Тема:

Преобразование суммы тригонометрических функций в произведение и произведения в сумму.

Свойства четности и нечетности

$\cos(-\alpha) = \cos\alpha$

четная

$\sin(-\alpha) = -\sin\alpha$

нечетная

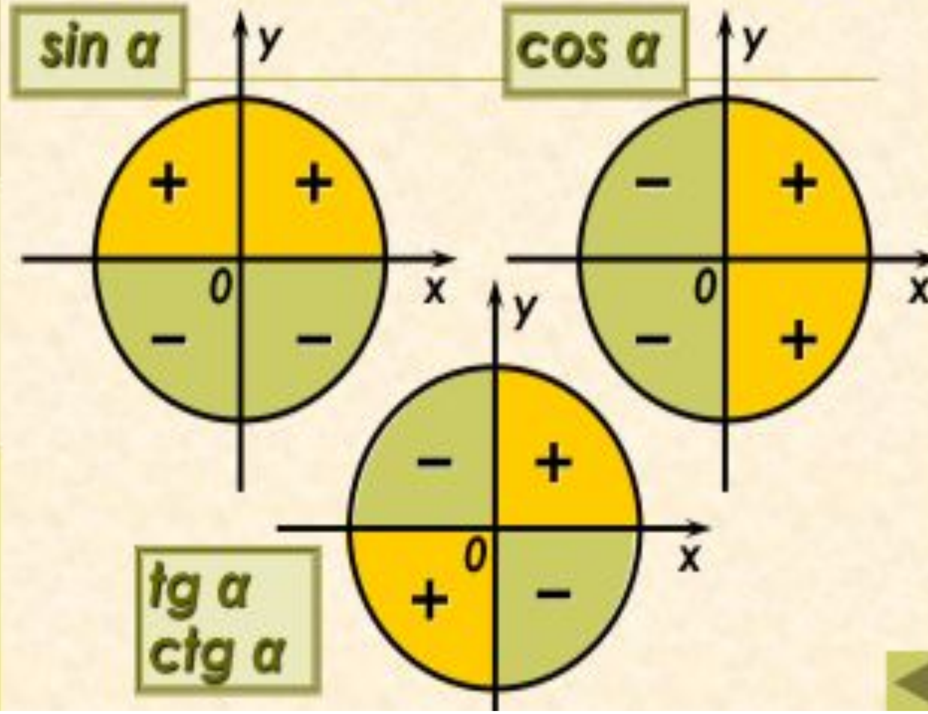
$tg(-\alpha) = -tg\,\alpha$

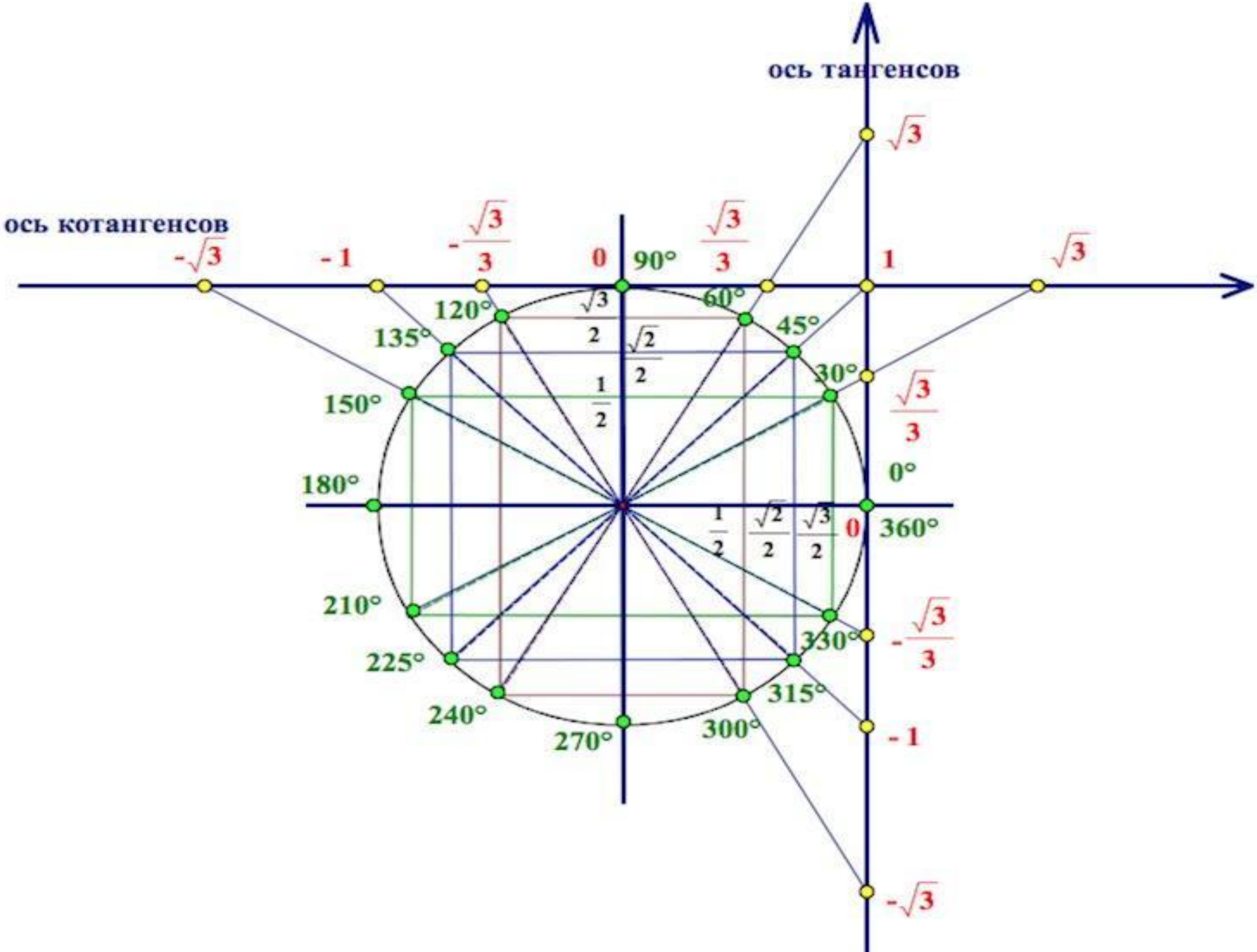
нечетная

$ctg(-\alpha) = -ctg\,\alpha$

нечетная

Знаки синуса и косинуса





ось тангенсов
ось котангенсов
$\sqrt{3}$
$-\sqrt{3}$
- 1
$-\frac{\sqrt{3}}{3}$
0
90°
$\frac{\sqrt{3}}{3}$
1
$\sqrt{3}$
120°
60°
$\frac{\sqrt{3}}{2}$
$\frac{\sqrt{2}}{2}$
135°
45°
30°
$\frac{1}{2}$
150°
$\frac{\sqrt{3}}{3}$
0°
180°
$\frac{1}{2}$
$\frac{\sqrt{2}}{2}$
$\frac{\sqrt{3}}{2}$
0
360°
210°
$-\frac{\sqrt{3}}{3}$
330°
225°
315°
240°
- 1
270°
300°
$-\sqrt{3}$

Формулы суммы

$$\mathrm{Sin}(\alpha) + \mathrm{Sin}(\beta) = 2 \cdot \mathrm{Sin}\left(\frac{\alpha + \beta}{2}\right) \cdot \mathrm{Cos}\left(\frac{\alpha - \beta}{2}\right)$$

