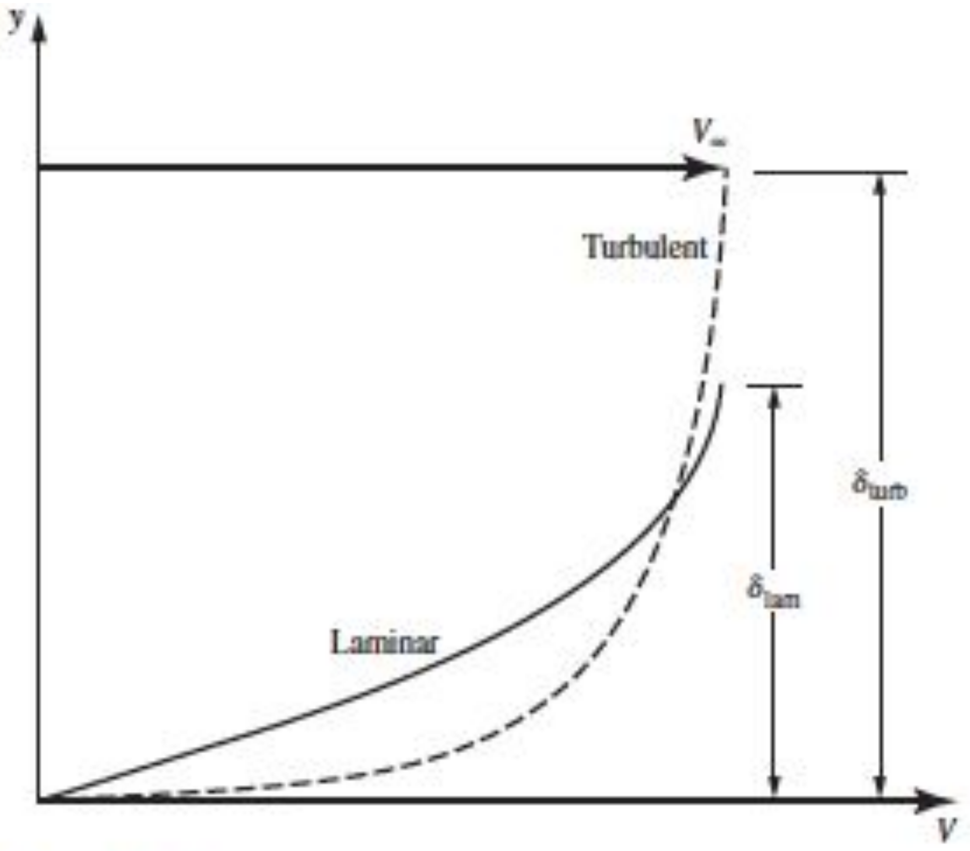



**Figure 1.53** Velocity profiles for laminar and turbulent boundary layers. Note that the turbulent boundary layer thickness is larger than the laminar boundary layer thickness.

# Chapter 1: Introduction

- By Eq.1.59: $(\tau_w)_{laminer} < (\tau_w)_{turbulent}$
  The important fact: The shear stress of laminar flow is smaller than turbulent flow !
- The types of the wall boundary layer determine the character of the frictional force on the aircraft——the frictional drag produced by laminar flow is smaller relatively!
- There is similar result for “aero-heating”: Turbulent flow is larger than laminar flow, even much huge. For hypersonic flow, even up to more than 10 times!

# Chapter 1: Introduction

Homework: 1.11, 1.12

Questions for thinking:

1. After released, the hydrogen ball will continue to go up, what will happen in the end? Please analyze the reason;

2. Deduce the Pascal law by Hydrostatic equation.

## 1.12 Applied aerodynamics: Aerodynamic coefficients——

Magnitudes and variations

Applied aerodynamics: For the practical evaluation on the aerodynamic properties of aircrafts and design works.

(1) Configuration and performance; (2) Properties of flow field; (3) Components design; (4) Drag reducing design; (5) Type modified design;(6) New conceptual aircraft , etc.

In fact, aerodynamics evaluations run through the whole developing process of aircraft, and are gradually coupled with the analysis for every other subjects(such as structural dynamics or flight dynamics etc.) tightly, in order to solve conceptual design, components design, aero-elastic design, flight quality( maneuverability, stability), structural strength( fatigue lifetime), heath monitor, human factor and other problems.

# Chapter 1: Introduction

- Lift, Drag and moment coefficient are the most frequently used technical terms in outflow aerodynamics.
- It is very important to master the magnitude concept of the actual typical values of aerodynamic coefficients.
- Question: Is it meaningful for drag coefficient to take the value in [$10^{-5}$,1000] ?
- The order of magnitude of the aerodynamic coefficients for commonly used configurations are listed below.
- Question: What are the typical drag coefficients of different aerodynamic configurations?

# Chapter 1: Introduction

- Fig.1.54, Typical drag coefficients of some bodies (fixed attitude) moving in a low speed.
- By Sec. 1.7:
- $C_D = f(Re, M_\infty)$ , then, $C_D = f(Re)$ .
- Precondition for simplification ?
- Re = ?

- Comparisons between Case *a*, *b* and *c*:

Re all are $10^5$, specific lengths *d*, and the specified areas are $S=d\cdot 1$. From *a* to *c*, the wake area shrinks and decreases, while the drag coefficient becomes smaller. $C_D$=2——1.2——0.12.

