BBA182 Applied Statistics
Week 5 (2) Probabilities

DR SUSANNE HANSEN SARAL

EMAIL: SUSANNE.SARAL@OKAN.EDU.TR

HTTPS://PIAZZA.COM/CLASS/IXRJ5MMOX1U2T8?CID=4#

WWW.KHANACADEMY.ORG

Where do probabilities come from?

Two different ways to determine probabilities:

1. **Objective approach:**
   a. **Relative frequency approach,** derived from historical data
   b. **Classical or logical approach** based on logical observations, ex. Tossing a fair coin

2. **Subjective approach**, based on personal experience

Types of Probability
Relative frequency approach

Objective Approach:

a) *Relative frequency*

- We calculate the relative frequency (percent) of the event:

$$P\text{ (event)} = \frac{\text{Number of occurrences of the event}}{\text{Total number of trials or outcomes}}$$

Objective probability –
The Relative Frequency

Hospital Unit	Number of Patients	Relative Frequency
Cardiac Care	1,052	11.93 %
Emergency	2,245	25.46 %
Intensive Care	34	3.86 %
Maternity	552	6.26 %
Surgery	4,630	52.50 %
Total:	**8,819**	**100.00 %**

$$P\text{ (cardiac care)} = \frac{1052}{8819}$$

1052 ← Number of patients admitted to cardiac care

8819 ← Total number of patient admitted to the hospital

Objective probability assessment –
...pproach

Ex

The 2 probability rules are satisfied:

Individual probabilities are all between 0 and 1

$0 \leq P\,(\text{event}) \leq 1$

Total of all event probabilities equals 1

$\sum P\,(\text{event}) = 1.00$

H		...elative
C...		
E...		
I...		
Maternity	552	6.26 %
Surgery	4,630	52.50 %
Total:	**8,819**	**100.00 %**

Types of Probability
Classical approach

Objective Approach:

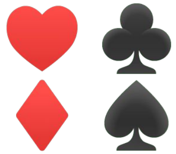
b) ***Classical approach:***

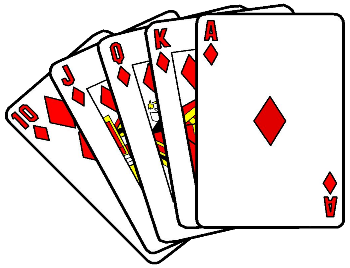
$$P\text{ (head)} = \frac{1}{2}$$

- 1 ← Number of ways of getting a head
- 2 ← Number of possible outcomes (head or tail)

$$P\text{ (spade)} = \frac{13}{52}$$

- 13 ← Number of chances of drawing a spade
- 52 ← Number of possible outcomes





Subjective approach to assign probabilities

We use the **subjective approach :**

- No possibility to use the classical approach nor the relative frequency approach.
- No historic data available
- New situation that nobody has been in so far

The probability will differ between two people, because it is subjective.

Types of Probability

Subjective Approach:

Based on the experience and judgment of the person making the estimate:

- Opinion polls (broad public)
- Judgement of experts (professional judgement)
- Personal judgement

Interpreting probability

No matter what method is used to assign probabilities, we interpret the probability, using the relative frequency approach for an **infinite** number of experiments.

The probability is only an estimate, because the relative frequency approach defines probability as the "long-run" relative frequency.

The larger the number of observations the better the estimate will become.

Ex.: Tossing a coin, birth of a baby, etc.

Head and tail will only occur 50 % in the long run

Girl and boy will only occur 50 % in the long run

Probability rules continued
Rule 1 and 2

1. If A is any event in the sample space S, then a probability is a number between 0 and 1

$$0 \leq P(A) \leq 1$$

2. The probability of the set of all possible outcomes must be 1

P(S) = 1 **P(S) = Σ P(Oi) = 1 ,** where **S** is the sample space

Objective probability assessment – … pproach

Ex…

Individual probabilities are all between 0 and 1

$0 \leq P\ (\text{event}) \leq 1$

Total of all event probabilities equals, S

$P(s) = \sum P\ (\text{event, O}) = 1.00$

H…		…elative
C…		
E…		
In…		
Maternity	552	6.26 %
Surgery	4,630	52.50 %
Total:	**8,819**	**100.00 %**

Probability rules. Rule 3
Complement rule

Suppose the probability that you win in the lottery is 0.1 or 10 %.

