

Лекция 10

Волны в связанных системах

1. Представление связанных осцилляторов
 - а. Волновод а анизотропной плазмой в представлении связанных осцилляторов*
 - б. Представление связанных осцилляторов для магнитоактивной плазмы*
 - в. Представление связанных осцилляторов в общем случае*
2. Пучково-плазменная система в представлении связанных осцилляторов
3. Основные уравнения электроники высоких частот
 - а. Общий вид уравнений. Дисперсионное уравнение*
 - б. Одночастичные и коллективные процессы*
 - в. Метод медленных амплитуд при одночастичных и коллективных процессах*
4. Волны и взаимодействие волн в периодических структурах
 - а. Поперечные электромагнитные волны в среде с периодической диэлектрической проницаемостью*
 - б. Обобщение на случай волн произвольной природы*
 - в. Брэгговское отражение*

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial z^2} \right) A_E(t, z) = k_{\perp} c \frac{\partial A_P}{\partial t},$$

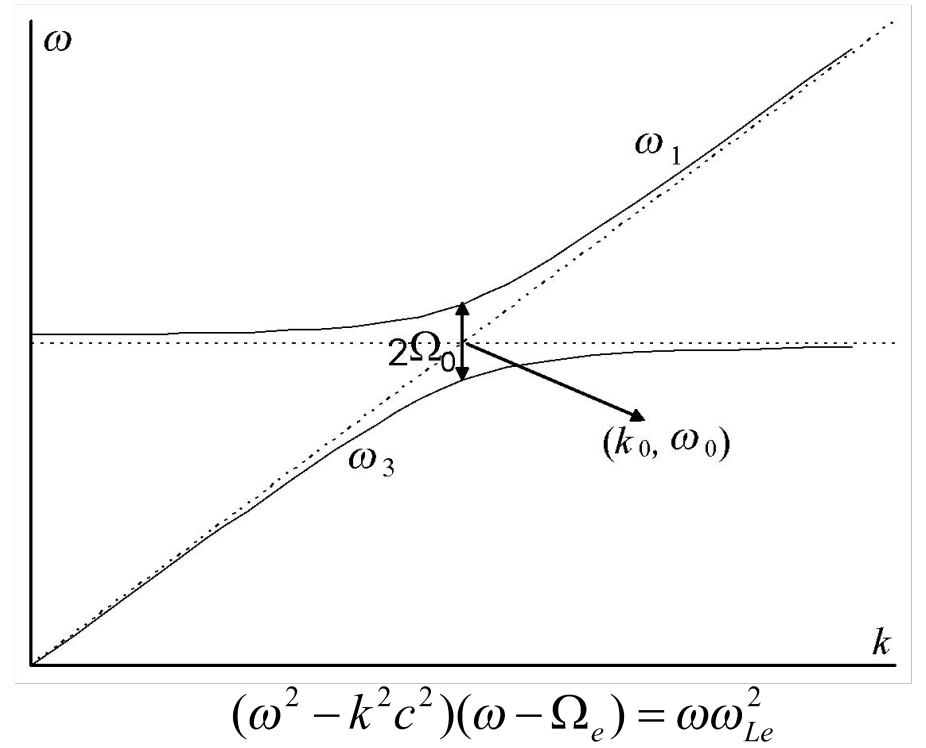
$$\left(\frac{\partial^2}{\partial t^2} + \omega_{Le}^2 \right) A_P(t, z) = -k_{\perp} c \frac{\partial A_E}{\partial t}.$$

$$D_E(\omega, k) \equiv -\omega^2 + k^2 c^2 = 0, \quad \omega_0 = \omega_p, \quad k_0 = \omega_{Le}/c$$

$$D_P(\omega, k) \equiv -\omega^2 + \omega_{Le}^2 = 0.$$

$$\omega = \omega_0 + \Omega, \quad |\Omega| \ll \omega_0$$

$$\Omega^2 = \frac{1}{4} k_{\perp}^2 c^2 \equiv \Omega_0^2 \rightarrow \Omega_{1,2} = \pm \Omega_0$$



