PART 1:
FINANCIAL PLANNING

Chapter 3

Understanding the Time Value of Money

Learning Objectives

- Explain the mechanics of compounding.
- Use a financial calculator to determine the time value of money.
- Understand the power of time in compounding.
- Explain the importance of the interest rate in determining how an investment grows.
- Calculate the present value of money to be received in the future.
- Define an annuity and calculate its compound or future value.

Compound Interest and Future Values

- Compound interest is interest on interest.
- If you take interest earned on an investment and reinvest it, you earn interest on the principal and the reinvested interest.
- The amount of interest grows, or compounds.

How Compound Interest Works

- How does $100 placed in a savings account at 6% grow at the end of the year?
- $106 = 100 + 6

- $FV_1 = PV + (i)$
- FV_1 = the future value of the investment at the end of year 1
- i = the annual interest rate, based on the beginning balance and paid at the end of the year
- PV = the present value or current value in today's dollars

How Compound Interest Works

- What will the account look like at the end of the second year if the interest is reinvested?

  PV = $106

  i = 6%

- $FV_2 = FV_1 + (1 + i)^n$

  $FV_2 = 106 + (1.06) =$ $112.36

- $FV_n = PV\ (1 + i)^n$
- FV_n = the future value of the investment at the end of *n* years
- *i* = the annual interest rate, based on the beginning balance and paid at the end of the year
- *PV* = the present value or current value in today's dollars

How Compound Interest Works

- Example: You receive a $1000 academic award this year for being the best student in your personal finance course. You place it in a savings account paying 5% interest compounded annually. How much will your account be worth in 10 years?

- $FV_n = PV + i$

  PV = $1000

  i = 5%

  n = 10 years

- FV_{10} = 1628.89

The Future-Value Interest Factor

- Calculating future values by hand can be difficult.
  Use a calculator or tables.
- The future-value interest factor, found in a table, replaces the $(1 + i)^n$ part of the equation.

The Future-Value Interest Factor

- The amounts in the table represent the value of $1 compounded at rate of *i* at the end of *n*th year.
- $FVIF_{i,n}$ is multiplied by the initial investment to calculate the future value of that investment.

The Future-Value Interest Factor

- Previous example: What is the future value of investing $1000 at 5% compounded annually for 10 years?
- Using Table 3.1, look for the intersection of the n = 10 row and the 5% column.
- The FVIF = 1.629
- $1000 x 1.629 = $1629

The Rule of 72

- How long will it take to double your money?
- The Rule of 72 determines how many years it will take for a sum to double in value by dividing the annual growth or interest rate into 72.

The Rule of 72

- Example: If an investment grows at an annual rate of 9% per year, then it should take 72/9 = 8 years to double.
- Use Table 3.1 and the future-value interest factor: The FVIF for 8 years at 9% is 1.993 (or $1993), nearly the approximated 2 ($2000) from the Rule of 72 method.

Compound Interest with Nonannual Periods

- Compounding periods may not always be annually.
- Compounding may be quarterly, monthly, daily, or even a continuous basis.
- The sooner interest is paid, the sooner interest is earned on it, and the sooner the benefits or compounding is realized.
- Money grows faster as the compounding period becomes shorter.

Compounding and the Power of Time

- Manhattan was purchased in 1626 for $24 in jewelry and trinkets.
- Had that $24 been invested at 8% compounded annually, it would be worth over $120.6 trillion today.
- This illustrates the incredible power of time in compounding.

The Importance of the Interest Rate

- The interest rate plays a critical role in how much an investment grows.
- Consider the “daily double” where a penny doubles in value each day. By the end of the month, it will grow to over $10 trillion.
- Albert Einstein called compound interest “the eighth wonder of the world.”

Present Value

- Present value is the value of today's dollars of money to be received in the future.
- Present value strips away inflation to see what future cash flows are worth today.
  - Allows comparisons of dollar values from different periods.

Present Value

- Finding present values means moving future money back to the present.
- This is the inverse of compounding.
- The “discount rate” is the interest rate used to bring future money back to present.