What is the probability then that you don't win in the lottery?

Probability rules. Rule 3
Complement rule

The set of outcomes that are not in the event A, but are in the sample space is called the “complement” of event A and is denoted $\bar{A}$.

The probability of an event occurring is 1 minus the probability that it does not occur:

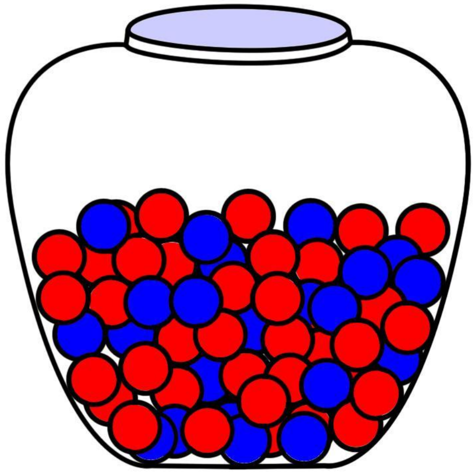
$$P(A) = 1 - P(\bar{A})$$

Remember: Sample space , S = 1.0

Probability rule 4

Multiplication rule – calculating joint probabiliti

Independent events



$$P(A \cap B) = P(A) \times P(B)$$

A jar contains 5 red marbles and 5 blue marbles, a total of 10 marbles.

Two marbles will be randomly chosen, one at a time. The first marble will be put back into the jar after the first trial (with replacement), then the second marble is picked.

What is the probability that 2 red marbles will be randomly chosen in two consecutive trials?

Multiplication Rule for **independent events**

(continued)

The probability to chose two red marbles in a row is the same, because we put the red marble back into the jar.

That is:

First trial: $P(R) = \frac{5}{10}$

Second trial: $P(RR) = \frac{5}{10}$

Therefore:

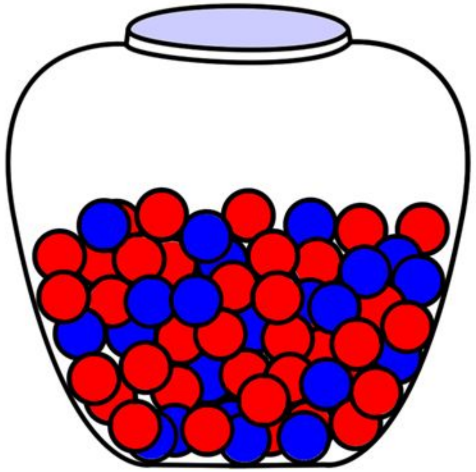
$$P(R \cap RR) = P(R)P(RR) = \left(\frac{5}{10}\right)\left(\frac{5}{10}\right) = \frac{25}{100} = .25$$

The probability that two red marbles are chosen consecutively is 25 %.

Independent events

Events are independent from each other when the probability of occurrence of the first event ***does not affect*** the probability of occurrence of the second event.

The probability of occurrence of the second event will be the same as for the first event.



lultiplication rule – calculating joint probabilities
Dependent events

$$P(A \cap B) = P(A) \times P(B|A)$$

A jar contains 5 red marbles and 5 blue marbles a total of 10 marbles.

Two marbles will be randomly chosen one at a time. The first marble will **not** be replaced after it has been removed from the jar.

What is the probability that 2 red marbles will randomly be chosen?

Multiplication rule – Dependent events

(continued)

Let, R represent the event that the **first** marble chosen is red

RR the event that the **second** marble chosen is also red.