$$A_E(t, z) = \tilde{A}_E(t) \exp(-i\omega_0 t + ik_0 z),$$

$$A_P(t, z) = \tilde{A}_P(t) \exp(-i\omega_0 t + ik_0 z).$$

$$\frac{d\tilde{A}_E}{dt} = \Omega_0 \tilde{A}_P, \quad \frac{d\tilde{A}_P}{dt} = -\Omega_0 \tilde{A}_E.$$

$$\tilde{A}_E(t=0) = A_{E0}, \quad \tilde{A}_P(t=0) = A_{P0}$$

$$A(\Omega) = \int_0^{\infty} A(t) \exp(i\Omega t) dt$$

$$A(t) = \frac{1}{2\pi} \int_{C(\Omega)} A(\Omega) \exp(-i\Omega t) d\Omega$$

$$i\Omega \tilde{A}_E(\Omega) + \Omega_0 \tilde{A}_P(\Omega) = -A_{E0},$$

$$i\Omega \tilde{A}_P(\Omega) - \Omega_0 \tilde{A}_E(\Omega) = -A_{P0}.$$

$$\tilde{A}_E(t) = A_{E0} \cos \Omega_0 t + A_{P0} \sin \Omega_0 t = |A_0| \sin(\Omega_0 t + \varphi_0),$$

$$\tilde{A}_P(t) = A_{P0} \cos \Omega_0 t - A_{E0} \sin \Omega_0 t = |A_0| \cos(\Omega_0 t + \varphi_0),$$

$$D_1(\hat{\omega}, \hat{k}) A_1(t, z) = S_1(\hat{\omega}, \hat{k}) A_2(t, z)$$

$$A_1(t, z) = \tilde{A}_1(t) \exp(-i\omega_0 t + ik_0 z)$$

$$D_2(\hat{\omega}, \hat{k}) A_2(t, z) = S_2(\hat{\omega}, \hat{k}) A_1(t, z)$$

$$A_2(t, z) = \tilde{A}_2(t) \exp(-i\omega_0 t + ik_0 z)$$

$$D_{1,2}(\hat{\omega}, k_0) = D_{1,2}\left(\omega_0 + i \frac{d}{dt}, k_0\right) \approx D_{1,2}(\omega_0, k_0) + i \frac{\partial D_{1,2}}{\partial \omega} \frac{d}{dt} = i \frac{\partial D_{1,2}}{\partial \omega} \frac{d}{dt},$$

$$S_{1,2}(\hat{\omega}, k_0) = S_{1,2}\left(\omega_0 + i \frac{d}{dt}, k_0\right) \approx S_{1,2}(\omega_0, k_0).$$

$$\frac{d\tilde{A}_1(t)}{dt} = -iS_1(\omega_0, k_0) \left[\frac{\partial D_1(\omega_0, k_0)}{\partial \omega} \right]^{-1} \tilde{A}_2(t),$$

$$\tilde{A}_{1,2}(t) = B_{1,2} \exp(-i\Omega t)$$

$$\frac{d\tilde{A}_2(t)}{dt} = -iS_2(\omega_0, k_0) \left[\frac{\partial D_2(\omega_0, k_0)}{\partial \omega} \right]^{-1} \tilde{A}_1(t).$$

$$\Omega^2 = S_1 S_2 \left(\frac{\partial D_1}{\partial \omega} \frac{\partial D_2}{\partial \omega} \right)^{-1} \equiv \Omega_0^2$$

$$\tilde{A}_1(t=0) = A_{10}, \quad \tilde{A}_2(t=0) = A_{20}$$

$$B_2 = \frac{1}{\Omega} S_2 \left(\frac{\partial D_2}{\partial \omega} \right)^{-1} B_1$$

$$\tilde{A}_1(t) = a \exp(-i\Omega_0 t) + b \exp(i\Omega_0 t)$$

$$\tilde{A}_2(t) = S_2 (\partial D_2 / \partial \omega)^{-1} \Omega_0^{-1} (a \exp(-i\Omega_0 t) - b \exp(i\Omega_0 t))$$

$$a = \frac{1}{2} \left(A_{10} + \Omega_0 S_2^{-1} (\partial D_2 / \partial \omega) A_{20} \right), \quad b = \frac{1}{2} \left(A_{10} - \Omega_0 S_2^{-1} (\partial D_2 / \partial \omega) A_{20} \right)$$

$$(\omega^2 - \omega_{Le}^2)((\omega - ku)^2 - \omega_{Lb}^2) - \omega_{Le}^2 \omega_{Lb}^2 = 0$$

$$\left(\left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial z} \right)^2 + \omega_{Lb}^2 \right) j_b = -\omega_{Lb}^2 j_e,$$