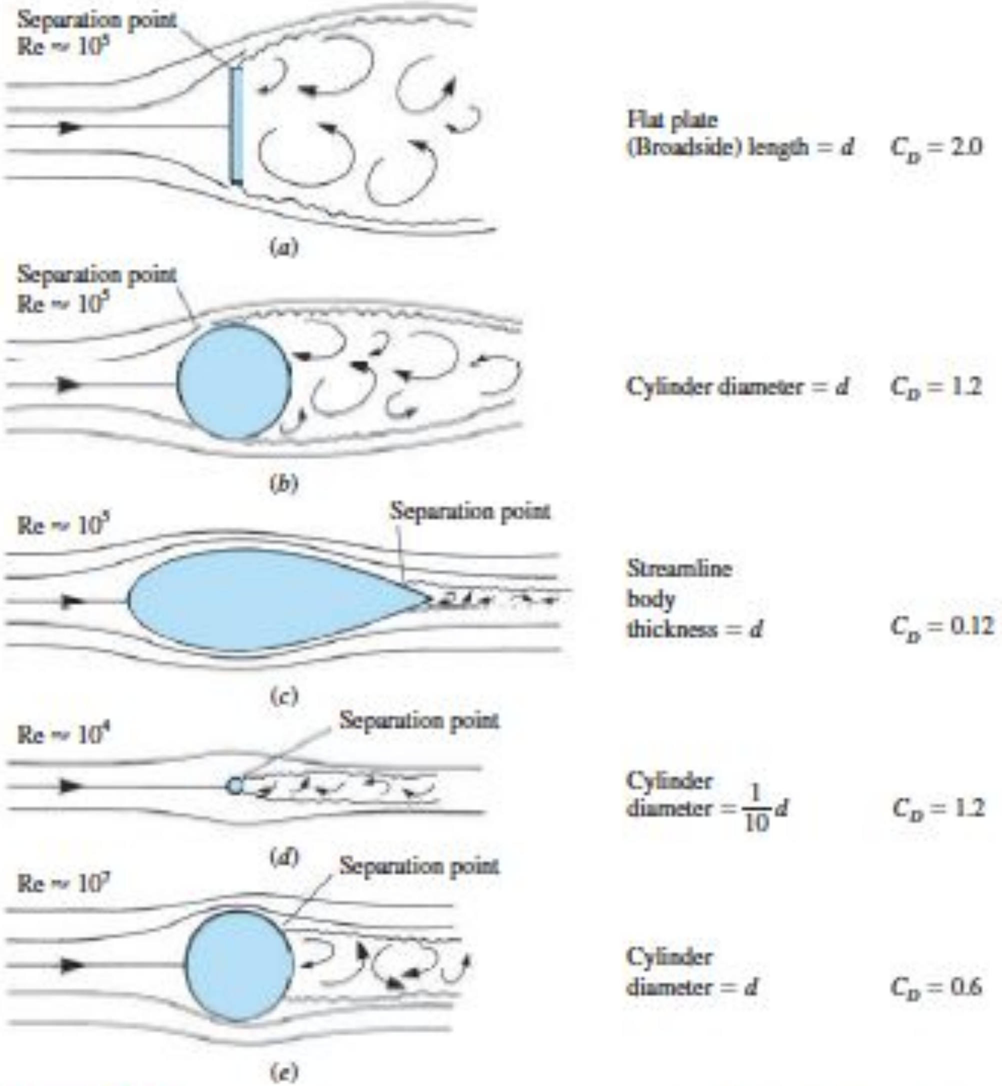



**Figure 1.54** Drag coefficients for various aerodynamic shapes. (*Source:* Talay, T. A., *Introduction to the Aerodynamics of Flight*, NASA SP-367, 1975).

- Comparisons between Case *b* and *d* :
  One order of magnitude difference in both Re and diameter, but $C_D$ same, all are 1.2. For cylinders, Re in the domain of $10^4$-$10^5$, $C_D$ values are relatively not inflected by the Reynolds number.
- Value of drag?

$$D' = q_\infty S C_D$$

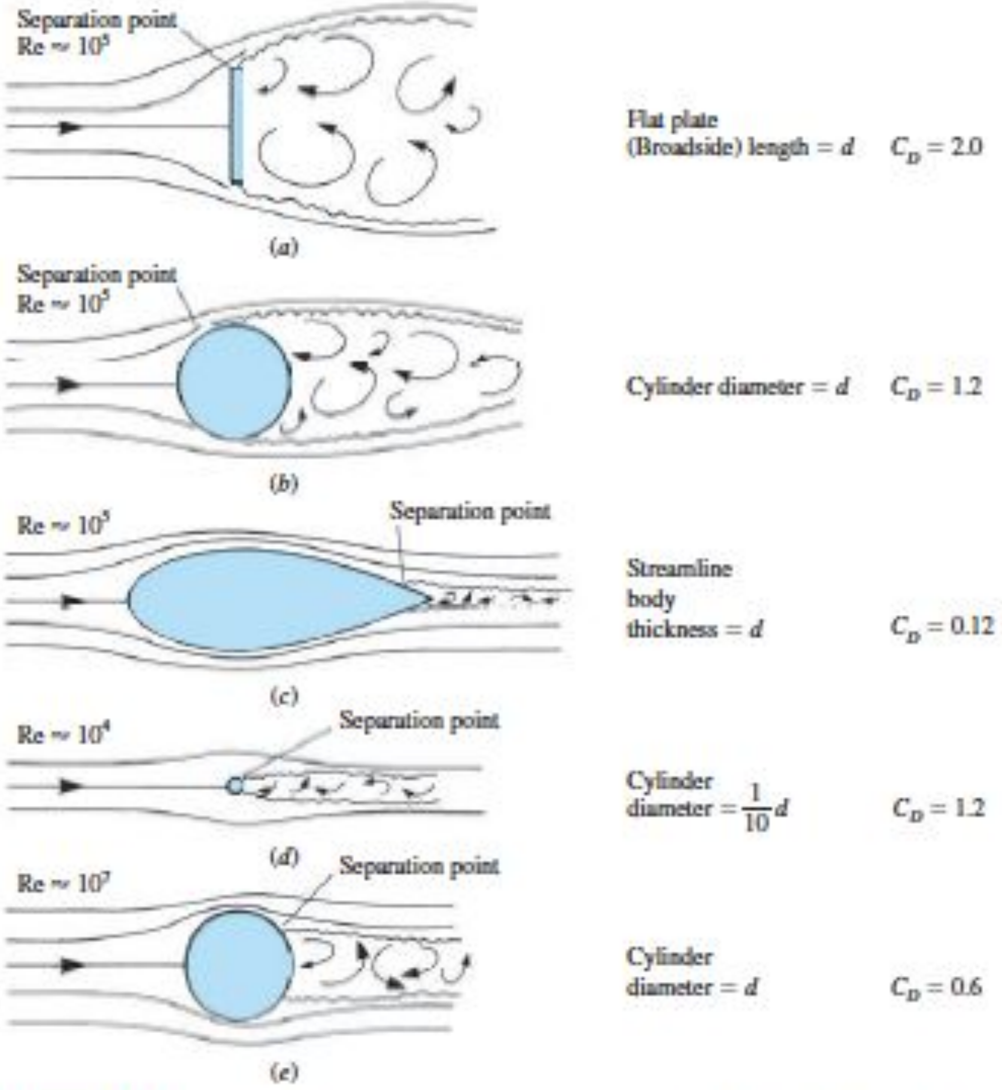



**Figure 1.54** Drag coefficients for various aerodynamic shapes. (*Source:* Talay, T. A., *Introduction to the Aerodynamics of Flight*, NASA SP-367, 1975).