$$\mathrm{Sin}(\alpha) - \mathrm{Sin}(\beta) = 2 \cdot \mathrm{Cos}\left(\frac{\alpha + \beta}{2}\right) \cdot \mathrm{Sin}\left(\frac{\alpha - \beta}{2}\right)$$

$$\mathrm{Cos}(\alpha) + \mathrm{Cos}(\beta) = 2 \cdot \mathrm{Cos}\left(\frac{\alpha + \beta}{2}\right) \cdot \mathrm{Cos}\left(\frac{\alpha - \beta}{2}\right)$$

$$\mathrm{Cos}(\alpha) - \mathrm{Cos}(\beta) = -2 \cdot \mathrm{Sin}\left(\frac{\alpha + \beta}{2}\right) \cdot \mathrm{Sin}\left(\frac{\alpha - \beta}{2}\right)$$

$$\mathrm{tg}(\alpha) + \mathrm{tg}(\beta) = \frac{\mathrm{Sin}(\alpha + \beta)}{\mathrm{Cos}(\alpha) \cdot \mathrm{Cos}(\beta)} \qquad \mathrm{tg}(\alpha) - \mathrm{tg}(\beta) = \frac{\mathrm{Sin}(\alpha - \beta)}{\mathrm{Cos}(\alpha) \cdot \mathrm{Cos}(\beta)}$$

$$\mathrm{ctg}(\alpha) + \mathrm{ctg}(\beta) = \frac{\mathrm{Sin}(\beta + \alpha)}{\mathrm{Sin}(\alpha) \cdot \mathrm{Sin}(\beta)} \qquad \mathrm{ctg}(\alpha) - \mathrm{ctg}(\beta) = \frac{\mathrm{Sin}(\beta - \alpha)}{\mathrm{Sin}(\alpha) \cdot \mathrm{Sin}(\beta)}$$