We want to calculate the joint probability:

$$P(R \cap RR) = P(R)P(RR|R)$$

There are 5 red marbles in the jar out of ten, the probability that the first marble chosen is

red: $P(R) = \frac{5}{10}$

Multiplication Rule - *Dependent events*

(continued)

After the first marble is chosen, there are only nine marbles left. Given that the first marble chosen is red, there are now only 4 red marbles left in the jar. It follows that:

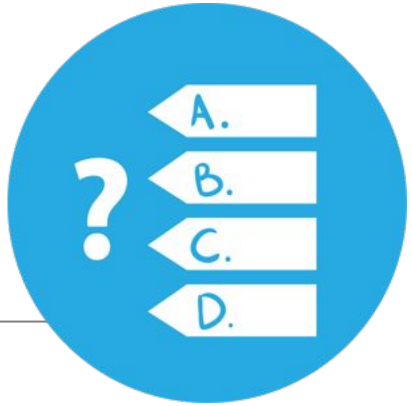
$$P(RR|R) = \frac{4}{9}$$

Thus the joint probability $P(R \cap RR)$ is:

$$P(R \text{ and } RR) = P(R)P(RR|R) = \left(\frac{5}{10}\right)\left(\frac{4}{9}\right) = \frac{20}{90} = .22 \text{ or } 22\%$$

The probability that two red marbles are chosen consecutively is 22 %

ultiple choice quiz: 1 correct 3 false



You are going to take a multiple choice exam. You did not have time to study and will therefore guess. The questions are **independent** from each other.

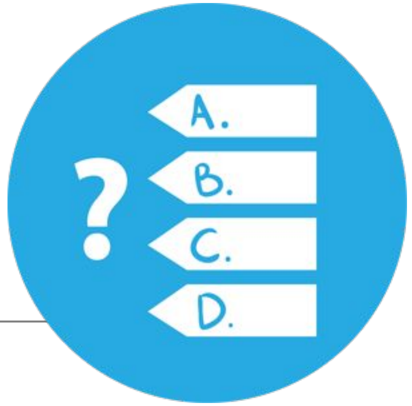
There are 5 multiples choice questions with 4 alternative answers. Only one answer is correct.

What is the probability that you will pick the right answer out of the 4 alternatives?

What is the probability that you will pick the wrong answer out of the 4 alternatives?

What is the probability that you will pick two answers correctly? What is the probability of picking two wrong answers? What is the probability that you will pick all the correct answers out of the 5 questions? What is the probability that you will pick all wrong answers out of the 5 questions?

ultiple choice quiz: 1 correct 3 false



You are going to take a multiple choice exam. You did not have time to study and will therefore guess. The questions are **independent** from each other.

There are 5 multiples choice questions with 4 alternative answers. Only one answer is correct.

What is the probability that you will pick the right answer out of the 4 alternatives? **P =** $\frac{1}{4} = 0.25$

What is the probability that you will pick the wrong answer out of the 4 alternatives? **P =** $\frac{3}{4} = 0.75$

What is the probability that you will pick two answers correctly? **P =** $(\frac{1}{4})(\frac{1}{4}) = 0.06$

What is the probability of picking two wrong answers? **P=** $(\frac{3}{4})(\frac{3}{4}) = .56$

What is the probability that you will pick all the correct answers out of the 5 questions? **P=**$(\frac{1}{4})^5 = 0.001$

What is the probability that you will pick all wrong answers out of the 5 questions? **P =** $(\frac{3}{4})^5 = .24$

Probability Rule 5: Addition rule for *mutually exclusive* events

A and B are **mutually exclusive events** in a sample space.

Then the probability of the union A ∪ B is the sum of their individual probabilities:

$$P(A \cup B) = P(A) + P(B)$$

Probability rule 5:
Addition rule for *mutually exclusive* events
Example

An online shop, **Netpoint**, a clothing retailer, receives 1’000 visits on a particular day. From past experience it has been determined that every 1’000 hits, result in 10 large sales of at least $ 500 and 100 sales of less than $ 500.

Assuming that all hits have the same probability of a sale = $\frac{1}{1000}$

(1) What is the probability of a large sale from a particular hit?

(2) What is the probability of a small sale?

(3) What is the probability of **any** sale?

ddition rule of ***mutually exclusive*** events: Example – **Definition of events**

Our single hit is selected over a total hit of 1’000 on a particular day.