# Chapter 1: Introduction

- Comparisons between Case *c* and *d* :

Same dynamic pressures, drags are same too; *c* body a streamline one and height is 10 times of *d*'s!

- The drag reducing effect of streamline one is significant!

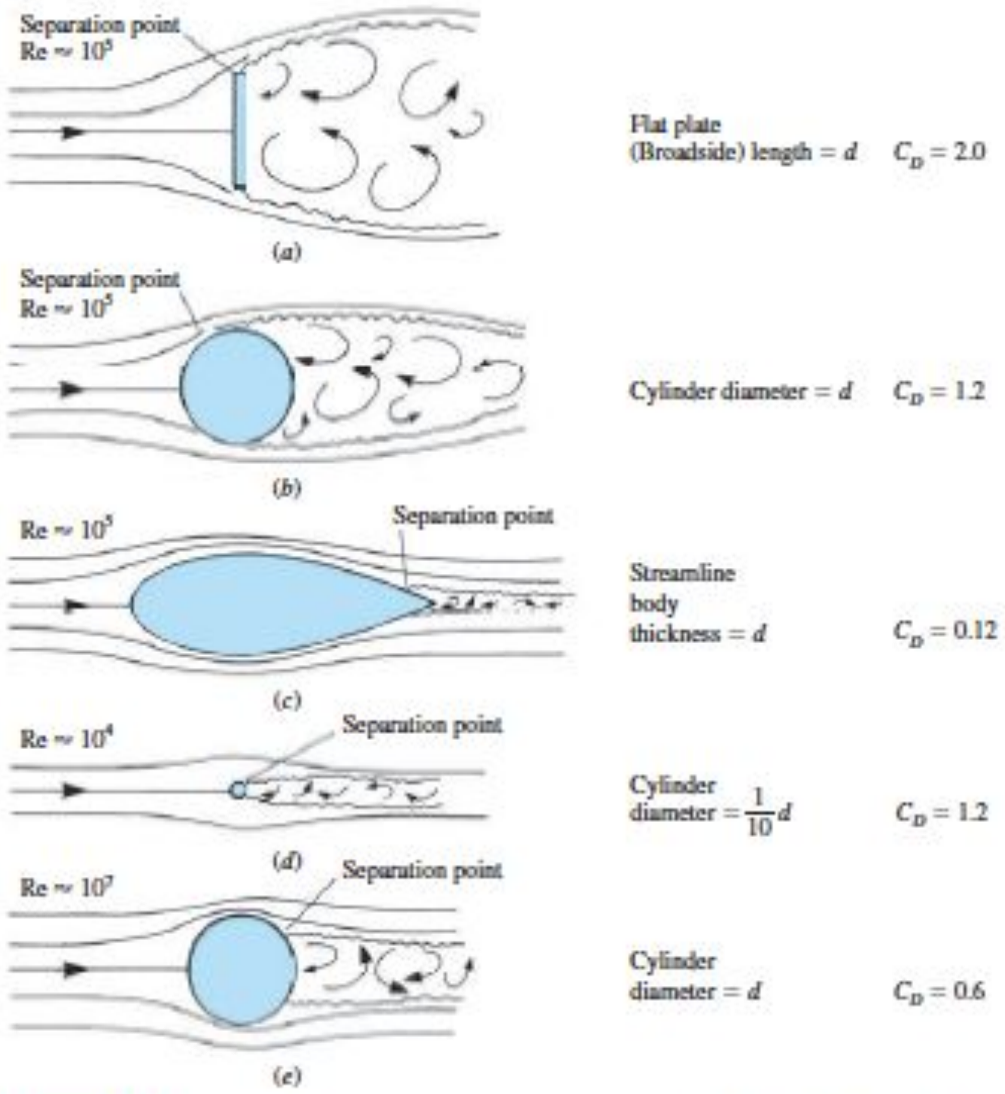



**Figure 1.54** Drag coefficients for various aerodynamic shapes. (*Source:* Talay, T. A., *Introduction to the Aerodynamics of Flight*, NASA SP-367, 1975).

# Chapter 1: Introduction

- Comparisons between Case *b* and *e* :
- Same diameter, but $Re_e=100\cdot Re_b$, and $C_{De}=2\cdot C_{Db}$

——the wake is relatively small

!(Part4)