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{Le}^2 \right) j_e = -\omega_{Le}^2 j_b.$$

$$\left(\left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial z} \right)^2 + \omega_{Lb}^2 \right) A_b = -\omega_{Lb}^2 S_b(\hat{\omega}, \hat{k}) A_w,$$

$$D_w(\hat{\omega}, \hat{k}) A_w = \omega_w^2 S_w(\hat{\omega}, \hat{k}) A_b.$$

$$D_w(\omega, k)((\omega - ku)^2 - \omega_{Lb}^2) - \omega_w^2 \omega_{Lb}^2 S_w(\omega, k) S_b(\omega, k) = 0$$

$$D_w(\omega, k) = 0$$

$$\omega_0/k_0 \approx u < c$$

$$\omega = \omega_0 + \Omega, \quad |\Omega| \ll \omega_0$$

$$D_b(\omega, k) \equiv (\omega - ku)^2 - \omega_{Lb}^2 = 0$$

$$\Omega(\Omega^2 \pm 2\Omega\omega_{Lb}) = \omega_w^2 \omega_{Lb}^2 S_w(\omega_0, k_0) S_b(\omega_0, k_0) \left(\frac{\partial D_w(\omega_0, k_0)}{\partial \omega} \right)^{-1} \equiv \tilde{\Omega} \omega_{Lb}^2$$

$$|\Omega| \gg \omega_{Lb} \rightarrow \Omega_m = \delta_m (\tilde{\Omega} \omega_{Lb}^2)^{1/3}, \quad m = 1, 2, 3$$

$$|\Omega| \ll \omega_{Lb} \rightarrow \Omega_{1,2} = \pm i \left(\frac{1}{2} \tilde{\Omega} \omega_{Lb} \right)^{1/2}$$

$$\frac{d^2 \tilde{A}_b}{dt^2} \boxtimes 2i\omega_{Lb} \frac{d\tilde{A}_b}{dt} = -\omega_{Lb}^2 S_b(\omega_0, k_0) \tilde{A}_w,$$

$$\frac{d\tilde{A}_b}{dt} = \boxtimes i \frac{1}{2} \omega_{Lb} S_b(\omega_0, k_0) \tilde{A}_w,$$

$$\frac{d\tilde{A}_w}{dt} = -i\omega_w^2 S_w(\omega_0, k_0) \left| \frac{\partial D_w(\omega_0, k_0)}{\partial \omega} \right|^{-1} \tilde{A}_b.$$

$$\frac{d\tilde{A}_w}{dt} = -i\omega_w^2 S_w(\omega_0, k_0) \left| \frac{\partial D_w(\omega_0, k_0)}{\partial \omega} \right|^{-1} \tilde{A}_b.$$

$$\tilde{A}_w = \frac{1}{2} (A_{w0} - i(\tilde{\Omega} S_b^{-1} \Omega_0^{-1}) A_{b0}) \exp(\Omega_0 t) + \frac{1}{2} (A_{w0} + i(\tilde{\Omega} S_b^{-1} \Omega_0^{-1}) A_{b0}) \exp(-\Omega_0 t) \quad \Omega_0 = \sqrt{(1/2) \tilde{\Omega} \omega_{Lb}}$$

$$\frac{\partial E_x}{\partial t} + \frac{c}{\varepsilon_0} \frac{\partial B_y}{\partial z} = 0, \quad \frac{\partial B_y}{\partial t} + c \frac{\partial E_x}{\partial z} = 0 \quad \varepsilon_0(z) = \varepsilon + h \cos \chi z \quad \mu = h/\varepsilon \ll 1$$