Let,

Event A: «selected hit results in a large sale» = 10

Event B: «selected hit results in a small sale» = 100

Event A ∪ B : Hit results in any sale = 110

What is P(A) ?

What is P(B) ?

What is the probability of any **sale**, A ∪ B (A and B are mutually exclusive events)?

Addition rule of ***mutually exclusive*** events: Example - **Solution**

P(A) = $\frac{N_A}{N}$ = $\frac{10}{1000}$ = 0.01 or 1 %

P(B) = $\frac{N_B}{N}$ = $\frac{100}{1000}$ = 0.10 or 10 %

P(A ∪ B) = $\frac{N_A+N_B}{1000}$ = $\frac{110}{1000}$ = 0.11 or 11 % (mutually exclusive)

Addition rule of ***mutually exclusive*** events: Class exercise

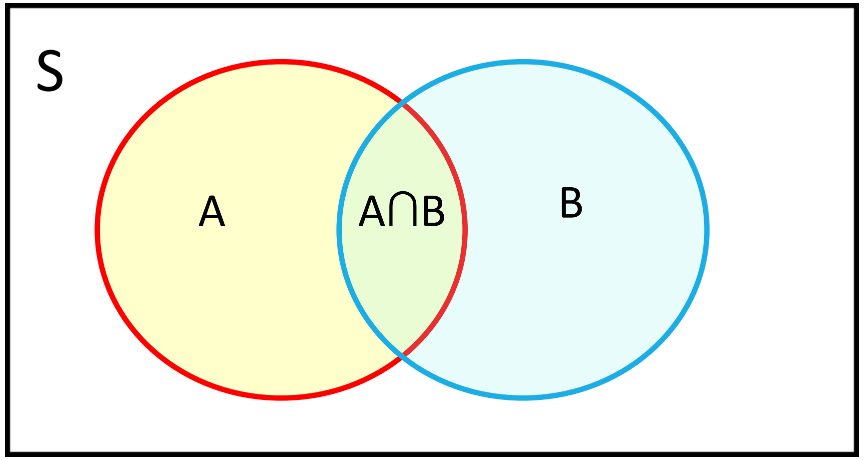
A corporation receives a shipment of 100 units of computer chips from a manufacturer.

Research indicates the probabilities of defective parts per shipment shown in the following table:

Number	0	1	2	3	> 3 defective
Probability	0.29	0.36	0.22	0.1	0.03

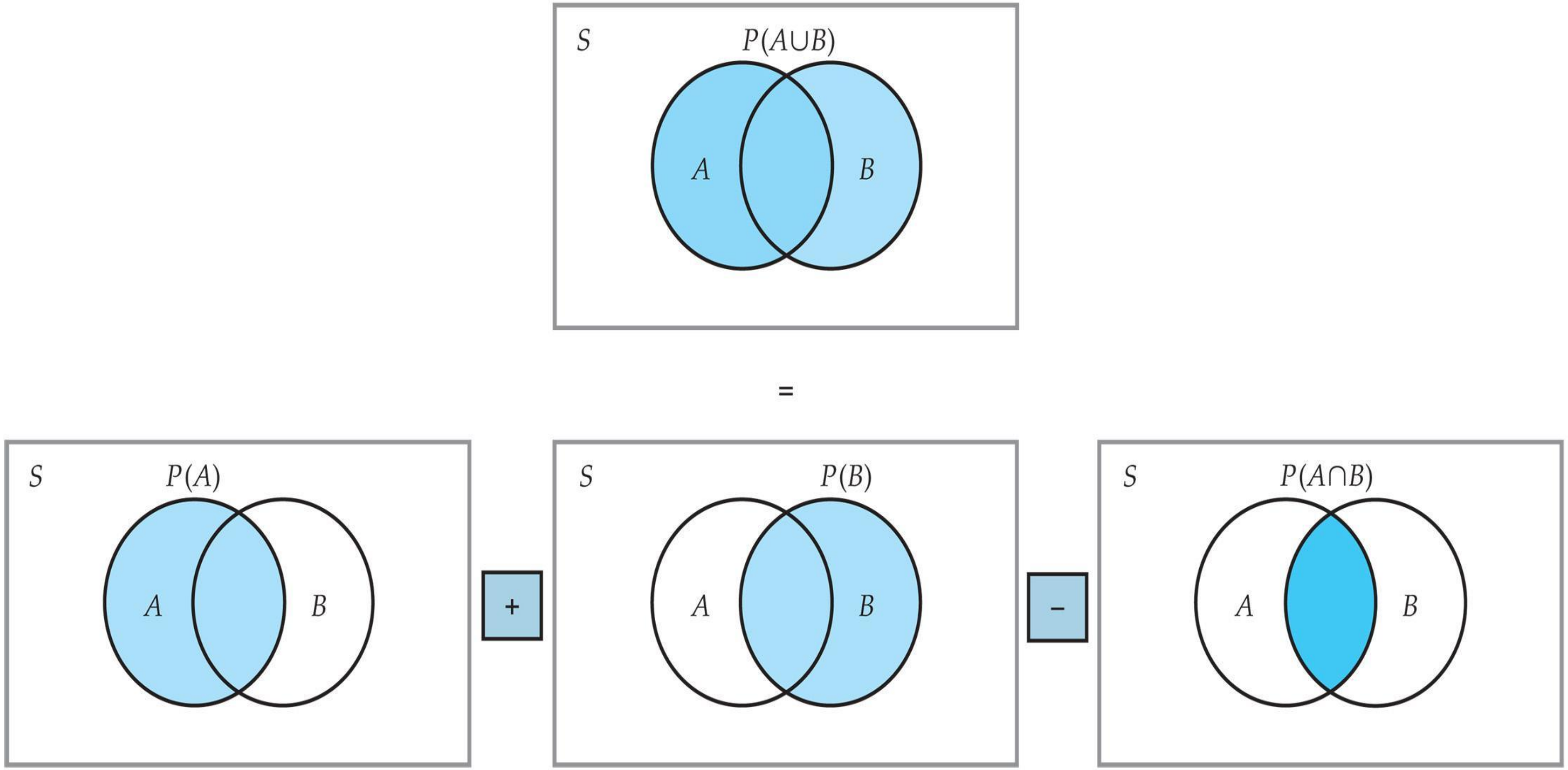
a. What is the probability that there will be fewer than three defective parts in a shipment? $P(x < 3)$
b. What is the probability that there will be more than one defective part in a shipment? $P(x > 1)$
c. The five probabilities in the table sum up to 1. Why must this be so?

Probability rule 6: Addition rule for ***non-mutually exclusive events***



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Probability rule 6: Addition rule for ***non-mutually exclusive*** events



Addition rule of ***mutually non-exclusive*** events rolling a dice

S = [1, 2, 3, 4, 5, 6]

A = [2, 4, 6]

B = [4, 5, 6]

$$A \cup B = [2, 4, 5, 6]$$

Addition rule of *mutually non-exclusive* events:

Example: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

A video store owner finds that 30 % of the customers entering the store ask an assistant for help, and that 20 % of the customers buy a video before leaving the store.

It is also found that 15 % of all customers both ask for assistance and make a purchase.

What is the probability that a customer does at least one of these two things?

Addition rule of *non-mutually exclusive* events:
Example:

A video store owner finds that 30 % of the customers entering the store ask an assistant for help, and that 20 % of the customers buy a video before leaving the store. It is also found that 15 % of all customers both ask for assistance and make a purchase.

What is the probability that a customer does at least one of these two things?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Addition rule of *non-mutually exclusive* events:

P(A U B) = P(A) + P(B) – P(A ∩ B) Class exercise

It was estimated that 30 % of all students in their 4^{th} year at a university campus were concerned about employment future. 25 % were seriously concerned about grades, and 20 % were seriously concerned about both.

What is the probability that a randomly chosen 4^{th} year student from this campus is seriously concerned with at least one of these two concerns?