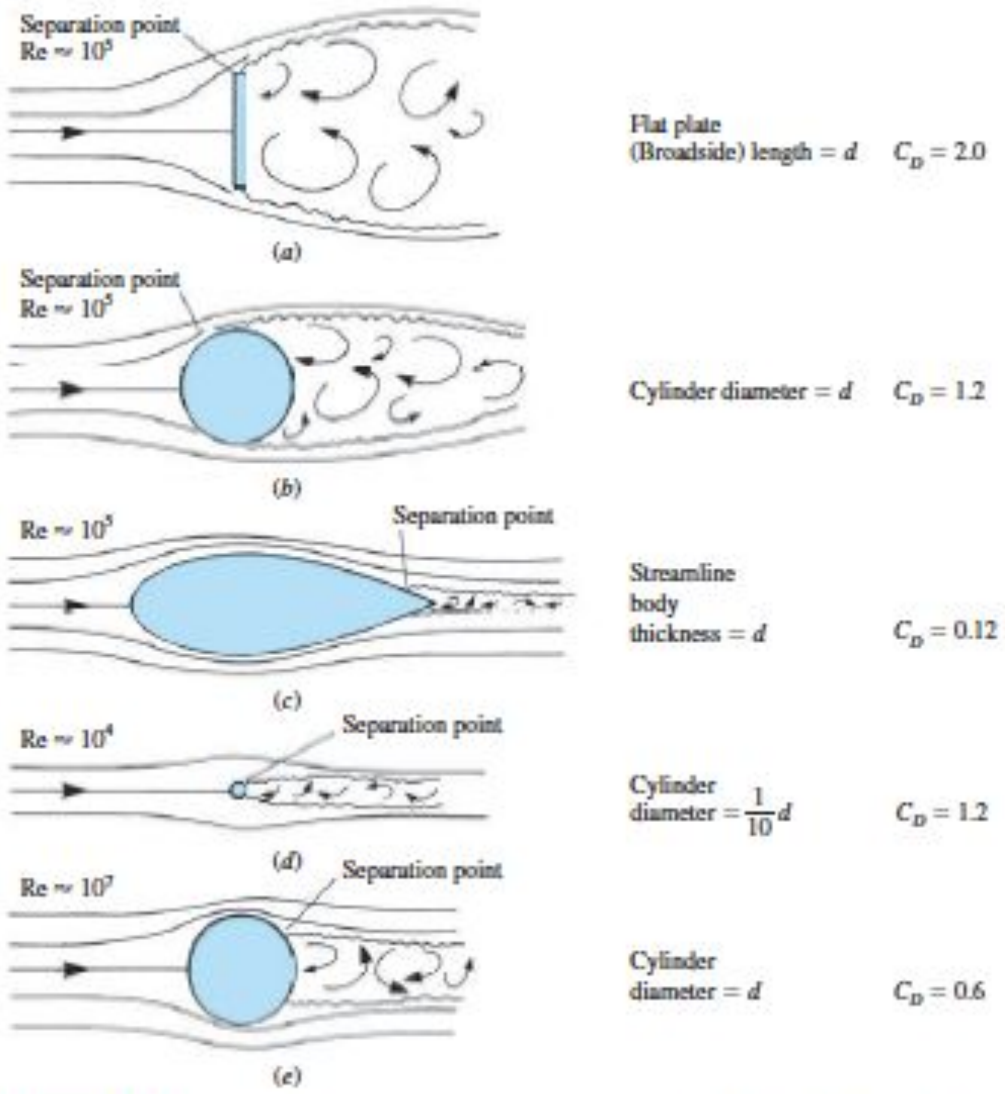



**Figure 1.54** Drag coefficients for various aerodynamic shapes. (*Source:* Talay, T. A., *Introduction to the Aerodynamics of Flight*, NASA SP-367, 1975).

Note: Character parameter(per unit span)—the maximum windward cross-sectional area. The drag coefficients vary from 2.0 to 0.12. Typical magnitude change.

The Reynolds numbers also change from tens of thousands to tens of millions. At standard sea level, for circle cylinder,

$$\rho_\infty = 1.23 kg/m^3;\ \mu_\infty = 1.7894\times10^{-5} kg/(m)(s)\ ;\ V_\infty = 45 m/s;\ \text{D}=1 m$$

then

$$\text{Re} = \frac{\rho_\infty V_\infty d}{\mu_\infty} = \frac{(1.23)(45)(1)}{1.789\times10^{-5}} = 3.09\times10^6$$

In practice, the orders of magnitude of usually used Reynolds number in aerospace are from millions to tens of millions .

# Chapter 1: Introduction

- Nature of drag: Axial force, Eq.1.8 becomes——

$$D' = \underbrace{\int_{LE}^{TE} -p_u \sin\theta \, ds_u + \int_{LE}^{TE} p_l \sin\theta \, ds_l}_{\text{pressure drag}}$$

$$+ \underbrace{\int_{LE}^{TE} \tau_u \cos\theta \, ds_u + \int_{LE}^{TE} \tau_l \cos\theta \, ds_l}_{\text{skin friction drag}}$$

- The aerodynamic drag on any body consists of pressure drag and friction drag .

# Chapter 1: Introduction

Fig.1.55, the comparison for the relative quantities of two types of drag of the above bodies.

Note:

1. The drags of upright plate and cylinder are mainly dominated by pressure drag;
2. The most part of drag of the streamline body is skin friction drag;
3. $\rho_\infty$、$\mu_\infty$ same, $v_\infty$ determined by *Re*, then $v_\infty$ in case *e* is much larger, and its drag is much larger than in case *b*.

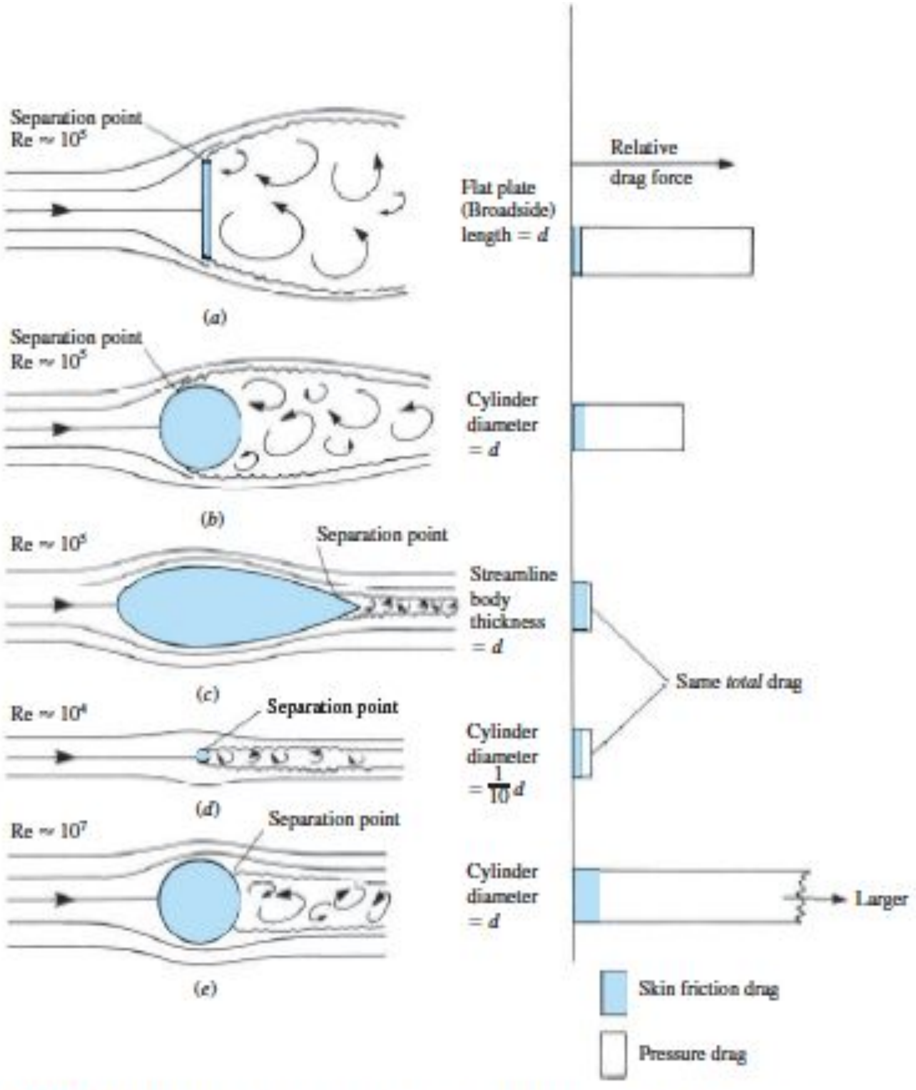



**Figure 1.55** The relative comparison between skin friction drag and pressure drag for various aerodynamic shapes. (*Source:* Talay, T. A., *Introduction to the Aerodynamics of Flight*, NASA SP-367, 1975).