1. $\sin 6x + \sin 4x = 2\sin\frac{6x+4x}{2}\cos\frac{6x-4x}{2} = 2\sin 5x\cos x;$

2. $\sin 43^\circ + \sin 17^\circ = 2\sin\frac{43^\circ+17^\circ}{2}\cos\frac{43^\circ-17^\circ}{2} =$

$= 2\sin 30^\circ\cos 13^\circ = 2\cdot\frac{1}{2}\cdot\cos 13^\circ = \cos 13^\circ.$

3. $\sin 3x - \sin 5x = 2\sin\frac{3x-5x}{2}\cos\frac{3x+5x}{2} =$

$= 2\sin(-x)\cos 4x = -2\sin x\cos 4x.$

4. $\cos\frac{\pi}{8} + \cos\frac{3\pi}{8} = 2\cos\frac{\frac{\pi}{8}+\frac{3\pi}{8}}{2}\cos\frac{\frac{\pi}{8}-\frac{3\pi}{8}}{2} =$

$= 2\cos\frac{\pi}{4}\cos\left(-\frac{\pi}{8}\right) = 2\cdot\frac{\sqrt{2}}{2}\cdot\cos\frac{\pi}{8} = \sqrt{2}\cos\frac{\pi}{8}.$

5. $\cos(2x+y) - \cos(4x-y) =$

$= -2\sin\frac{(2x+y)+(4x-y)}{2}\sin\frac{(2x+y)-(4x-y)}{2} =$

$= -2\sin 3x\sin(-x+y) = 2\sin 3x\sin(x-y).$

Формулы произведения

$$\sin(\alpha)\cdot\sin(\beta)=\frac{1}{2}(\cos(\alpha-\beta)-\cos(\alpha+\beta))$$

$$\cos(\alpha)\cdot\cos(\beta)=\frac{1}{2}(\cos(\alpha-\beta)+\cos(\alpha+\beta))$$

$$\sin(\alpha)\cdot\cos(\beta)=\frac{1}{2}(\sin(\alpha-\beta)+\sin(\alpha+\beta))$$

$$\operatorname{tg}(\alpha)\cdot\operatorname{tg}(\beta)=\frac{\operatorname{tg}(\alpha)+\operatorname{tg}(\beta)}{\operatorname{ctg}(\alpha)+\operatorname{ctg}(\beta)}$$

$$\operatorname{ctg}(\alpha)\cdot\operatorname{ctg}(\beta)=\frac{\operatorname{ctg}(\alpha)+\operatorname{ctg}(\beta)}{\operatorname{tg}(\alpha)+\operatorname{tg}(\beta)}$$

$$\operatorname{ctg}(\alpha)\cdot\operatorname{tg}(\beta)=\frac{\operatorname{ctg}(\alpha)+\operatorname{tg}(\beta)}{\operatorname{tg}(\alpha)+\operatorname{ctg}(\beta)}$$

1. $$\sin 5x \cos 3x = \frac{\sin(5x + 3x) + \sin(5x - 3x)}{2} = \frac{\sin 8x + \sin 2x}{2}.$$

2. $$\sin 3x \cos 5x = \frac{\sin(3x + 5x) + \sin(3x - 5x)}{2} =$$
$$= \frac{\sin 8x + \sin(-2x)}{2} = \frac{\sin 8x - \sin 2x}{2}.$$

3. $$\sin 27^\circ \sin 57^\circ = \frac{\cos(27^\circ - 57^\circ) - \cos(27^\circ + 57^\circ)}{2} =$$
$$= \frac{\cos(-30^\circ) - \cos 84^\circ}{2} = \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \cos 84^\circ\right).$$

$$\mathrm{Sin}(\alpha)\cdot\mathrm{Sin}(\beta) = \frac{1}{2}(\mathrm{Cos}(\alpha-\beta) - \mathrm{Cos}(\alpha+\beta))$$

$$\mathrm{Cos}(\alpha)\cdot\mathrm{Cos}(\beta) = \frac{1}{2}(\mathrm{Cos}(\alpha-\beta) + \mathrm{Cos}(\alpha+\beta))$$

$$\mathrm{Sin}(\alpha)\cdot\mathrm{Cos}(\beta) = \frac{1}{2}(\mathrm{Sin}(\alpha-\beta) + \mathrm{Sin}(\alpha+\beta))$$