Class exercise - solution

Let P(A) be the probability of the event A: The student is concerned about employment prospect: 30 %

Let P(B) be the probability of the event B: The student is concerned about the final grade, 25 %

$P(A \cap B)$ the probability of both: 20 %

$P(A) = 30\ \%$; $P(B) = 25\ \%$;

$P(A \cap B) = 20\%$

Addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= .30 + .25 - .20 = .35$$

Calculating probabilities of complex events

Now we will look at how to calculate the probability of more complex events from the probability of **related events**.

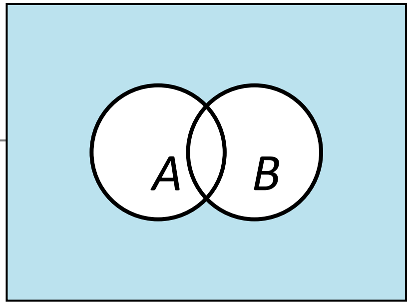
Example:

Probability of tossing a 3 with two dices is 2/36.

This probability is derived by combining two possible events:

tossing a 1 (1/36) and tossing a 2 (1/36)

How to calculate probabilities of intersecting events



Intersection of Events A and B - Events that are **not mutually exclusive**

The **intersection of events A and B** is the event that occurs when both A and B occur. It is denoted as

A and B ($A \cap B$)

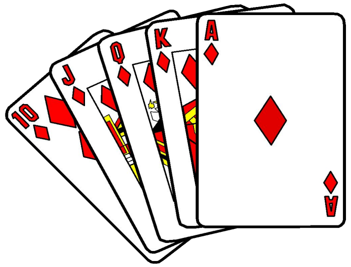
The probability of the intersection is called the **joint probability: P(A∩B)**

Drawing a Card – not mutually exclusive

Draw one card from a deck of 52 playing cards

A = event that a 7 is drawn

B = event that a heart is drawn



P (a 7 is drawn) = $P(A) = {}^{4}/_{52} = {}^{1}/_{13}$

P (a heart is drawn) = $P(B) = {}^{13}/_{52} = {}^{1}/_{4}$

- These two events are **not mutually exclusive** since a 7 of hearts can be drawn
- These two events are **not collectively exhaustive** since there are other cards in the deck besides 7s and hearts

Joint probabilities - A business application

A manufacturer of computer hardware buys microprocessors chips to use in the assembly process from two different manufacturers A and B.

Concern has been expressed from the assembly department about the reliability of the supplies from the different manufacturers, and a rigorous examination of last month's supplies has recently been completed with the results shown:

Manufacturer of computer hardware- Contingency table - joint probabilities

	Manufacturer A	Manufacturer B	Total
Chips found to be:			
Satisfactory	5828	3752	9580
Defective	119	198	317
Total:	5947	3950	9897

Manufacturer of computer hardware

Contingency table joint probabilities

It looks as if the assembly department is correct in expressing concern. Manufacturer B is supplying a smaller quantity of chips in total but more are found to be defective compared with Manufacturer A.

However, let us consider this in the context of the probability principles we have developed:

Relative frequency method (based on available data)

Manufacturer of computer hardware

Marginal probabilities

Let us consider the total of 9897 as a sample. Suppose we had chosen one chip at random from this sample. The following events and their probabilities can then be obtained:

Find the probability of the following – marginal probabilities :

Event A: the chip was supplied by Manufacturer A

Event B: the chip was supplied by Manufacturer B

Event C: the chip was satisfactory

Event D: the chip was defective

Marginal probabilities

	Manufacturer A	Manufacturer B	Total
Chips found to be:			
Satisfactory	0.589	0.379	0.968
Defective	0.012	0.020	0.032
Total:	0.601	0.399	1.000

Manufacturer of computer hardware

Joint probabilities

(Continued)

Let us consider the total of 9897 as a sample. Suppose we had chosen one chip at random from this sample. The following joint events and their probabilities can be obtained:

And the joint probabilities:

P(A and C) supplied by A and satisfactory

P(B and C) supplied by B and satisfactory

P(A and D) Supplied by A and defective

P(B and D) supplied by B and defective

Joint probabilities

	Manufacturer A	Manufacturer B	Total
Chips found to be:			
Satisfactory	0.589	0.379	0.968
Defective	0.012	0.020	0.032
Total:	0.601	0.399	1.000