# Chapter 1: Introduction

Thus, there are two types of typical aerodynamic shapes:

1. Blunt body: Most part of drag of the body (the radius of its head is relatively large) are pressure drag;
2. Streamline body: Most part of drag of the body are skin friction drag;

The drag of blunt body is much larger because its flow separates in its most area.

- Pressure drag: The pressure difference drag resulted by the separated flow, also called "shape drag".

- Variation of the plate drag at zero angle of attach with the Reynolds number ： The drag is totally produced by shear stress, and no any pressure in drag direction.
- Skin friction drag coefficient:

$$C_f = \frac{D'}{q_\infty S} = \frac{D'}{q_\infty c(1)}$$

Where, the reference area is the plane area per unit span.

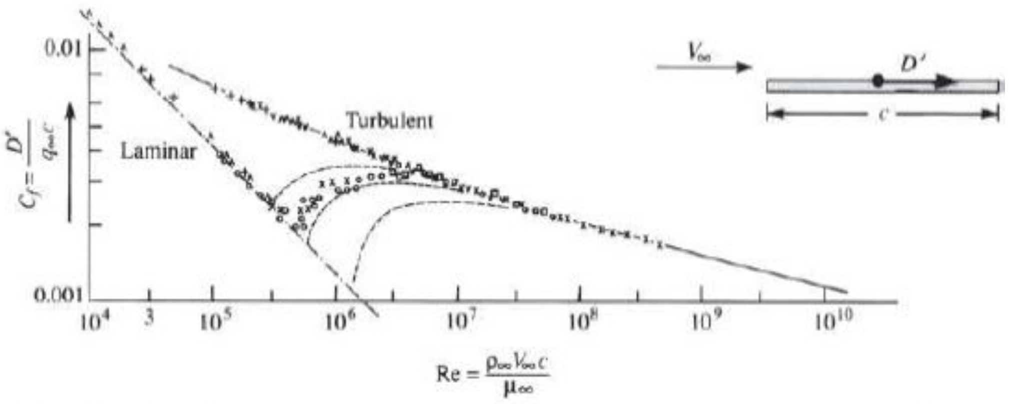



Figure 1.56 Variation of laminar and turbulent skin friction coefficient for a flat plate as a function of Reynolds number based on the chord length of the plate. The intermediate dashed curves are associated with various transition paths from laminar flow to turbulent flow.

The figure listed above indicates that:

1) $C_f$ strongly depends on Re, of which the character length $l$ is the chord length $c$ of the plate. As Re increases, $C_f$ decreases;
2) $C_f$ value is decided by the flow over the plate whether is laminar or turbulent. With the same Re, the turbulent is larger than the laminar;
3) $C_f$ The typical magnitude range is from 0.001 to 0.01 in a large Re domain.

The difference relative to upright plate?

# Chapter 1: Introduction

- NACA63-210 airfoil：
  With thickness, laminar airfoil—low angle—laminar flow—turbulent one at higher angle, the drag increases quickly. Minimum drag exists(0.0045)—“bottom of pot (barrel)”.
- For typical airfoils — 0.004~0.006 (dominated by friction)
- For the streamline body — flow separation—pressure drag—the drag coefficient increases with angle.