$$\Psi(t, z) = \{E_x(t, z), B_y(t, z)\} = \{E_x(\omega, k, z), B_y(\omega, k, z)\} \exp(-i\omega t + ikz)$$

$$E_x(\omega, k, z) = \sum_{n=-\infty}^{\infty} E_n(\omega, k) \exp(in\chi z), \quad B_y(\omega, k, z) = \sum_{n=-\infty}^{\infty} B_n(\omega, k) \exp(in\chi z)$$

$$E_x(\omega, k, z) = E_{-1} \exp(-i\chi z) + E_0 + E_1 \exp(i\chi z),$$

$$B_x(\omega, k, z) = B_{-1} \exp(-i\chi z) + B_0 + B_1 \exp(i\chi z).$$

$$[\omega^2 - (k - \chi)^2 c_0^2] E_{-1} + \omega^2 (\mu/2) E_0 = 0,$$

$$[\omega^2 - k^2 c_0^2] E_0 + \omega^2 (\mu/2) E_{-1} + \omega^2 (\mu/2) E_1 = 0,$$

$$[\omega^2 - (k + \chi)^2 c_0^2] E_1 + \omega^2 (\mu/2) E_0 = 0,$$

$$D_{-1} D_0 D_1 - \omega^4 \left(\frac{\mu}{2}\right)^2 (D_{-1} + D_1) = 0$$

$$D_n = \omega^2 - (k + n\chi)^2 c_0^2$$

$$\begin{matrix} D_0(\omega, k) \approx 0 \\ D_{\mathbb{N}1}(\omega, k) \neq 0 \end{matrix} \longrightarrow \omega_{1,2} = \pm k c_0 \left[1 + \frac{1}{4} \left(\frac{\mu}{2}\right)^2 \left(1 - \frac{\chi^2}{4k^2}\right)^{-1} \right]$$

$$\Psi(t, z) = \begin{Bmatrix} E_x(t, z) \\ B_y(t, z) \end{Bmatrix} = \sum_{m=1}^2 A_m \left\{ \begin{array}{l} \left[1 - \frac{\mu}{2} \frac{k^2}{\chi(2k - \chi)} \exp(-i\chi z) + \frac{\mu}{2} \frac{k^2}{\chi(2k + \chi)} \exp(i\chi z) \right] \\ \frac{kc}{\omega_m} \left[1 - \frac{\mu}{2} \frac{k(k - \chi)}{\chi(2k - \chi)} \exp(-i\chi z) + \frac{\mu}{2} \frac{k(k + \chi)}{\chi(2k + \chi)} \exp(i\chi z) \right] \end{array} \right\} \exp(-i\omega_m t + ikz).$$

$$D_0(\omega, k) \equiv \omega^2 - k^2 c_0^2 = 0, \quad \omega_0 = \chi c_0 / 2, \quad k_0 = \chi / 2$$