Present Value

- $PV = FV_n[1/(1 + i)^n]$
  - PV = present value of a sum of money.
  - FV = future value of investment at the end of n years.
  - n = number of years until payment will be received.
  - i = annual discount (or interest) rate.
- The present value of a future sum of money is inversely related to both the number of years until payment will be received and the discount rate.

Present Value

- Tables can be used to calculate the $[1/(1+i)^n]$ part of the equation.
- This is the present-value interest factor (PVIF).

Present Value

- Example: What is the present value of $100 to be received 10 years from now if the discount rate is 6%?
- Using Table 3.3, $n = 10$ row and $i = 6\%$ column, the PVIF is 0.558.
  - Insert $FV_{10} = \$100$ and $PVIF_{6\%, 10\ yr} = 0.558$ into the equation.
- The value in today's dollars of $100 future dollars is $55.80.

Present Value

- Example: You have been promised $500,000 payable 40 years from now. What is the value today if the discount rate is 6%?
- $PV = FV_n(PVIF_{i\%,\, n\ yrs})$
- Using Table 3.3, n = 40 row and i = 6% column, the PVIF is 0.097.
  - Multiply the $500,000 by 0.097.
- The value in today's dollars is $48,500.

Present Value

- You've just seen that $500,000 payable 40 years from now, with a discount rate of 6%, is worth $48,500 in today's dollars.
- Conversely, if you deposit $48,500 in the bank today, earning 6% interest annually, in 40 years you would have $500,000.
- There is really only one time value money equation.

Annuities

- An annuity is a series of equal dollar payments coming at the end of each time period for a specific time period.
- Pension funds, insurance obligations, and interest received from bonds are annuities.

Compound Annuities

- A compound annuity involves depositing an equal sum of money at the end of each year for a certain number of years, allowing it to grow.
- Constant periodic payments may be for an education, a new car, or any time you want to know how much your savings will have grown by some point in the future.

Compound Annuities

- Example: You deposit $500 at the end of each year for the next 5 years. If the bank pays 6% interest, how much will you have at the end of 5 years?

- Future value of an annuity = annual payment x future value interest factor of an annuity.
- Use Table 3.6, column i = 6%, row n = 5, the FVIFA is 5.637.
- $500 x 5.637 = $2,818.50 at the end of 5 years.

Compound Annuities

- Example: You need $10,000 for education in 8 years. How much must you put away at the end of each year at 6% interest to have the college money available?
- You know the values of *n*, *i*, and FV_n, but don't know the *PMT*.
- You must deposit $1010.41 at the end of each year at 6% interest to accumulate $10,000 at the end of 8 years.

Compound Annuities

- Example: You deposit $2000 in an IRA at the end of each year, and it grows at 10% per year. How much will you have after 40 years?

- $FV_n = PMT\ (FVIFA_{i\%,\ n\ years})$
- The future value after 40 years of an annual deposit of $2000 per year is $885,160.

Present Value of an Annuity

- To compare the relative value of annuities, you need to know the present value of each.
- Use the present-value interest factor for an annuity $PFIVA_{i,n}$.

Present Value of an Annuity

- Example: You are to receive $1,000 at the end of each year for the next 10 years. If the interest rate is 5%, what is the present value?
- Using Table 3.7, row n = 10, i = 5%.
- The present value of this annuity is $7722.

Amortized Loans

- Annuities usually involve paying off a loan in equal installments over time.
- Amortized loans are paid off this way.
- Examples include car loans and mortgages.

Amortized Loans

- Example: You borrow \$6000 at 15% interest to buy a car and repay it in 4 equal payments at the end of each of the next 4 years. What are the annual payments?
- PV=\$6000, i=15%, n=4.
- Substituting into the equation the *PMT* would be \$2101.58.

Perpetuities

- A perpetuity is an annuity that continues forever.
- Every year this investment pays the same dollar amount and never stops paying.
- Present value of a perpetuity = payment/discount rate.

Perpetuities

- What is the present value of a perpetuity that pays a constant dividend of $10 per share forever, if the discount rate is 5%?

- *PV* = present value of the perpetuity
- *PP* = the annual dollar amount provided by the perpetuity.
- *i* = the annual interest (or discount) rate.
- $10/0.05 = $200