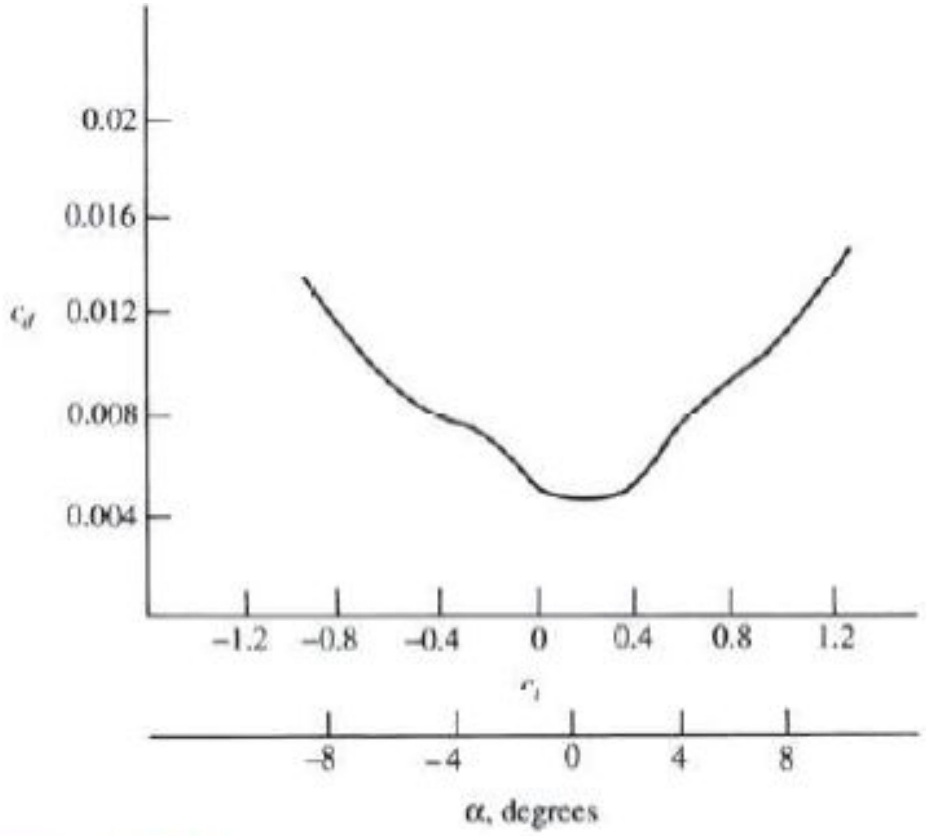


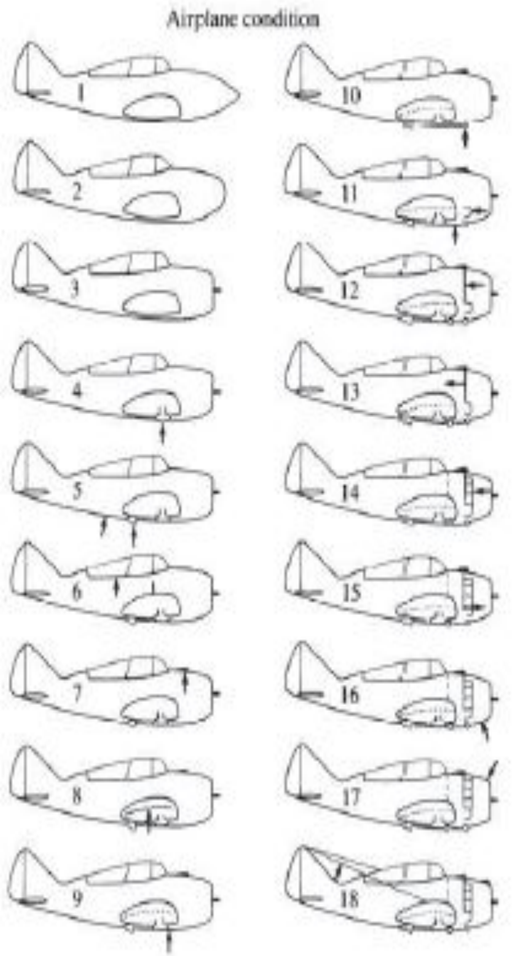

Figure 1.57 Variation of section drag coefficient for an NACA 63-210 airfoil. Re = $3 \times 10^6$.

# Chapter 1: Introduction

- The drag on a low speed airplane :

  Seversky P-35, the typical fighter in 1930s. Fig.1.58 is the detail drag variation for the aircraft in its design process.

  Note: condition 1 is clean aerodynamic configuration, $C_L$=0.15, $C_D$=0.0166. From 2 to 18, conventional and operational changes to equip the plane , all the additions make the drag coefficient increase up to more than 65%, as all are equipped, it reaches to 0.0275, this is the typical value for this type of aircraft.




| Condition number | Description | $C_D$ ($C_L = 0.15$) | $\Delta C_D$ | $\Delta C_D$, %[a] |
|---|---|---|---|---|
| 1 | Completely faired condition, long nose fairing | 0.0166 | | |
| 2 | Completely faired condition, blunt nose fairing | 0.0169 | | |
| 3 | Original cowling added, no airflow through cowling | 0.0186 | 0.0020 | 12.0 |
| 4 | Landing-gear seals and fairing removed | 0.0188 | 0.0002 | 1.2 |
| 5 | Oil cooler installed | 0.0205 | 0.0017 | 10.2 |
| 6 | Canopy fairing removed | 0.0203 | −0.0002 | −1.2 |
| 7 | Carburetor air scoop added | 0.0209 | 0.0006 | 3.6 |
| 8 | Sanded walkway added | 0.0216 | 0.0007 | 4.2 |
| 9 | Ejector chute added | 0.0219 | 0.0003 | 1.8 |
| 10 | Exhaust stacks added | 0.0225 | 0.0006 | 3.6 |
| 11 | Intercooler added | 0.0236 | 0.0011 | 6.6 |
| 12 | Cowling exit opened | 0.0247 | 0.0011 | 6.6 |
| 13 | Accessory exit opened | 0.0252 | 0.0005 | 3.0 |
| 14 | Cowling fairing and seals removed | 0.0261 | 0.0009 | 5.4 |
| 15 | Cockpit ventilator opened | 0.0262 | 0.0001 | 0.6 |
| 16 | Cowling venturi installed | 0.0264 | 0.0002 | 1.2 |
| 17 | Blast tubes added | 0.0267 | 0.0003 | 1.8 |
| 18 | Antenna installed | 0.0275 | 0.0008 | 4.8 |
| | Total | | 0.0109 | |

[a]Percentages based on completely faired condition with long nose fairing.

**Figure 1.58** The breakdown of various sources of drag on a late 1930s airplane, the Seversky XP-41 (derived from the Seversky P-35 shown in Figure 3.2). [*Source:* Experimental data from Coe, Paul J., "Review of Drag Cleanup Tests in Langley Full-Scale Tunnel (From 1935 to 1945) Applicable to Current General Aviation Airplanes," NASA TN-D-8206, 1976].

# Chapter 1: Introduction

- Question: As Mach number increases, how will the drag coefficient change ?
- Northrop T-38A jet trainer, see Fig.1.60.

Zero-lift drag coefficient $C_{DL=0}$ : The corresponding drag coefficient when the lift is just zero for an aircraft at a small angle of attack.

- Before the quick increase of the drag, $C_D \approx 0.015$, lower than that of P35 due to its good aerodynamic shape.（0.86）