Interpretation of the *joint probabilities* in the example

The joint probability that a chip is defective ***and*** that it is delivered from Manufacturer A is 0.012

The joint probability that a chip is satisfactory ***and*** it is delivered by Manufacturer A is 0.589

The probability that a chip is satisfactory ***and*** it is delivered by Manufacturer B is 0.379

The probability that a chip is defective ***and*** it is delivered by Manufacturer B is 0.020

OKAN ÜNİVERSİTESİ İSTANBUL

otations for the *marginal and joint events*

Let,

A1 = The chip is delivered from Manufacturer A

A2 = The chip is delivered from Manufacturer

B1 = The chip is satisfactory

B2 = The chip is defective

Therefore:

P(A1 ∩ B1) = .589

P(A2 ∩ B1) = .379

P(A1 ∩ B2) = .012

P(A2 ∩ B2) = .029

	Manuf. A, A1	**Manuf B, A2**	**Total**
Chips found to be:			
Satisfactory, B1	0.589 (A1∩ $B1$)	0.379 (A2∩ $B1$)	0.968 (B1)
Defective, B2	0.012 (A1∩ $B2$)	0.020 (A2∩ $B2$)	0.032 (B2)
Total:	0.601(A1)	0.399 (A2)	1.000

Marginal probabilities

The joint probabilities in the table allow us to calculate marginal probabilities:

Marginal probabilities, computed by adding across rows or down columns. They are called so, because they are calculated in the margin of the table:

Example:

The chip was delivered by manufacturer A:

Marginal probability P(A1) : P(A1 ∩ B1) + P (A1 ∩ B2) = .589 + .012 = .601

	Manuf. A, A1	**Manuf B, A2**	**Total**
Chips found to be:			
Satisfactory, B1	0.589 ($A1 \cap B1$)	0.379 ($A2 \cap B1$)	0.968 (B1)
Defective, B2	0.012 ($A1 \cap B2$)	0.020 ($A2 \cap B2$)	0.032 (B2)
Total:	0.601(A1)	0.399 (A2)	1.000

The following contingency table shows opinion about global warming among U.S. adults, broken down by political party affiliation.

		Opinion on Global warming		
		Nonissue	Serious concern	Total
Political party	Democratic	85	415	**500**
	Republican	290	210	**500**
	Independent	70	130	**200**
	Total	**445**	**755**	**1200**

		Opinion on Global warming		
		Nonissue	Serious concern	Total
Political party	Democratic	85	415	**500**
	Republican	290	210	**500**
	Independent	70	130	**200**
	Total	**445**	**755**	**1200**

A) What is the probability that a U.S. adult selected at random believes that global warming is a serious problem?

B) What type of probability did you find in part A? (marginal or joint probability)

C) What is the probability that a U.S. adult selected at random is a Republican and believes that global warming is a serious issue?

D) What type of probability did you find in part C?

		Opinion on Global warming		
		Nonissue	Serious concern	Total
	Democratic	7%	35%	**42%**
	Republican	24%	18%	**5%**
Political party	Independent	6%	11%	**17%**
	Total	**37%**	**63%**	**100%**

A) What is the probability that a U.S. adult selected at random believes that global warming is a serious problem? 63 %

B) What type of probability did you find in part A? (marginal or joint probability) Marginal probability

C) What is the probability that a U.S. adult selected at random is a Republican and believes that global warming is a serious issue? 18 %

D) What type of probability did you find in part C? Joint probability

A Probability Table

Marginal probabilities and **joint probabilities** for two events A and B are summarized in this table:

	B	$\overline{B}$	
A	$P(A \cap B)$	$P(A \cap \overline{B})$	$P(A)$
$\overline{A}$	$P(\overline{A} \cap B)$	$P(\overline{A} \cap \overline{B})$	$P(\overline{A})$
	$P(B)$	$P(\overline{B})$	$P(S) = 1.0$