$$D_{-1}(\omega, k) \equiv \omega^2 - (k - \chi)^2 c_0^2 = 0.$$

$$E_0(t, z) = \tilde{A}_0(t) \exp\left(i \frac{\chi}{2} (z - c_0 t)\right),$$

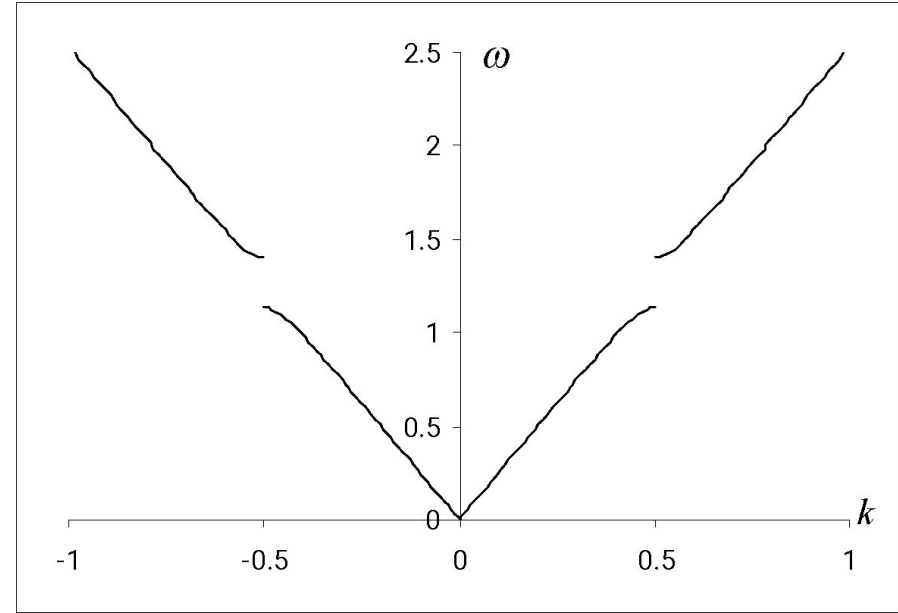
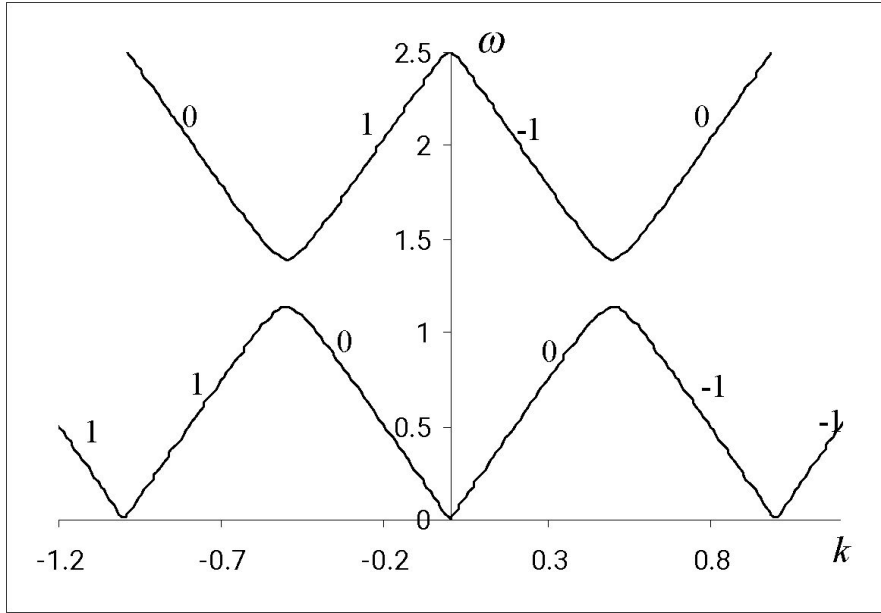
$$E_{-1}(t, z) = \tilde{A}_{-1}(t) \exp\left(-i \frac{\chi}{2} (z + c_0 t)\right),$$

$$\tilde{A}_0(t) = B_0 \sin(\Omega_0 t + \varphi_0), \quad \tilde{A}_{-1}(t) = B_0 \cos(\Omega_0 t + \varphi_0).$$

$$D_{-1} D_0 - \omega^4 (\mu/2)^2 = 0$$



$$\Omega^2 = \frac{1}{4} \omega_0^2 (\mu/2)^2 \equiv \Omega_0^2 \rightarrow \Omega_{1,2} = \pm \Omega_0$$



$$D(\hat{\omega}, \hat{k})A(t, z) + \mu\eta(z)Q(\hat{\omega})A(t, z) = 0$$

$$\eta(z) = \sum_{m=-\infty}^{\infty} \theta_m \exp(im\chi z)$$

$$A(t, z) = \exp(-i\omega t + ikz) \sum_{n=-\infty}^{\infty} A_n \exp(in\chi z) \quad D(\omega, k + n\chi)A_n + \mu Q(\omega) \sum_{m=-\infty}^{\infty} \theta_m A_{n-m} = 0, \quad n = 0, \pm 1, \dots$$