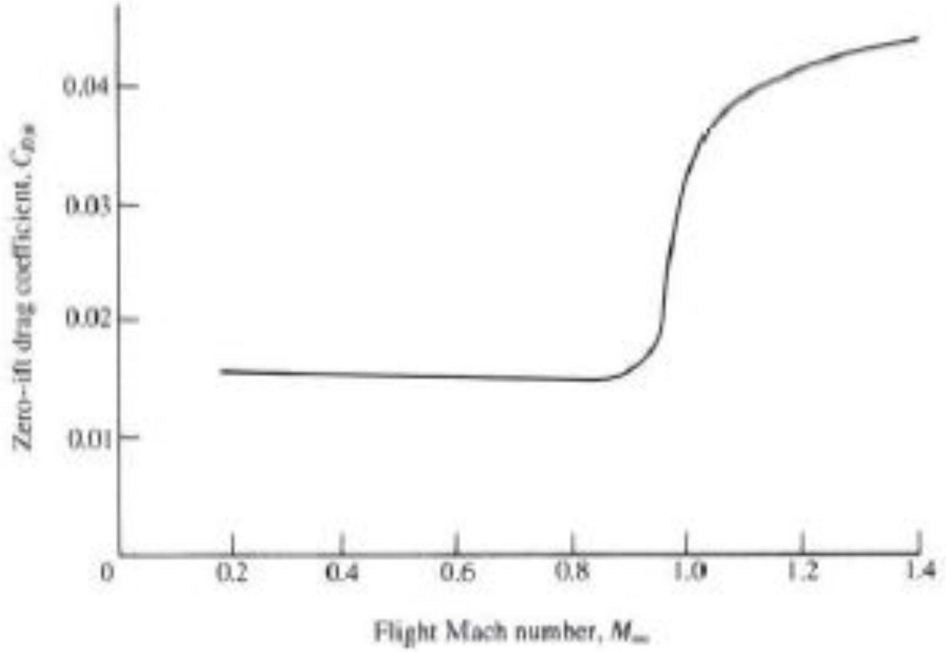



**Figure 1.60** Zero-lift drag coefficient variation with Mach number for the T-38 (*Courtesy of the U.S. Air Force*).

# Chapter 1: Introduction

- Typical order of magnitude of lift coefficient:

  Fig.1.61-NACA63-210 airfoil 's lift curve—$c_l$ in the range of [-1.0, 1.5], AOA — -12° to +14°.

- Lift-to-drag ratio L'/D'($c_l / c_d$):

  Important index; the ratio —

  $c_l / c_d$=0.6/0.0046=130（4 °）

  To produce enough lift by overcoming drag as little as possible!

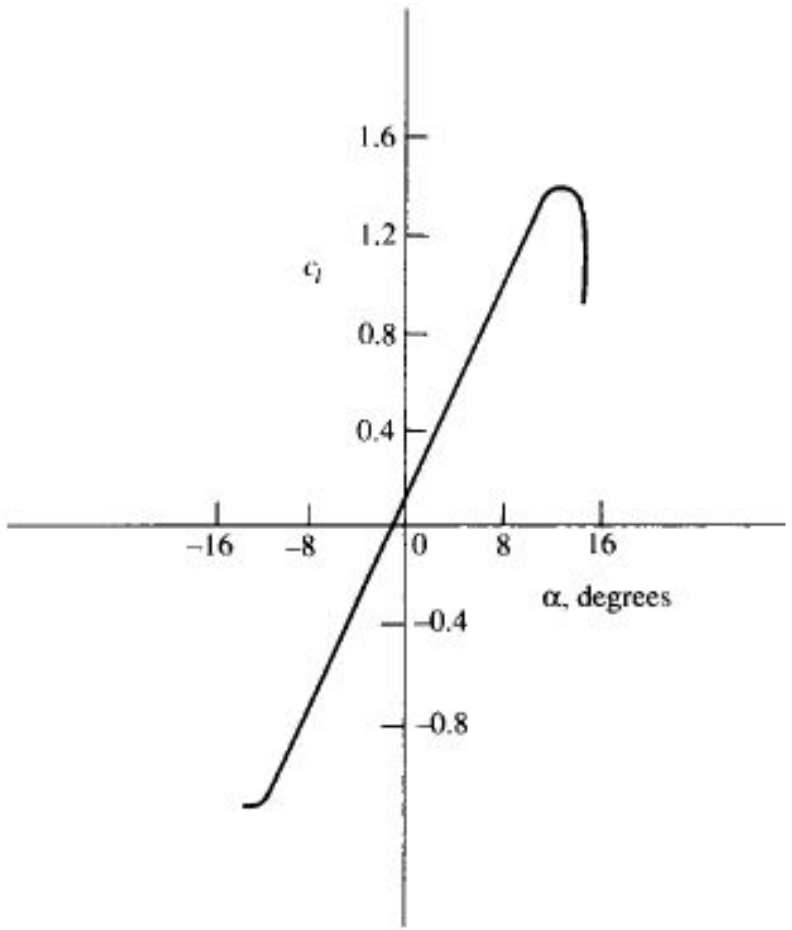



**Figure 1.61** Variation of section lift coefficient for an NACA 63-210 airfoil. Re = 3 × $10^6$. No flap deflection.

# Chapter 1: Introduction

- Lift coefficient for full aircraft:
  Fig.1.62- $C_L$-$\alpha$ curve for T38 :
  added the influence of the flap deflection angle variation.
- Lift to drag ratio for full aircraft:
  About 10. More components and the 3D flow effect of wing tip—induced drag(extra pressure drag), thus, much smaller than airfoil. For B-52 boomer, it has its maximum value 21.6.

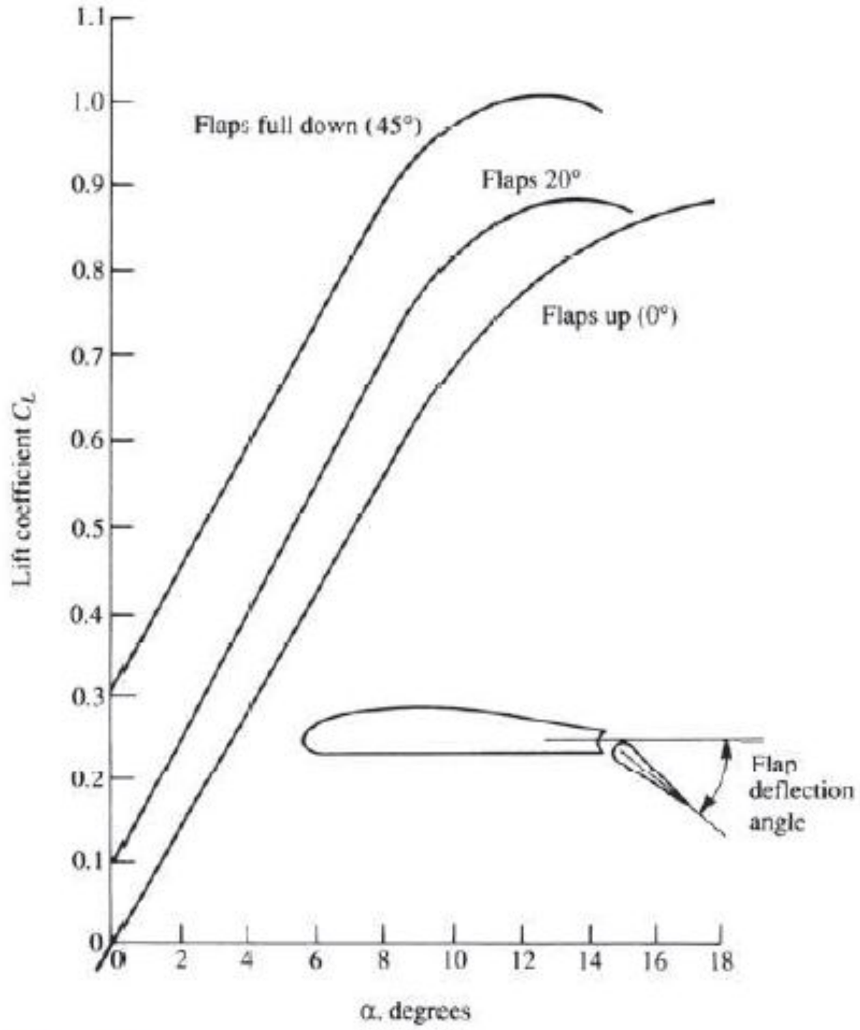



**Figure 1.62** Variation of lift coefficient with angle of attack for the T-38. Three curves are shown corresponding to three different flap deflections. Freestream Mach number is 0.4. (*Courtesy of the U.S. Air Force*).

The order of magnitude of moment coefficient :

Fig.1.63-$c_{m.c/4}$ curve for NACA63-210 airfoil. Mainly negative value, magnitude —-0.035 or so, typical one.

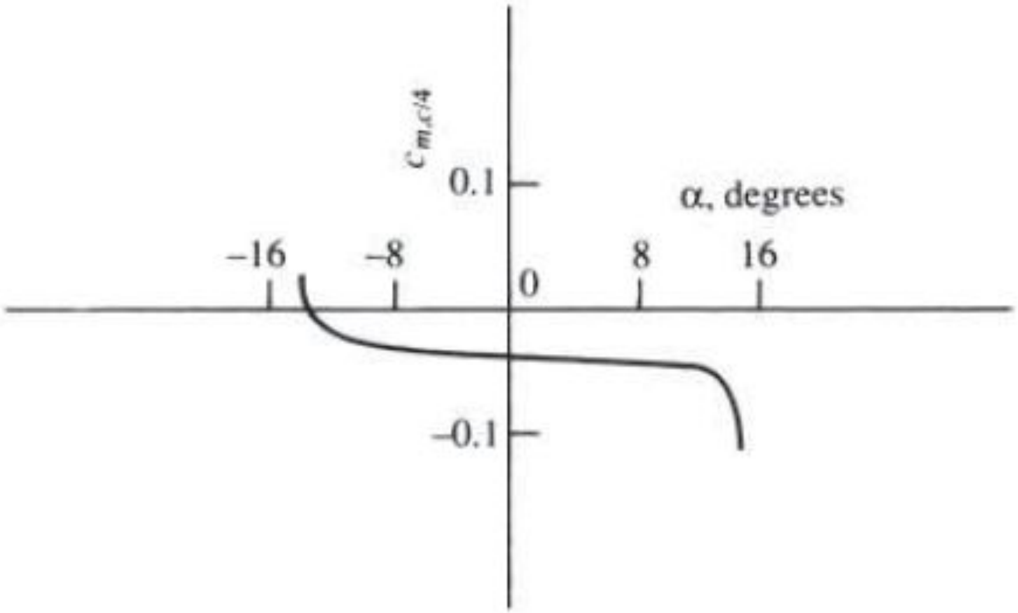



**Figure 1.63** Variation of section moment coefficient about the quarter chord for an NACA 63-210 airfoil. Re = $3 \times 10^6$.

# Chapter 1: Introduction

Homework: p1.16、p1.18

Thinking work:

Can all the drag come down to pressure drag or friction drag from the view of body itself？

# Chapter 1: Introduction

The key points and difficult points in this chapter

**Key points:**

1. Fundamental aerodynamics variables;
2. Aerodynamic forces and moments
3. Pressure center
4. Similar concepts, theorems, criterion and derived methods
5. Flow type
6. Order of  magnitudes of aerodynamic forces

**Difficult points:**

7. Concept of pressure center
8. Similar criterion and derived methods

## Summation

The normal, axial, lift, drag, and moment coefficients for an aerodynamic body can be obtained by integrating the pressure and skin friction coefficients over the body surface from the leading to the trailing edge. For a two-dimensional body,

$$c_n = \frac{1}{c}\left[\int_0^c (C_{p,l} - C_{p,u})\,dx + \int_0^c \left(c_{f,u}\frac{dy_u}{dx} + c_{f,l}\frac{dy_l}{dx}\right) dx\right] \quad (1.15)$$

$$c_a = \frac{1}{c}\left[\int_0^c \left(C_{p,u}\frac{dy_u}{dx} - C_{p,l}\frac{dy_l}{dx}\right) dx + \int_0^c (c_{f,u} + c_{f,l})\,dx\right] \quad (1.16)$$

$$c_{m_{\mathrm{LE}}} = \frac{1}{c^2}\left[\int_0^c (C_{p,u} - C_{p,l})x\,dx - \int_0^c \left(c_{f,u}\frac{dy_u}{dx} + c_{f,l}\frac{dy_l}{dx}\right) x\,dx + \int_0^c \left(C_{p,u}\frac{dy_u}{dx} + c_{f,u}\right) y_u\,dx + \int_0^c \left(-C_{p,l}\frac{dy_l}{dx} + c_{f,l}\right) y_l\,dx\right] \quad (1.17)$$

$$c_l = c_n \cos\alpha - c_a \sin\alpha \quad (1.18)$$

$$c_d = c_n \sin\alpha + c_a \cos\alpha \quad (1.19)$$

## Summation

The center of pressure is obtained from

$$x_{cp} = -\frac{M'_{LE}}{N'} \approx -\frac{M'_{LE}}{L'} \qquad (1.20) \text{ and } (1.21)$$

The criteria for two or more flows to be dynamically similar are:

1. The bodies and any other solid boundaries must be geometrically similar.
2. The similarity parameters must be the same. Two important similarity parameters are Mach number $M = V/a$ and Reynolds number $\text{Re} = \rho V c/\mu$.

If two or more flows are dynamically similar, then the force coefficients $C_L$, $C_D$, etc., are the same.


Q
A