.....						
.....	D_{-2}	$\mu'\theta_{-1}$	$\mu'\theta_{-2}$	$\mu'\theta_{-3}$	$\mu'\theta_{-4}$
.....	$\mu'\theta_1$	D_{-1}	$\mu'\theta_{-1}$	$\mu'\theta_{-2}$	$\mu'\theta_{-3}$
.....	$\mu'\theta_2$	$\mu'\theta_1$	D_0	$\mu'\theta_{-1}$	$\mu'\theta_{-2}$
.....	$\mu'\theta_3$	$\mu'\theta_2$	$\mu'\theta_1$	D_1	$\mu'\theta_{-1}$
.....	$\mu'\theta_4$	$\mu'\theta_3$	$\mu\theta_2$	$\mu'\theta_1$	D_2
.....						

$$= 0$$

$$D_n = D(\omega, k + n\chi)$$

$$D_0 = D(\omega, k)$$

$$\mu' = \mu Q(\omega)$$

$$D_{-2}D_{-1}D_0D_1D_2 - \mu^2 Q^2(\omega) \{ |\theta_1|^2 [D_0(D_1D_2 + D_{-1}D_{-2}) + D_{-2}D_2(D_{-1} + D_1)] +$$

$$+ |\theta_2|^2 [D_{-2}D_0D_2 + D_{-1}D_1(D_{-2} + D_2)] + |\theta_3|^2 D_0(D_1D_{-2} + D_{-1}D_2) + |\theta_4|^2 D_{-1}D_0D_1 \} = 0$$

$$D_0 = D_{\pm 1} = 0 \quad D_0D_{\pm 1} - \mu^2 Q^2(\omega_0) |\theta_1|^2 = 0 \quad \Delta_1 = 2\Omega = 2\mu \theta_1 Q(\omega_0) \left(\frac{\partial D_0(\omega_0, k_0)}{\partial \omega_0} \right)^{-1}$$

$$D_{-2}D_{-1}D_0D_1D_2 - \mu^2 Q^2(\omega) |\theta_1|^2 [D_0(D_{-1}D_{-2} + D_1D_2) + D_{-2}D_2(D_{-1} + D_1)] +$$

$$+ \mu^4 Q^4(\omega) |\theta_1|^4 (D_0 + D_{-2} + D_2) = 0$$

$$\Delta_2 = \frac{1}{4\sqrt{3}} \mu^2 \omega_0$$

$$\Delta_2 = 2\mu^2 |\theta_2|^2 Q^2(\omega_0) \left(\sqrt{|D_{-1}(\omega_0, k_0)D_1(\omega_0, k_0)|} \left| \frac{\partial D_0(\omega_0, k_0)}{\partial \omega_0} \right| \right)^{-1}$$

$$k = k_0 + \kappa, \quad |\kappa| \ll k_0 = \chi/2 \quad \kappa = \pm i\delta_B, \quad \delta_B = \mu \left| \theta_1 Q(\omega_0) \left(\frac{\partial D(\omega_0, k_0)}{\partial k_0} \right)^{-1} \right|$$

$$A(t, z) = A_+(z) \exp(-i\omega t + ikz) + A_-(z) \exp(-i\omega t - ikz)$$

$$\frac{dA_+}{dz} \exp(ikz) - \frac{dA_-}{dz} \exp(-ikz) + i\mu Q(\omega) \left(\frac{\partial D}{\partial k} \right)^{-1} [\theta_1 \exp(i\chi z) + \theta_1^* \exp(-i\chi z)] [A_+ \exp(ikz) + A_- \exp(-ikz)] = 0.$$

$$\frac{dA_+}{dz} + i\mu Q(\omega_0) \left(\frac{\partial D(\omega_0, k_0)}{\partial k_0} \right)^{-1} \theta_1 A_- = 0,$$

$$A_+(z) = B \exp(\delta_B z) + C \exp(-\delta_B z),$$

$$\frac{dA_-}{dz} - i\mu Q(\omega_0) \left(\frac{\partial D(\omega_0, k_0)}{\partial k_0} \right)^{-1} \theta_1^* A_+ = 0.$$

$$A_-(z) = \zeta [B \exp(\delta_B z) - C \exp(-\delta_B z)],$$

$$A(t, z) = C \exp(-\delta_B z) \exp(-i\omega_0 t + ik_0 z) - \zeta C \exp(-\delta_B z) \exp(-i\omega_0 t - ik_0 z)$$

